Quantifying the Mesoscopic Nature of Einstein-Podolsky-Rosen Nonlocality

M. D. Reid[1,2,*] and Q. Y. He[3,4,†]

[1]*Centre for Quantum and Optical Science, Swinburne University of Technology, Melbourne, Victoria 3122, Australia*
[2]*Institute of Theoretical Atomic, Molecular and Optical Physics (ITAMP), Harvard University, Cambridge, Massachusetts 02138, USA*
[3]*State Key Laboratory of Mesoscopic Physics, School of Physics, Peking University, Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*
[4]*Collaborative Innovation Center of Extreme Optics, Shanxi University, Taiyuan 030006, China*

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Evidence for Bell's nonlocality is so far mainly restricted to microscopic systems, where the elements of reality that are negated predetermine results of measurements to within one spin unit. Any observed nonlocal effect (or lack of classical predetermination) is then limited to no more than the difference of a single photon or electron being detected or not (at a given detector). In this paper, we analyze experiments that report the Einstein-Podolsky-Rosen (EPR) steering form of nonlocality for mesoscopic photonic or Bose-Einstein condensate systems. Using an EPR steering parameter, we show how the EPR nonlocalities involved can be quantified for four-mode states, to give evidence of EPR-nonlocal effects corresponding to a two-mode number difference of 10^5 photons, or of several tens of atoms (at a given site). Applying to experiments, we also show how the variance criterion of Duan, Giedke, Cirac and Zoller for EPR entanglement can be used to determine a lower bound on the number of particles in a pure two-mode EPR-entangled or steerable state.

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Introduction.—In 1935, Einstein, Podolsky, and Rosen (EPR) presented a seemingly compelling argument that quantum mechanics was incomplete [1]. In their gedanken experiment, properties of a system B can be predicted ultraprecisely, by the measurements of a distant observer, popularly called Alice. EPR assumed no "spooky action-at-a-distance" to argue that Alice's measurement is noninvasive and, therefore, that Alice's prediction represents a predetermined property (an "element of reality") of system B. Further, they showed that the set of all such predetermined properties could not be consistent with any local quantum state description for B; thus, they concluded that quantum mechanics was an incomplete theory. The assumptions made in EPR's argument are collectively known as local realism (LR). Bell's theorem negated these premises by showing that LR could be falsified [2].

Understanding whether and how local realism fails macroscopically remains an open question in physics. Loophole-free experiments confirming Bell's theorem have so far been limited to microscopic systems; e.g., system B is a single photon or electron [3,4]. In these cases, the predetermined properties that EPR called elements of reality give predictions to within a single spin unit. Therefore, the failure of LR that is inferred from the experiments is a microscopic effect only, in the sense that this pertains only to predictions specified to an accuracy of one spin unit for a microscopic particle. Similar accuracies are required in almost all of the experiments predicted to violate LR for multiparticle systems [5,6].

By contrast, EPR's experiment (called an "EPR-steering" experiment [7–11]) has been investigated experimentally for mesoscopic optical fields [12–24], atomic ensembles [25–32], and, recently, for mesoscopic mechanical oscillators [33–41]. In many of these experiments, not only are the systems sizable, but the outcomes are over a larger range, corresponding to several or many spin units. Thus, it is possible to test for a mesoscopic EPR-nonlocal effect, where the predetermined elements of reality that are falsified give predictions with an indeterminacy of several spin units. One may then ask how much "spooky action-at-distance" is occurring in terms of spin units. How to perform the quantification is not obvious, however. It is not simply the size of the entangled system nor the range of outcomes. Previous measures inform us of how many atoms are mutually entangled [42,43], or what fraction of particle pairs behave locally versus nonlocally [44], but these need not imply large differences in the actual *outcomes* of observables due to nonlocal effects.

