How to Observe the Vacuum Decay in Low-Energy Heavy-Ion Collisions

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In slow collisions of two bare nuclei with the total charge larger than the critical value $Z_{\rm cr} \approx 173$, the initially neutral vacuum can spontaneously decay into the charged vacuum and two positrons. The detection of the spontaneous emission of positrons would be direct evidence of this fundamental phenomenon. However, the spontaneously produced particles are indistinguishable from the dynamical background in the positron spectra. We show that the vacuum decay can nevertheless be observed via impact-sensitive measurements of pair-production probabilities. The possibility of such an observation is demonstrated using numerical calculations of pair production in low-energy collisions of heavy nuclei.

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In relativistic quantum mechanics, the energy levels of hydrogenlike ions are described by the Dirac equation. For the pointlike nucleus this equation has a solution for the $1s$ state only if the nuclear charge Z is not greater than $Z_0 = 137$. Therefore the energy of this state is bounded from below by $E(Z_0) = 0$. However, for an extended nucleus, $E(Z)$ decreases further as Z increases and eventually crosses the value $-mc^2$ at the critical charge $Z_{\rm cr} \approx 173$ [1–7]. After the crossing, the level "dives" into the negative-energy Dirac continuum and becomes a resonance. If this supercritical resonance state was initially vacant then it can be occupied by two electrons from the negative-energy continuum with emission of two positrons [2–6]. This process can be interpreted as a spontaneous decay of the old neutral vacuum with the formation of a new "charged" vacuum.

Obviously, the required critical charge $Z_{\rm cr} \approx 173$ is much larger than the charge of the heaviest nuclei produced so far. However, two heavy colliding ions can form a quasimolecular system with a total charge $Z_{\rm tot} = Z_1 + Z_2$ large enough for the ground state to reach the negative-energy continuum. The observation of the electron-positron pairs spontaneously produced during the collision would be the direct evidence of vacuum decay. But in heavy-ion collisions the pair production is also induced by the ion dynamics. In order to detect the vacuum decay, one has to distinguish the spontaneous pair production from the dynamical one.

The experiments on low-energy heavy-ion collisions were intensively performed many years ago at GSI (Darmstadt, Germany). However, no sign of the spontaneous pair production or the diving phenomenon had been found [6,8]. There are several proposals for investigation of supercritical collisions at the upcoming accelerator facilities [9–11], which will allow us to perform the experiments on an entirely new level. In particular, experiments on low-energy collisions of heavy bare nuclei are anticipated at these facilities. But so far it is not clear whether or not there exists a theoretical possibility of the diving phenomenon detection.

To date, pair production in low-energy ion collisions has been investigated using various theoretical approaches [12–24]. As was found by the Frankfurt group, the pair-production probability as a function of the total nuclear charge and the impact parameter has no threshold effects at the border of the supercritical region, where the spontaneous mechanism should start to work [17]. It was also shown that the energy-differential spectra of the emitted positrons do not exhibit any feature which can be associated with the spontaneous pair production. The calculations were performed using so-called monopole approximation, in which only the spherical part of the two-center ion potential is taken into account. Recently the obtained results were confirmed with the monopole approximation [20] as well as beyond it [22–24].

The absence of any signature of the spontaneous mechanism in the calculated pair-production probabilities and in the positron spectra led the Frankfurt group to the conclusion that the vacuum decay could only be observed in collisions with nuclear sticking, in which the nuclei are bound to each other for some period of time by nuclear forces [25]. In such collisions, there should be a visible effect of vacuum decay due to the increase of diving time. In numerical calculations, the nuclear sticking can be taken into account via introducing the time delay at the point of the closest nuclear approach. It was demonstrated that the time delay leads to the enhancement of the pair-production probability in the supercritical case that can be explained only with the spontaneous mechanism (see, e.g., Ref. [18]). However, to date there is no robust evidence of the existence of sufficiently long nuclear sticking.

In this Letter, we show that the vacuum decay can be detected experimentally even without any nuclear sticking. This idea of detection is based on the different behavior of the pair-production probability as a function of nuclear velocities in the supercritical and subcritical cases.

