Vanishing Hall Response of Charged Fermions in a Transverse Magnetic Field

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We study the Hall response of two-dimensional lattice systems of charged fermions in a transverse magnetic field, in the ballistic coherent limit. We identify a setup in which this response vanishes over wide regions of parameter space: the "Landauer-Büttiker" setup commonly studied for coherent quantum transport, consisting of a strip contacted to biased ideal reservoirs of charges. We show that this effect does not rely on particle-hole symmetry, and is robust to a variety of perturbations including variations of the transverse magnetic field, chemical potential, and temperature. We trace this robustness back to a topological property of the Fermi surface: the number of Fermi points with positive velocity of the system. We argue that the mechanism leading to a vanishing Hall response applies to noninteracting and interacting systems alike, which we verify in concrete examples using density-matrix renormalization group simulations.

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Transport properties induced by electromagnetic fields are an active area of study in condensed matter physics. The Hall response, σ_{H}, is of particular interest: It represents the off-diagonal response of a current density $\mathbf{J}$ to an electric field $\mathbf{E}$, $\sigma_{\mathrm{H}} = \varepsilon_{ij}\sigma_{ij}$, where $J_i = \sigma_{ij}E_j$ and ε_{ij} is the Levi-Civita symbol. The Hall response probes important geometric or topological properties of quantum systems: the Fermi-surface curvature of metals under weak magnetic fields [1–3], the Berry curvature of anomalous Hall systems [4], and related topological invariants of band insulators [5,6]. Studies of σ_{H} are ubiquitous in fields focused on topological quantum matter [7] and synthetic realizations thereof [8,9].

Scattering is essential in conventional studies of σ_{H}: in the two-dimensional (2D) Hall effect [10], e.g., Boltzmann-type approaches [11] that reproduce the observed Hall constant, R_{H}, for weak magnetic fields B: $R_{\mathrm{H}} \equiv -\sigma_{\mathrm{H}}/(\sigma_{xx}\sigma_{yy}B) \sim -1/(ne)$, where n is the density of carriers with charge e, and $x(y)$ denote the longitudinal (transverse) direction. Scattering also explains the plateaus of quantized Hall conductance ($\sigma_{\mathrm{H}} = \nu e^2/h$ for filling factor ν) appearing in strong-field regimes [12–14].

As ballistic systems become more accessible experimentally [8,15,16], new challenges are emerging for theory beyond Boltzmann-type approaches, despite past efforts in mesoscopic systems [17,18]. For example, σ_{ii} can be infinite in clean interacting systems, even at a finite temperature [19,20]. The connection between Hall response and carrier density is not even clear in the presence of interactions [21–24]. Recent progress was made with the calculation of R_{H} in dissipative metallic systems [25], where σ_{ii} is finite at zero frequency [26]. Nevertheless, the Hall response of coherent ballistic systems remains largely unexplored.

In this Letter, we identify a ballistic setup in which charged fermions in a transverse magnetic field can exhibit a strictly vanishing Hall response. We demonstrate this effect in noninteracting 2D lattice systems at zero temperature, and extend our results to interacting analogs using a density-matrix renormalization group (DMRG). We show that the Hall response vanishes under a wide variety of perturbations: variations in magnetic field, chemical potential, temperature, and particle-hole symmetry breaking. We relate this remarkable robustness to the topological nature of the key property underpinning the effect: the number of Fermi points with positive velocity of the system.

Hall response of ballistic systems.— We consider lattice systems in a 2D strip geometry (Fig. 1), with edges in the y direction. Edges imply that the transverse current J_y vanishes in the low-frequency limit $\omega \to 0$ of the longitudinal electric field E_x. The Hall response is then described by the transverse polarization difference $\Delta P_y(x,t) = \int_{t_0}^{t} dt' J_y(x,t')$. We set $P_y(x,t_0) = 0$ at time t_0 right before E_x is applied (corresponding to a gauge choice [29–31]), and denote $\Delta P_y(x,t) \equiv P_y(x,t)$.