The situation is clear if the physical quantities measured by observers Bob and Alice (at different locations) have two mesoscopically distinct outcomes $+$ and $-$, e.g., N particles in an *up* position versus N particles in a *down* position ($N \gg 1$) [45,46]. One may then extend EPR's premises to define δ-scopic local realism (δLR), which asserts the following [47,48]: (1) any measurement by Alice cannot instantly induce a change of magnitude δ from $+$ to $-$ (or vice versa) to the outcome of the measurement at Bob's location, and (2) if the outcome $+$ or $-$ for Bob's system can be predicted with certainty by Alice, then Bob's system is always predetermined to be in a state that gives *either* the result + or −. If the measurement at B is the number of particles *up* minus the number of particles *down*, then $\delta = 2N$. The failure of such premises then implies a "spooky action" nonlocal effect of size δ. However, examples of EPR systems with just two outcomes $\pm$ separated by a large δ are limited.

In this paper, we present a practical approach to test EPR premises based on δLR. We consider a version of EPR's argument based on the premise of δLR, where the separation of outcomes + and − is quantified by δ but where there is a continuous range of outcomes. Our analysis leads to a criterion that is sufficient to demonstrate EPR δ-scopic nonlocality, thus revealing an inconsistency between the completeness of quantum mechanics and the validity of δLR. The size of δ quantifies, in part (1) of the δLR premise above, the upper bound on the amount of change that can occur to Bob's system due to Alice's measurements. Failure of δLR where δ is large therefore implies a large nonlocal effect. We analyze EPR experiments that have a mesoscopic, continuous range of outcomes for Alice and Bob's measurements to present preliminary evidence for quantifiable mesoscopic EPR nonlocalities.

Quantifying the EPR paradox.—The EPR argument can be generalized to pairs of measurements $\{X_A, P_A\}$ and $\{X_B, P_B\}$ on two spatially separated systems A and B. We consider X_B, P_B to be scaled, noncommuting observables satisfying $[X_B, P_B] = 2$, so that the Heisenberg uncertainty relation is $\Delta X_B \Delta P_B \geq 1$. To demonstrate the paradox, one measures the variances $V_B(X_B|X_A)$ and $V_B(P_B|P_A)$ of the respective conditional distributions $P(X_B|X_A)$ and $P(P_B|P_A)$ [9]. Here, $P(X_B|X_A)$ is the probability for result X_B, given a measurement X_A. The average conditional variance $(\Delta_{\mathrm{inf}} X_B)^2 = \sum_{X_A} P(X_A) V_B(X_B|X_A)$ determines the accuracy of inference of the results for X_B, based on the measurements at A. The $\Delta_{\mathrm{inf}} P_B$ is defined similarly. Using EPR's logic, these inference variances define the average indeterminacy of the two respective elements of reality, for X_B and P_B. If

$$\varepsilon \equiv \Delta_{\mathrm{inf}} X_B \Delta_{\mathrm{inf}} P_B < 1 \tag{1}$$

then an EPR paradox arises since the simultaneous predetermination for X_B and P_B is more accurate than allowed by the uncertainty principle [9,13]. The condition (1) is a condition for "EPR steering of system B" [8,10].

We now construct a quantified version of the EPR argument by relaxing EPR's premises. The assumptions of δ_X-scopic local realism (δ_XLR) assert two premises. (1) A measurement made at A *might* disturb the system B, so the outcome for a simultaneous measurement X on B can be altered. However, any change to the outcome cannot be greater (in magnitude) than δ_X. (2) If the value for a physical quantity X is predictable, without disturbing the (predictions for X of the) system by more than δ_X, then the value of that physical quantity is a predetermined property of the system (the element of reality for X), with the predetermined value being given to within $\pm\delta_X$ of the predicted value. We refer to δ_X as the degree of "nonlocal indeterminacy," with respect to the EPR observable X.