Let us first consider hypothetical collisions with the modified velocity [20]:

$$\dot{R}_\alpha(t) = \alpha \dot{R}(t). \quad (1)$$

Here $R(t)$ is the internuclear distance which depends on time in accordance with the classical Rutherford scattering:

$$R = a(e \cosh \xi + 1),$$
$$t = \sqrt{\frac{M_r a^3}{Z_1 Z_2}}(e \sinh \xi + \xi), \quad (2)$$

where

$$a = \frac{Z_1 Z_2}{2E}, \qquad e = \left(1 + \frac{b^2}{a^2}\right)^{1/2}, \qquad \xi \in (-\infty, \infty), \quad (3)$$

E is the collision energy in the center-of-mass frame, M_r is the reduced mass of the nuclei, and b is the impact parameter. Varying the parameter α, we can change the nuclear velocity in numerical calculations. Figure 1 presents the pair-production probability P as a function of α obtained in Ref. [20]. The calculations were performed for subcritical Fr-Fr and supercritical U-U head-on collisions of bare nuclei at energy about the Coulomb barrier. As one can see from the figure, the behavior of the curves at small values of α is remarkably different. As α decreases, $P(\alpha)$ decreases in the subcritical case and drastically increases in the supercritical one, which indicates the existence of the spontaneous pair production mechanism. It should be emphasized that the subcritical curve *always* rises with increase of α, and the supercritical curve has quite a simple shape with one minimum.

Of course, it is impossible to modify the collisions according to Eq. (1) in real experiments. However, there exists a way to investigate the dependence of the pair-production probability on the ion velocity using the pure Rutherford kinematics defined by Eq. (1). Let us fix the

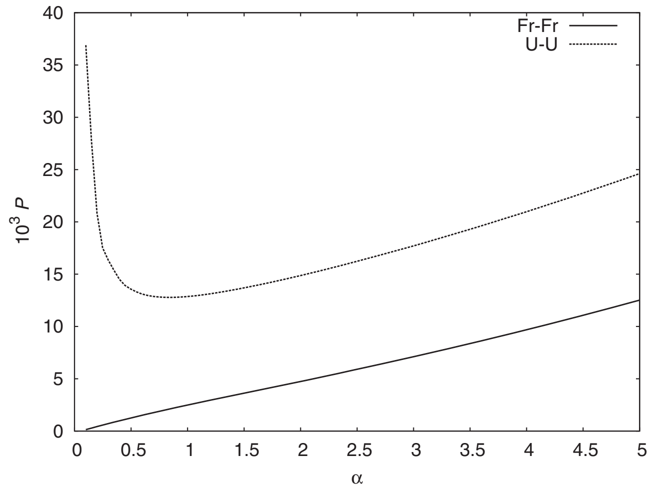

FIG. 1. Pair-production probability P in the hypothetical head-on collision of bare nuclei with the modified dependence of the internuclear distance on time $R_\alpha(t)$, defined by Eq. (1), as a function of α. The solid line shows the results for the Fr-Fr (subcritical) collision at $E = 674.5$ MeV; the dashed line corresponds to the U-U (supercritical) collision at $E = 740$ MeV. The results were obtained in Ref. [20].

nuclear charges Z_1, Z_2, and the distance of the closest nuclear approach

$$R_{\rm min} = a(e + 1). \quad (4)$$

One can vary the collision energy E with changing the impact parameter b according to the equation

$$b^2 = R_{\rm min}^2 - R_{\rm min} \frac{Z_1 Z_2}{E}, \quad (5)$$

with fixed $R_{\rm min}$. The collision energy is bounded from below by the value

$$E_0 = \frac{Z_1 Z_2}{R_{\rm min}}, \quad (6)$$

which corresponds to the head-on collision ($b = 0$). Using Eqs. (5) and (2), for the range of available energies, $E \geq E_0$, one can define the set of functions $R_E(t)$ which have the same minimum but different durations of the supercritical regime. For the case of U-U collision, these functions are displayed in Fig. 2. It can be seen that the supercritical time period decreases monotonically with increase of E. Employing the defined set of $R_E(t)$ it is possible to investigate the pair-production probability as a function of nuclear velocity keeping the range of internuclear distances fixed [$R_{\rm min} \leq R(t) < \infty$]. The major limitation of this approach is that the nuclei cannot be slowed down more than they are allowed to by the condition $E \geq E_0$.