The relation between P_y and σ_{H} can be derived using linear response theory [32]: writing $E_x = -\partial_t A_x$ (with $e = \hbar = c = 1$), one finds

$$P_y(k,\omega) = -\sigma_{\mathrm{H}}(k,\omega)A_x(k,\omega), \tag{1}$$

where k is the crystal momentum along x. This can be seen as a Kubo formula for the polarization induced by a time-dependent vector potential, $P_y(x,t) = i\sum_{x'}\int dt'\theta(t-t')\langle[P_y(x,t), J_x(x',t')]\rangle A_x(x',t')$ [see the Supplemental Material for details [33]].

Equation (1) allows for *very different* Hall responses σ_{H}, depending on the nature of E_x, for the *same*

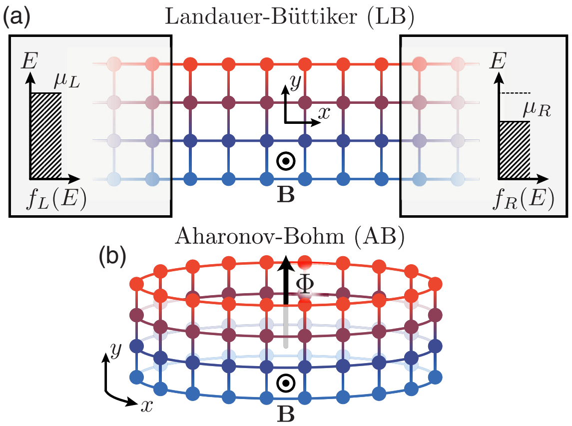

FIG. 1. (a) Landauer-Büttiker setup enabling a vanishing Hall response: a ballistic lattice system is connected to ideal reservoirs (in gray) with weakly biased chemical potentials $\mu_R < \mu_L$ [corresponding to Fermi-Dirac distributions $f_{L/R}(E)$]. (b) Ballistic Aharonov-Bohm setup where the Hall response is, in contrast, generically finite (with persistent current along x induced by a magnetic flux Φ).

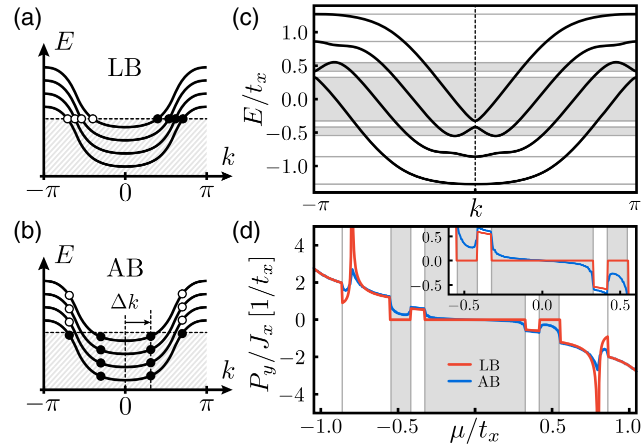

FIG. 2. (a), (b) Schematic band structures showing the key single-particle states for the Hall response: in the LB setup, the current $J_x \neq 0$ is induced by occupied states (full dots) with velocity $v_k > 0$ in a small energy window $[\mu_R, \mu_L]$ around the chemical potential μ (horizontal dashed line). Left-moving states at μ are empty. In the AB setup, instead, $J_x \neq 0$ is induced by the spectral flow $\Delta k = \Phi/N_x$ of *all* states with threaded magnetic flux Φ. (c) Band structure of the HH model computed for $N_y = 4$, $B = 0.7$, and $t_y = 0.5 t_x$. Horizontal lines (dark gray) indicate energies at which the number $c(\mu)$ of Fermi points with $v_k > 0$ changes, with $c(\mu) = N_y$ in shaded (light gray) regions. See the Supplemental Material [33] for other parameter regimes including $N_y \to \infty$. (d) Hall response of the system in (c) in LB vs AB setups: when $c(\mu) = N_y$, P_y/J_x strictly vanishes in the LB setup, while it only goes to zero at particle-hole symmetry ($\mu = 0$), in the AB setup (see inset enlargement).