The assumption of δLR changes the condition for an EPR paradox, making it more difficult to demonstrate the paradox, because some degree of "spooky action-at-a-distance" has been permitted in the assumptions. Applying δ_XLR, the indeterminacy in the predictions for X_B associated with the element of reality has increased, but by a limited amount only. We show in the Supplemental Material [49] that the maximum value of this indeterminacy becomes [49]

$$(\Delta_{\mathrm{inf},\delta_X} X_B)^2 = (\Delta_{\mathrm{inf}} X_B)^2 + \delta_X^2 + 2\delta_X \sum_{X_A, X_B} P(X_A, X_B) |X_B - \langle X_B | X_A \rangle|, \tag{2}$$

where $P(X_A, X_B)$ is the joint probability. Defining $\Delta_{\mathrm{inf},\delta_P} P_B$ in a similar manner, the experimental realization of

$$\varepsilon_\delta \equiv \Delta_{\mathrm{inf},\delta_X} X_B \Delta_{\mathrm{inf},\delta_P} P_B < 1 \tag{3}$$

will therefore imply an inconsistency between the premise of δLR and the completeness of quantum mechanics. Here, $\delta = (\delta_X, \delta_P)$. The calculation of ε_δ is straightforward once the distributions $P(X_A, X_B)$ and $P(P_A, P_B)$ are known. When $\delta = 0$, Eq. (3) reduces to the standard EPR condition (1). The inequality is progressively more difficult to satisfy as δ increases.

Gaussian δ-scopic EPR nonlocality.—We consider EPR experiments based on field modes at locations A, B. $X_{A/B}$, $P_{A/B}$ are defined according to $a = (X_A + iP_A)/2$ and $b = (X_B + iP_B)/2$, where a, b are the annihilation operators of each mode. The δ-scopic EPR inequality reduces to Eq. (3). A widely used source of EPR-correlated fields is the parametric amplifier, the ideal output of which is the two-mode squeezed state [9,13]. Here, the conditionals $P(X_B|X_A)$ and $P(P_B|P_A)$ are Gaussian. Moreover, a Gaussian profile is maintained in nonideal situations where losses and thermal noise are present [13,54,55].

Assuming Gaussianity, the prediction of ε_δ given measured values of $\Delta_{\mathrm{inf}} X_B$ and $\Delta_{\mathrm{inf}} P_B$ is straightforward. Using Eq. (2) and that, for a Gaussian distribution $\langle |X_B - \mu_X| \rangle = \Delta_{\mathrm{inf}} X_B \sqrt{2/\pi}$ [where μ_X is the mean of $P(X_B|X_A)$], we find $\varepsilon_\delta = \sigma^2 + \delta^2 + 2\delta\sigma\sqrt{2/\pi}$ [49]. For the sake of simplicity, we have taken $\sigma = \Delta_{\mathrm{inf}} X_B = \Delta_{\mathrm{inf}} P_B$ and $\delta = \delta_X = \delta_P$. We see that $\varepsilon < [-\delta\sqrt{2/\pi} + \sqrt{2\delta^2/\pi - (\delta^2 - 1)}]^2$ will be sufficient to imply the δ-scopic EPR nonlocality.

Extensive data have been reported for continuous variable EPR experiments [12–27] (see Fig. 1). Gaussian distributions are predicted in almost all cases plotted

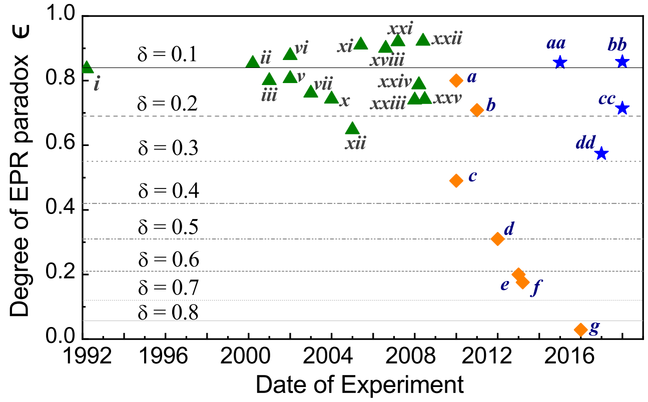

FIG. 1. The δ-scopic EPR nonlocality is realized ($\epsilon_\delta < 1$) when ϵ is below the line shown, for the given δ. Data $i - xxv$ are for the experiments referenced in Fig. 9 of Ref. [13], while data a [15], b [16], c [22], d [23], e [19], f [20], g [24], aa [25], bb [26], cc [27], dd [32] are later experiments. All results determine EPR correlations between spatially-separated optical fields, except those given by blue stars, which determine correlations between mesoscopic groups of cold atoms (aa, bb, cc) or between hybrid systems (dd). The cold-atom groups have negligible (aa) or small separations $\sim 10\ \mu$m (bb, cc).