In order to find the desired difference in pair production between subcritical and supercritical systems, we performed

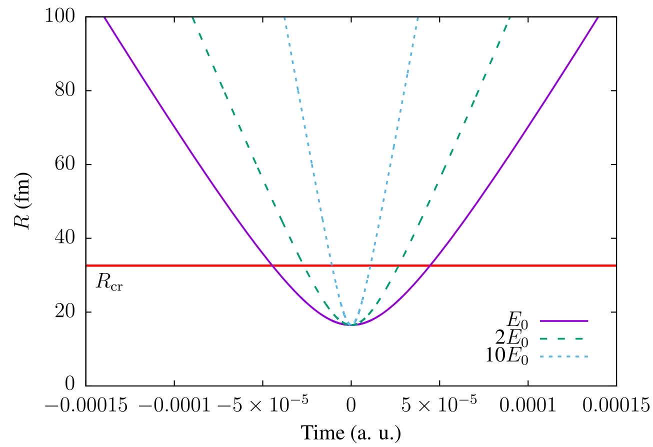

FIG. 2. The internuclear distance R for U-U collision as a function of time for different values of the collision energy with the fixed distance of the closest approach $R_{min} = 16.5$ fm, E_0 is the energy of the head-on collision. The red horizontal line corresponds to the critical distance $R_{cr} \approx 32.6$ fm and indicates the border between subcritical and supercritical regimes.

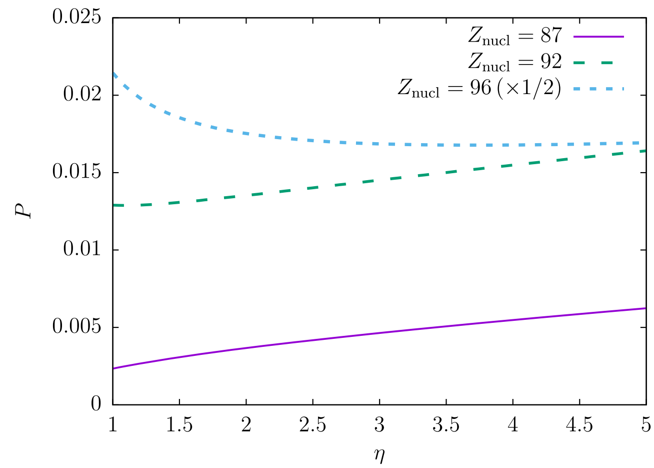

FIG. 3. The pair-production probability in the collision of two identical nuclei with $Z_1 = Z_2 = Z_{nucl}$ as a function of the ratio $\eta = E/E_0$, where E is the collision energy and E_0 is the energy of the head-on collision. The results for $Z_{nucl} = 96$ are multiplied by factor 0.5.

calculations using the method described in Ref. [20]. The method is based on the numerical solving of the time-dependent Dirac equation in the monopole approximation, according to which the two-center nuclear potential $V_{TC}(\boldsymbol{r}, t)$ is approximated by its spherically symmetric part

$$V_{mon}(r, t) = \frac{1}{4\pi} \int d\Omega V_{TC}(\boldsymbol{r}, t). \tag{7}$$

This approximation allows us to consider the radial Dirac equation instead of the two-center one. The corresponding electron wave function can be represented as

$$\psi_{\kappa m}(\boldsymbol{r}, t) = \begin{pmatrix} \frac{G_\kappa(r,t)}{r} \chi_{\kappa m}(\Omega) \\ i \frac{F_\kappa(r,t)}{r} \chi_{-\kappa m}(\Omega) \end{pmatrix}, \tag{8}$$

where $\chi_{\pm\kappa m}(\Omega)$ are the spherical spinors, $F_\kappa(r, t)$ and $G_\kappa(r, t)$ are the small and large radial components, respectively, m is the projection of the total angular momentum, and κ is the relativistic angular quantum number. We take into account the electronic states with $\kappa = \pm 1$, which are expected to give the major contribution to the pair production. Since there is no coupling between these two sets of states, the corresponding contributions can be calculated independently.