longitudinal current J_x. Here we consider a paradigmatic setup for coherent quantum transport: a system with two ends in the x direction, where E_x (or J_x) is generated by ideal contacts to two external reservoir of charges (left and right) with chemical potentials μ_L and μ_R [Fig. 1(a)]. In this "Landauer-Büttiker" (LB) setup [34], J_x is related to the potential difference $eV \equiv \mu_\mathrm{L} - \mu_\mathrm{R}$ via the conductance G of the system: $J_x = GV$. Without interactions, the polarization $P_y \equiv P_y^\mathrm{LB}$ in this setup can be obtained from conventional scattering theory [35], with conductance G derived from the Landauer formula. Kubo's formalism [Eq. (1)] provides an instructive equivalent approach [36]. As we detail in the Supplemental Material [33], the LB setup can be described by $A_x(x,t) = -Ve^{-i\omega t}\delta(x)$, i.e., by a potential drop of amplitude V at the position $x = 0$ of contact between the system and the left reservoir. Since $A_x(x,t)$ is local, the stationary P_y^LB takes the form of an *integral* of the Hall response *over all momenta* [33]:

$$\frac{P_y^\mathrm{LB}}{J_x} = -G^{-1}\lim_{\omega\to 0}\frac{1}{2\pi}\int dk e^{ikx}\frac{\sigma_\mathrm{H}(k,\omega)}{\omega + i0^+}, \qquad (2)$$

where $i0^+$ is a small positive imaginary part.

To illustrate the strong differences that can arise in Hall response between ballistic coherent systems, we consider an additional "Aharonov-Bohm" (AB) setup: a contactless ring where J_x is induced by a time-dependent magnetic flux [Fig. 1(b)]. In that case, $A_x(x,t)$ corresponds to the vector potential describing the inserted flux, i.e., $A_x(x,t) = e^{i\omega t}\Phi/N_x$, where N_x is the number of sites along x. The flux induces a persistent current [37,38] $J_x = D\Phi/N_x$, where D is the Drude weight [39], generating a *reactive* Hall response [40,41] [Fig. 2(b)]. In contrast to Eq. (2), and in agreement with known results [42], the stationary $P_y \equiv P_y^\mathrm{AB}$ found here depends on the *zero-momentum* component of σ_H [33]:

$$\frac{P_y^\mathrm{AB}}{J_x} = -D^{-1}\lim_{\omega\to 0}\sigma_\mathrm{H}(0,\omega). \qquad (3)$$

Hall response in the LB setup.—We now detail the LB setup and derive an explicit formula for P_y^LB at zero temperature, in the low-bias limit $\mu_\mathrm{L} \to \mu_\mathrm{R} \equiv \mu$. Our results apply to a broad variety of lattice models. For clarity, however, we take the viewpoint of the Harper-Hofstadter (HH) model [43]. Specifically, we consider fermions on a square lattice with Hamiltonian $H_\mathrm{HH} = -\sum_{x,y}[t_x e^{iBy} c_{x,y}^\dagger c_{x+1,y} + t_y c_{x,y}^\dagger c_{x,y+1}]/2 + \mathrm{H.c.}$, in the Landau gauge, where $c_{x,y}^\dagger$ creates a fermion on site (x,y), B is the magnetic flux per plaquette, and $t_x(t_y)$ is the nearest-neighbor hopping amplitude in the $x(y)$ direction [Fig. 1(a)]. In this minimal model, the system can be seen as N_y coupled longitudinal wires: its spectrum ε_k can be regarded as N_y bands $t_x\cos[k - yB/(N_y - 1)]$, shifted by $yB/(N_y - 1)$ in momentum (with $y = 0, 1, \ldots, N_y - 1$), and hybridized by t_y.

As we demonstrate below, the response P_y^LB vanishes identically when the system's spectrum is symmetric under

$k \to -k$, and the number $c(\mu)$ of Fermi points with velocity $v_k \equiv \partial\varepsilon_k/\partial k > 0$ is equal to N_y. Remarkably, these conditions are satisfied in wide regions of parameter space, for weak and strong B. We distinguish two main scenarios, in particular: (i) the "weak-field" regime ($B \lesssim 1/N_y$), where all bands $t_x \cos[k - yB/(N_y - 1)]$ hybridize in the first Brillouin zone, and (ii) the "strong-field" regime, where bands hybridize after backfolding into the first Brillouin zone. We focus on (i) in what follows, and extend our discussion to (ii) in the Supplemental Material [33].