(including the data indicated by g) as has been verified experimentally [13,55]. For rigorous testing, a full construction of the distributions with spacelike separated measurement events is required [2,4]. With this proviso, we note that the recently achieved values of the EPR parameter $\varepsilon \sim 0.176$ [20] will imply a δ-scopic EPR nonlocality, with $\delta \sim 0.633$.

To determine the significance of the value of δ, one needs to resort to the details of the individual experiments. The nonlocal indeterminacy δ is given relative to the quantum noise level, which for the optical experiments is usually considered microscopic. On the other hand, entanglement has now been detected between two mechanical oscillators [33,34,38] and between an oscillator and a field [35]. Entanglement, however, does not imply the EPR steering condition (1). It has been proposed to detect the EPR condition (1) for these cases [33,36,40,41], where X_B, P_B refer to the quadratures of the phonon modes of the oscillator. Equation (3) enables a quantification of the EPR nonlocality that would be observed in such an experiment. Here, δ is quantifiable at the Planck scale [56], and it corresponds to a nonlocal indeterminacy with respect to mechanical motion.

EPR nonlocality using Schwinger spins.—For some experiments, the quantum noise level and hence δ may correspond to a large number of photons. This case is understood by considering the Heisenberg relation $\Delta J^Z \Delta J^Y \geq |\langle J^X \rangle|/2$ for spin systems (here $(\Delta x)^2$ is the variance of x), where measurements are made of the spin components $J^{X,Y,Z}$. For high spins, $|\langle J^X \rangle|$ can be large.

Indeed, EPR states exist for which $|\langle J^X \rangle|$ is a scalable large number. In these cases, the EPR observables are two-mode Schwinger spins, defined as $J_A^X = (a_+^\dagger a_- + a_+ a_-^\dagger)/2$, $J_A^Y = (a_+^\dagger a_- - a_+ a_-^\dagger)/2i$, $J_A^Z = (a_+^\dagger a_+ - a_-^\dagger a_-)/2$, and $J_B^X = (b_+^\dagger b_- + b_+ b_-^\dagger)/2$, $J_B^Y = (b_+^\dagger b_- - b_+ b_-^\dagger)/2i$, $J_B^Z = (b_+^\dagger b_+ - b_-^\dagger b_-)/2$, where $a_\pm$, $b_\pm$ are annihilation operators for four modes [47]. The four modes are created from spatially separated modes a, b prepared in an EPR state $|\psi\rangle$. Each mode a, b interferes (via a beam splitter) with an intense "local oscillator" field (denoted by annihilation mode operators b_{LO}, a_{LO}). This creates a macroscopic photonic state $|\psi\rangle_M$ involving four fields $a_\pm = (a \pm a_{\mathrm{LO}})/\sqrt{2}$, $b_\pm = (b \pm b_{\mathrm{LO}})/\sqrt{2}$ at sites A and B, respectively. The fields at each site pass through second polarizing beam splitters set at respective angles θ_A and θ_B. The number of particles in each arm is detected as a large number, and the difference gives a measure of J^Z, J^Y, or J^Z depending on the choice of θ_A, θ_B. Based on the Heisenberg uncertainty relation, the EPR criterion is

$$\Delta_{\mathrm{inf},\delta_J}(J_B^Z)\Delta_{\mathrm{inf},\delta_J}(J_B^Y) < |\langle J_B^X \rangle|/2, \tag{4}$$

which normalizes to Eq. (3) on defining $X_B/P_B = J_B^{Z/Y}/\sqrt{|\langle J_B^X \rangle|/2}$ and $\delta = \delta_J/\sqrt{|\langle J_B^X \rangle|/2}$. Here, $|\langle J_B^X \rangle| = |\langle b_{\mathrm{LO}}^\dagger b_{\mathrm{LO}} - b^\dagger b \rangle/2|$, which becomes $\langle b_{\mathrm{LO}}^\dagger b_{\mathrm{LO}} \rangle/2$ since $\langle b^\dagger b \rangle / \langle b_{\mathrm{LO}}^\dagger b_{\mathrm{LO}} \rangle$ is small. The intensity of the local oscillator is macroscopic, and $\delta_J \sim \delta\sqrt{\langle b_{\mathrm{LO}}^\dagger b_{\mathrm{LO}} \rangle/4}$ (the nonlocal indeterminacy in the values of J_B^X, J_B^Y) can therefore also be large.