For simplicity, we consider the collision of two identical bare nuclei with $Z_1 = Z_2 = Z_{nucl}$. The closest nuclear approach is fixed to $R_{min} = 16.5$ fm. At such a distance the nuclei are about 1–2 fm away from touching each other. The calculations are performed for subcritical and supercritical collisions at different energies E for different values of Z_{nucl}. In Fig. 3, we present the obtained results for the pair-production probability P as a function of the $\eta = E/E_0$ ratio for Fr-Fr ($Z_{nucl} = 87$), U-U ($Z_{nucl} = 92$), and Cm-Cm ($Z_{nucl} = 96$) collisions. The Fr-Fr system is subcritical (it is the heaviest subcritical system), the U-U and Cm-Cm systems are supercritical. As can be seen from the figure, the Fr-Fr curve goes monotonically down with a decrease of E as in the case of the modified collisions (see Fig. 1). Such a behavior takes place for all collisions with $Z_{nucl} \leq 87$. In contrast, the pair-production probability in the supercritical Cm-Cm collision starts to increase as η approaches unity. In the U-U collision, which is also supercritical, the function $P(\eta)$ has only a slight increase as $\eta \to 1$ but exhibits clearly different behavior compared to the subcritical case.

To clarify this point, let us consider the U-U collision in more detail. It should be noted that, in our calculations, the total pair-production probability is the sum of two independent contributions: $P_{\kappa=-1}$ and $P_{\kappa=1}$, which correspond to creation of particles in the states with $\kappa = -1$ and $\kappa = 1$, respectively. Only the channel with $\kappa = -1$ is supercritical, because it includes the diving $1s$ state. In Fig. 4, we depict the calculated values of $P_{\kappa=-1}$, $P_{\kappa=1}$, and the total probability for the U-U collision. The curve corresponding to the supercritical ($\kappa = -1$) results has a rather pronounced minimum while the subcritical ($\kappa = 1$) one is monotonic. But in the sum $P_{\kappa=-1} + P_{\kappa=1}$ for $\eta \to 1$, an increase in $P_{\kappa=-1}$ and a decrease in $P_{\kappa=1}$ almost cancel each other out, which leads to a much less pronounced minimum.

It is clear that all the calculations can be easily extended to asymmetric collisions of bare nuclei. In Fig. 5, we present the corresponding results for the pair-production probability as a function of η for the U-Cm collision ($Z_1 + Z_2 = 188$). As one can see, there is a clear signal of the spontaneous pair production for this system as well.

So far we considered only collisions of bare nuclei. On the one hand, the calculations for collisions of bare nuclei

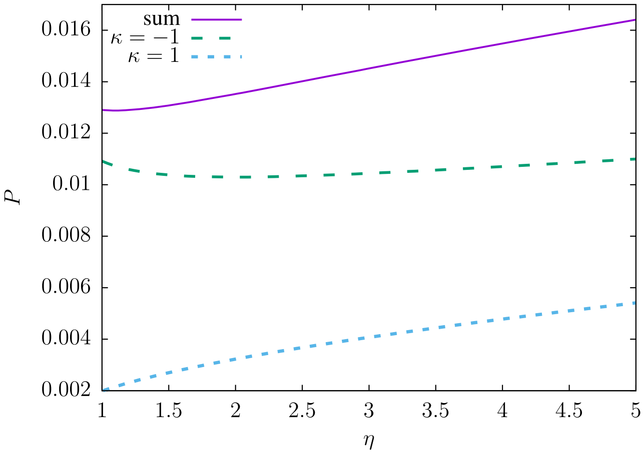

FIG. 4. The total pair-production probability in U-U collision, and contributions from channels with $\kappa = \pm 1$ as functions of the ratio $\eta = E/E_0$, where E is the collision energy and E_0 is the energy of the head-on collision.

are the simplest to demonstrate the principle possibility of vacuum decay detection in the proposed scenario and, on the other hand, the experiments with such systems would be most favorable for such detection. Proposals for experimental investigation of collisions of bare nuclei up to Cm-Cm system were considered, e.g., in Ref. [10]. However, we would like to note that the same scenario can be potentially used for collisions of bare nuclei with atoms having the filled K shell. An estimation of the bound quasimolecular level occupation probability in collisions of bare uranium nuclei on uranium and curium atoms with a filled K shell, based on the methods developed in Refs. [26–28], revealed that the filled K shell can only lead to a decrease of the pair-production probability roughly by a factor within the range 0.25–0.6. We also do not expect that a possible energy dependence of the corresponding suppression factor will qualitatively change the main conclusions concerning the observation of the effect of interest. We note that even if such a dependence is noticeable, it can be analyzed and isolated by means of more accurate calculations of the effects of the filled K shell. The refined treatment requires more elaborate many-electron two-center calculations of the pair-creation probabilities and is currently under way.