In the HH model, symmetry under $k \to -k$ arises from a combination of time reversal (TR) and spatial inversion in the y direction. This effective TR symmetry is described by the operator $\Theta = I_y\mathcal{K}$, where I_y permutes positions y around the center of the system, and $\mathcal{K}$ describes complex conjugation. As $[H_{\mathrm{HH}}, \Theta] = 0$, the action of Θ on an eigenstate $|\psi_k(E)\rangle$ of H_{HH} with momentum k and energy E yields a (non-necessarily distinct [44]) eigenstate $\Theta|\psi_{k'}(E)\rangle$ with $k' = -k$ and identical energy. As in Eq. (2) [33], the Hall response can be derived using scattering theory: in the low-bias, zero-temperature limit, the conductance reads $G = G_0 \sum_j T_j$, where $G_0 = e^2/h = 1/(2\pi)$ is the conductance quantum, and T_j is the transmission probability, at the chemical-potential energy μ, of scattering modes $\psi_j(x, y)$ incoming from the left reservoir. We consider infinite reservoirs described by H_{HH} (with chemical potentials μ_L and μ_R, respectively), so that scattering modes have a similar form as the system's eigenmodes. In that case, $T_j = 1$ for all modes available at μ. Relevant modes have an asymptotic form $\psi_j(x \to -\infty, y) = e^{ik_{F,j}x} w_j(y)/v_{F,j}$, where $k_{F,j}$ ($v_{F,j}$) are Fermi momenta (velocities), and $w_j(y)$ transverse wave functions. The conductance reduces to $G = c(\mu)G_0$, where $c(\mu)$ is the number of Fermi points with positive velocity $v_k = \partial\varepsilon_k/\partial k > 0$, as in Fig. 2(a). Equation (2) becomes

$$\frac{P_y^{\mathrm{LB}}(\mu)}{J_x} = \frac{1}{c(\mu)G_0} \sum_{j=1}^{c(\mu)} \sum_y y \frac{w_j(y)^2}{v_{F,j}}, \tag{4}$$

as derived in the Supplemental Material [33] (with more explicit expressions for finite B and $N_y = 2$, or $B \to 0$ and arbitrary $N_y \geq 2$).

We used the simulation package KWANT [46] to verify our formulas, compute P_y^{LB} for arbitrary B and N_y, and compare P_y^{LB} to P_y^{AB}. Our results are illustrated in Fig. 2(d) for the weak-field regime, and in the Supplemental Material [33] for the strong-field regime (including a discussion of the limit $N_y \to \infty$). They demonstrate two key points: first and foremost, P_y^{LB} *vanishes identically* whenever $c(\mu) = N_y$, irrespective of the specific value of B or μ [see, e.g., the region around $\mu/t_x = \pm 0.5$ in Fig. 2(d)], and of particle-hole symmetry (generically absent here). Second, the responses P_y^{LB} to P_y^{AB} are strikingly different, as hinted by Eqs. (2) and (3). Intuitively, this arises from the fact that LB and AB stationary states are different [Figs. 2(a) and 2(b)], with distinct polarizations P_y, though they carry the same current J_x. Specifically, each conduction channel j gives a contribution $P_{y,j} = \mathcal{C}_j J_{x,j}$ to P_y, proportional to the current $J_{x,j}$ carried by the channel [33]. The factor $\mathcal{C}_j$ does not depend on the origin of $J_{x,j}$. Crucially, however, every channel carries a *different* current $J_{x,j} = v_{F,j}\Phi/(\pi N_x)$ in the AB setup, whereas all channels carry the same one in the LB case. Accordingly, the two responses coincide when $c(\mu) = 1$, and generically differ otherwise [33]. They also share the same sign, set by the particle (+) or hole (−) nature of charge carriers, leading to their vanishing at particle-hole symmetry [$\mu = 0$ in Fig. 2(d)]. Moreover, both responses are discontinuous at transitions between distinct $c(\mu)$.