EPR nonlocality for Schwinger spins has been realized in the experiments of Bowen *et al.* [14], where $a_\pm$ ($b_\pm$) correspond to two orthogonal horizontally ("H" or "x") and vertically ("V" or "y") polarized field modes at A (B). From the description of their experiment [14,49], $|\langle J_B^X \rangle| \sim 10^{11}$ photons, implying a δ_J of order 10^5 photons. The relative value $\delta_J / |\langle J_B^X \rangle|$ is, however, small.

Analogy to Schrödinger cat.—The Schwinger-spin experiment provides a simple parallel to Schrödinger's cat gedanken experiment [45,47]. In the original cat paradox, a macroscopic superposition is created by the process of measurement, which couples the microscopic system (prepared in a superposition state) to a measurement apparatus. In the experiment, the microscopic EPR state $|\psi\rangle$ is indeed coupled to a macroscopic system (the local oscillator fields) at each site, and a four-mode amplified state $|\psi\rangle_M$ is produced that enables a macroscopic readout of $X_{A/B}$ and $P_{A/B}$ of the original fields.

The many-particle state $|\psi\rangle_M$ is created prior to the measurements $J_A^{\theta_A}$, $J_B^{\theta_B}$ (θ_A, $\theta_B = X, Y$ or Z), and it is this feature that enables the demonstration of mesoscopic nonlocality. The $|\psi\rangle_M$ is a superposition of states with definite outcomes for $J_B^{\theta_B}$, where those outcomes are given by $J_B^{\theta_B} = EX_B/2$ or $EP_B/2$ (here, $E^2 = \langle b_{\mathrm{LO}}^\dagger b_{\mathrm{LO}} \rangle$), depending on θ_B. The superposition $|\psi\rangle_M$ comprises many states that have a large range of continuous outcomes for $J_B^{\theta_B}$, rather than just two distinct states as in Schrödinger's case. In the Supplemental Material, we prove that the observation of the δ_J-scopic EPR nonlocality can only arise if $|\psi\rangle_M$ comprises at least two states that differ in outcome for $J_B^{\theta_B}$ by at least δ_J [49]. Such states (when δ_J is large) give a nonzero mesoscopic quantum coherence and signify a generalized Schrödinger-cat paradox (refer Refs. [57–60]). These states, for which EPR nonlocality is also demonstrated, are well nested *within* the overall superposition state, however. For an experimental value $\epsilon \sim 0.42$ (where $\delta \sim 0.4$), their separation is typically $\delta_J = 0.4E/2$, whereas the state $|\psi\rangle_M$ predicts a Gaussian distribution for $J_B^{\theta_B}$, with $\Delta J_B^{\theta_B} \sim 1.3E/2$.

The δ-scopic EPR nonlocality manifests without the significant decoherence that normally prevents formation of Schrödinger-cat states because the separation δ_J between states of the superposition is not amplified relative to the quantum noise level. The EPR steering parameter ϵ_δ is unchanged, consistent with the requirement that entanglement cannot be created by local entangling transformations, such as produced by beam splitters [61].