In Fig. 6, for symmetric collisions, we show the derivative $dP/d\eta$ taken at $\eta = 1$ as a function of Z_{nucl}. As one can see from the figure, the function changes its behavior after the transition to the supercritical domain. It starts to decrease and finally crosses the zero line that corresponds to the appearance of the minimum on the graph of $P(\eta)$. The derivative becomes negative at $Z_{\mathrm{nucl}} \approx 92$.

From comparing the subcritical and supercritical scenarios, we conclude that there is the qualitative difference in behavior of the pair-production probability in the subcritical and the supercritical cases. If the distance of the closest approach is fixed, the increase of this probability with a decrease of the collision energy can be observed only in the supercritical collisions. Moreover, even a pronounced decrease of $dP/d\eta$ at $\eta \approx 1$ as a function of Z_{nucl}, which takes place already at $Z_{\mathrm{nucl}} = 92$ (see Fig. 6), must be considered as a clear evidence of the vacuum decay at supercritical field.

Although the calculations in the present Letter are mainly restricted to the monopole approximation, our recent study [22–24] clearly showed that effects beyond the monopole approximation only slightly change the pair-creation probabilities in the region of small impact parameters, where the derivative presented in Fig. 6 is calculated. We can state that effects beyond the monopole approximation will not change the main results obtained in this Letter. We believe, however, that further studies of

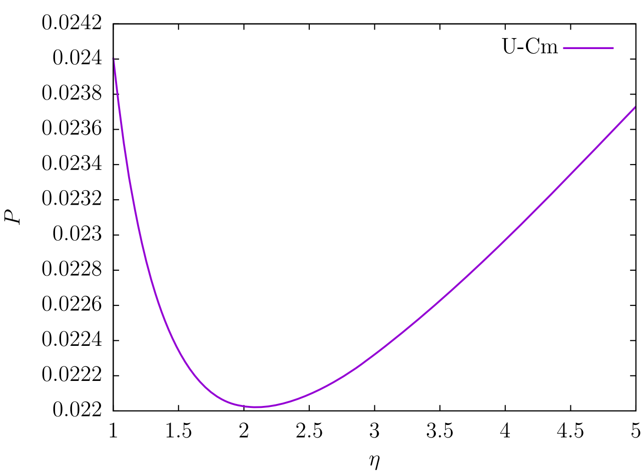

FIG. 5. The total pair-production probability in U-Cm collision as a function of the ratio $\eta = E/E_0$, where E is the collision energy and E_0 is the energy of the head-on collision.

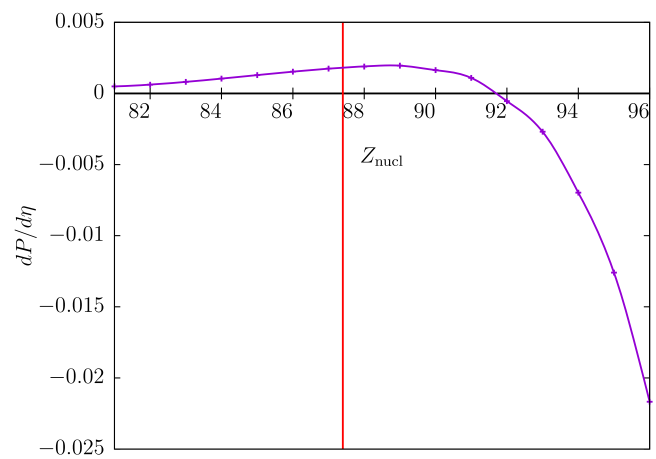

FIG. 6. The derivative $dP/d\eta$ taken at $\eta = 1$ as a function of Z_{nucl}, where $\eta = E/E_0$, E is the collision energy, E_0 is the energy of the head-on collision, P is the pair-production probability, and Z_{nucl} is the charge of each colliding nucleus. The red vertical line marks the border between subcritical and supercritical domains.

the pair-production probabilities and the corresponding positron spectra beyond the monopole approximation can be very useful for finding the most promising experimental scenarios that allow for the determination of the angular distribution of the emitted positrons [24].

We hope that the results obtained in this Letter will promote new efforts for the experimental detection of the vacuum decay in a supercritical Coulomb field. In particular, such experiments seem feasible with the CRYRING facility at GSI/FAIR [29,30], where storing of bare uranium nuclei at low energies is anticipated in the near future.

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