Topological origin of the vanishing Hall response.— We now demonstrate that $P_y^{\mathrm{LB}} = 0$ due to (i) the topological nature of $c(\mu)$, and (ii) the traceless nature of the operator $\hat{P}_y$ describing the polarization. The number $c(\mu)$ of Fermi points with $v_k > 0$ is topological in the sense that it corresponds to the *central charge* of the system (the number of gapless modes with $v_k > 0$, in a Luttinger-liquid interpretation). The polarization operator is $\hat{P}_y = eY$, where $Y \equiv \sum_{x,y} y c_{x,y}^\dagger c_{x,y}$ describes the "center-of-mass" position along y. To ensure that $\langle\psi_i^{\mathrm{LB}}(\mu)|\hat{P}_y|\psi_i^{\mathrm{LB}}(\mu)\rangle = 0$ in the initial state $|\psi_i^{\mathrm{LB}}(\mu)\rangle \equiv |\psi_i\rangle$ with zero bias ($V, J_x = 0$), corresponding to our gauge choice for the polarization, we set $y = 0$ at the center of the system. The operator $\hat{P}_y$ then satisfies $I_y^T \hat{P}_y I_y = -\hat{P}_y$. It is traceless, and $P_y^{\mathrm{LB}} = 0$ at zero bias is ensured by the symmetry between k and $-k$: indeed, $|\psi_i\rangle$ is the many-body ground state of H_{HH} with single-particle states occupied symmetrically around $k = 0$, up to the chemical potential μ. It is symmetric under Θ (i.e., $\Theta|\psi_i\rangle = \pm|\psi_i\rangle$), such that $\langle\psi_i|\hat{P}_y|\psi_i\rangle = \langle\psi_i|\Theta^\dagger\hat{P}_y\Theta|\psi_i\rangle = \langle\psi_i|I_y^T\hat{P}_y I_y|\psi_i\rangle = -\langle\psi_i|\hat{P}_y|\psi_i\rangle$.

When applying a finite bias $V \neq 0$ to generate a stationary current J_x in the "final" state $|\psi_f^{\mathrm{LB}}(\mu)\rangle \equiv |\psi_f\rangle$, the symmetry Θ breaks: the state $|\psi_f\rangle$ is a many-body stationary state with single-particle states occupied symmetrically around $k = 0$ *except* at μ where single-particle states are occupied where $v_k > 0$ only. By symmetry, noncanceling contributions to the polarization can only come from these $c(\mu)$ Fermi points. We index the latter as $j = 1, 2, \ldots, c(\mu)$, and denote by $|j\rangle$ the corresponding single-particle states ($|j\rangle \equiv |k_{F,j}, s_j\rangle$, here, where $k_{F,j}$ and s_j are the Fermi momentum and band index of the Fermi point j). In this picture, the polarization becomes

$$P_y^{\mathrm{LB}}(\mu) = \sum_{j=1}^{c(\mu)} n_j \langle j|\hat{P}_y|j\rangle, \tag{5}$$

where $n_j = 1$ is the occupation of $|j\rangle$.

We can now show that $P_y^{\mathrm{LB}}(\mu)$ vanishes when $c(\mu) = N_y$: the states $|j\rangle$ in Eq. (5) belong to the eigenspace of H_{HH} with energy μ, and are characterized by distinct momenta. Since they are not related by symmetry [47], they form a basis for a Hilbert (sub)space of dimension $c(\mu)$. Therefore, when $c(\mu) = N_y$, Eq. (5) becomes

$$P_y^{\mathrm{LB}}(\mu)|_{c(\mu)=N_y} = \sum_{j=1}^{N_y} \langle j|\hat{P}_y|j\rangle = \mathrm{Tr}\hat{P}_y = 0. \qquad (6)$$

This demonstrates our main result: the existence of a conservation law for the Hall response of the LB setup. Note that other, potentially observable conservation laws can be derived from the tracelessness of $\hat{P}_y$: in particular, replacing the set $\{|j\rangle\}$ by a basis of Bloch eigenstates $\{|k,s\rangle\}$ (with momentum k and band index $s = 1, 2, \ldots, N_y$), one finds $P_y(k) \equiv \sum_{s=1}^{N_y} \langle k,s|\hat{P}_y|k,s\rangle = 0$, meaning that the Hall response of a system with N_y bands vanishes in momentum sectors k where bands are equally occupied. This conservation law corresponds to the known zero-sum rule for the Berry curvature of all eigenstates of a Hamiltonian [48]. Equation (6) can be seen as an analog with fixed energy, instead of fixed k.