EPR nonlocality between distinct atom groups.—The experiments of Refs. [28–31] investigate EPR entanglement between two spatially separated macroscopic atomic ensembles, A and B. In their experiment, $J_{A/B}^{X,Y,Z}$ are the collective spins of each ensemble, defined relative to two atomic levels. The observation of the condition $D \equiv \{[\Delta(X_A - X_B)]^2 + [\Delta(P_A + P_B)]^2\}/4 < 1$ implies entanglement between subsystems A and B [62]. For spins, this entanglement condition becomes [63]

$$D \equiv \frac{[\Delta(J_A^Z + J_B^Z)]^2 + [\Delta(J_A^Y + J_B^Y)]^2}{|\langle J_A^X\rangle| + |\langle J_B^X\rangle|} < 1. \tag{5}$$

Measurements give $D \sim 0.8$ for thermal atomic ensembles [28,29]. The value of $D < 0.5$ would imply an EPR steering nonlocality (4) [13,64,65]. For a rigorous demonstration of EPR nonlocality, it is, however, necessary to measure the EPR observables independently and locally so that information is gained simultaneously about each of $J_A^{\theta_A}$ and $J_B^{\theta_B}$.

EPR steering correlations meeting the condition (1) have, however, recently been observed for matter-wave fields created with Bose-Einstein condensates (BEC) [25–27, 66,67]. In the experiments of Refs. [25,26,67], twin atom-beam states are generated by a parametric interaction [67]. Atoms are created in pairs, one into each group A and B that correspond to different spins. The atom field quadratures $X_{A/B}$, $P_{A/B}$ are measured by an atomic homodyne method, where the local oscillators are a different, larger group consisting of $E = \langle a_{\mathrm{LO}}^\dagger a_{\mathrm{LO}}\rangle \sim \langle b_{\mathrm{LO}}^\dagger b_{\mathrm{LO}}\rangle > 10^2$ atoms [67]. Experiments of Piese *et al.* observe EPR correlations (1) between mesoscopic atomic groups A and B with $\epsilon \sim 0.85$ (with no spatial separation) [25]. Recently, Kunkel *et al.* observed an EPR steering with $\epsilon \sim 0.71$ between atom groups spatially separated by $\sim 10\ \mu$m [27]. Assuming results are unchanged if the experiments were reconfigured along the lines of the Schwinger-spin experiments, and that the distributions are approximately Gaussian, these results suggest nonlocalities with $\delta_J > E\delta > 20$ atoms.

Using a different method of generation, Fadel *et al.* observed EPR steering correlations ($\epsilon \sim 0.74$) between the Schwinger spins J_A^Z, J_B^Z (and J_A^Y, J_B^Y) of two atomic groups separated by 4 μm [26]. Here, group A (B) consists of two BEC components, $a_\pm$ ($b_\pm$). Their measurement of spins (J^X or J^Y) is achieved with a variable pulse rotation θ, in analogy to the polarizing beam splitter of Bowen *et al.* (although without independent selection of the two measurement angles). The BEC experiments thus reveal quantifiable nonlocal indeterminacies in the *atom* number differences $J_B^{\theta_B}$ at the given site. The nonlocality being tested here is whether an action at the site A can create a change in the number of atoms between the groups b_+ and b_-, at site B.

Quantification of number of bosons in the two-mode entangled state.—The large values of δ_J arise for four-mode states. However, the two-mode EPR state $|\psi\rangle$ can itself be constrained to have a certain degree of "largeness." For any entangled pure state $|\psi\rangle$, the mean total number of bosons is $\bar{n} = \langle\psi|a^\dagger a + b^\dagger b|\psi\rangle$. The value of D places a lower bound on $\bar{n}$. Using the identity $|\langle ab\rangle| \leq \sqrt{(\langle a^\dagger a\rangle + 1)\langle b^\dagger b\rangle}$ [68], we find $D \geq D_{\bar{n}}^{(l)}$ [49], where

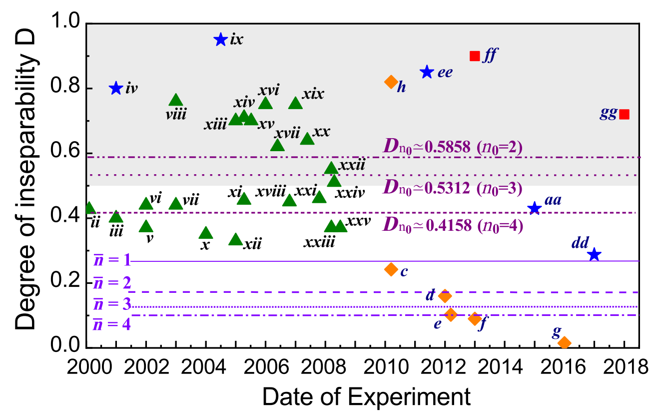

FIG. 2. The entanglement parameter D is plotted versus the date for a sample of experiments. Data ($ii - xxv$) are the values reported for atomic (stars) and optical (triangles) experiments, as listed in Fig. 9 of Ref. [13]. Data c [22], d [23], e [19], f [20], g [24], h [18] are for optical experiments (diamonds); data aa [25], dd [32], ee [29] are for atomic or hybrid experiments (stars); and ff [35], gg [38] are for mechanical-oscillator experiments (squares). Note that $D < D_{\bar{n}}^{(l)}$ implies the EPR state $|\psi\rangle$ has a mean number of bosons greater than $\bar{n}$. The $D_{\bar{n}}^{(l)}$ are plotted for $\bar{n} = 1, 2, 3, 4$. Here, $D < D_{n_0}$ requires states of more than n_0 bosons. We plot D_{n_0} for $n_0 = 2, 3, 4$. For $n_0 = 10$, $D_{n_0} \simeq 0.2228$.

$D_{\bar{n}}^{(l)} = 1 + \bar{n} - \bar{n}\sqrt{1 + 2/\bar{n}}$ decreases with $\bar{n}$ and is achieved for the two-mode squeezed state, for which $D = (1 - x)/(1 + x)$. In an experiment, the two-mode system is generally not a pure state. Then, the measured $\langle a^\dagger a + b^\dagger b\rangle$ does not reflect the mean number of bosons in an entangled state because there may be components of the mixture that are not entangled. However, the observation of $D < D_{\bar{n}}^{(l)}$ certifies that a pure entangled state $|\psi\rangle$ with $\langle a^\dagger a + b^\dagger b\rangle > \bar{n}$ must be a component of the mixed state [49]. Similarly, by expanding all pure states in the basis of number states $|i\rangle_a|j\rangle_b$, we prove in the Supplemental Material that the value D places a lower bound on the minimum number of bosons $n_0 = i + j$ contributing a nonzero term to the expansion. If the bosons are atoms, the state $|i\rangle_a|j\rangle_b$ has an entanglement depth of $n_0 = i + j$, meaning all n_0 atoms are mutually entangled [42,43,49].

Experimental values of D are plotted in Fig. 2. The values $D < 0.2228$ confirm two-mode optical EPR states $|\psi\rangle$ involving more than 10 photons ($n_0 > 10$). These states are different from states constructed from photon pairs, for which $n_0 \leq 2$. Where $D < 0.5$, the two modes of the state $|\psi\rangle$ are both EPR steerable [69]. Measurements by Piese *et al.* [25] observe $D < 0.43$, implying two-way EPR steerable states $|\psi\rangle$ with more than 3 atoms (if spatial separation could be achieved) [49].

Conclusion.—We have given evidence for a mesoscopic EPR nonlocality that "delocalizes" $\delta_J \sim 10^5$ photons between two polarization modes at a given site. This represents a tenth of the full range of outcomes (defined as that within 3 standard deviations of the mean) for the polarization photon-number difference at the site. Recent experiments with Bose-Einstein condensates show similar EPR nonlocalities involving four atomic modes. This motivates new experiments where it may be feasible to demonstrate an EPR nonlocality "delocalizing" $\delta_J \sim 10$ atoms across two highly occupied atomic modes at a given site. The criteria presented in this paper may also have a practical application. The curves of Fig. 1 can be used to detect a genuine EPR effect, even when a causal effect is present. If the maximum disturbance due to the causal effect can be quantified (to be δ_C say), then an EPR nonlocality can be deduced if $\epsilon_\delta < 1$ where $\delta > \delta_C$. An example of such a causal effect ("cross-talk") is given in Ref. [27].

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*mdreid@swin.edu.au
†qiongyihe@pku.edu.cn

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