Robustness to perturbations.— The vanishing of P_y^{LB} at $c(\mu) = N_y$ is protected against temperature by an energy gap $\Delta\mu$ corresponding to the smallest chemical-potential variation required to change $c(\mu)$. More precisely, the Hall response is suppressed as $e^{-\beta|\Delta\mu|}$ at finite temperature $T = 1/\beta$ [33]. We emphasize that the gap $\Delta\mu$ need not close with increasing N_y. In fact, in the above HH model, the gap around $\mu = 0$ is $\Delta\mu \approx (t_x - t_y) - |\mu|$ approximately independent of N_y when $t_y \lesssim t_x$ [33].

Deviations from a strictly vanishing Hall response are expected in the presence of generic disorder, as this breaks the symmetry Θ connecting momentum sectors k and $-k$. Disorder in quasi-1D systems generally leads to Anderson localization [49]. Nevertheless, if the scattering region connecting the reservoirs is shorter than the localization length (scaling as $N_y t_x^2/W^2$ with disorder strength W [50]), disorder remains a weak perturbation. In that case, deviations of the disorder-averaged polarization $\langle P_y\rangle$ from zero scale as W^2/t_x^2, with large fluctuations around the average (as do conductance fluctuations in disordered systems [51]); see the Supplemental Material for details [33].

Generalization to interacting systems.—Equation (6) applies whenever the current J_x is carried by $c = N_y$ independent, equally occupied fermionic channels, regardless of interactions. To demonstrate this, we consider the HH model on a two-leg ladder ($N_y = 2$), with additional intra- and interleg interactions described by Hamiltonian terms $U_\parallel \sum_{x,y=\pm1} n_{x,y} n_{x+1,y} + U_\perp \sum_x n_{x,1} n_{x,-1}$, where $n_{x,y}$ is the density on site (x,y). To simulate transport in the LB setup, we evolve the system with reservoirs

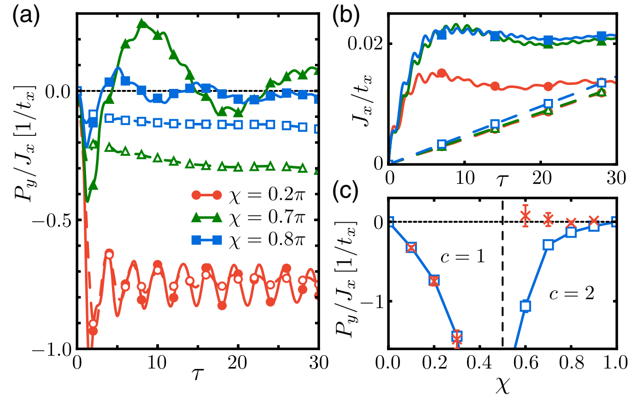

FIG. 3. Numerical TDMRG estimates of the LB and AB Hall responses of the interacting HH model, with $t_x = t_y = 1$, $U_\parallel = U_\perp = 1/2$, and $\epsilon = 0.01$, for 10 fermions in a system of length $L_x = 60$. (a) Evolution of P_y^{LB}/J_x (filled symbols) and P_y^{AB}/J_x (empty symbols) for a magnetic flux $\chi \equiv B/N_x = 0.2\pi$ (Luttinger-liquid phase with $c = 1$), and 0.7π and 0.8π ($c = 2$). Lines interpolate more data points than shown. (b) Time evolution of J_x for parameters as in (a). (c) Average of P_y^{LB}/J_x (×) and P_y^{AB}/J_x (□) over times $10 < \tau < 30$. The dashed line indicates the estimated transition between $c = 1$ and $c = 2$. Averages coincide for $\chi = 0.4\pi$, while no stationary regime was reached for 0.5π.

described by a quenched steplike potential $-\epsilon\sum_{x<L_{\mathrm{res}},y} n_{x,y} + \epsilon\sum_{x>L_{\mathrm{sys}}+L_{\mathrm{res}},y} n_{x,y}$, where $L_{\mathrm{sys/res}}$ denotes the length of the system and reservoirs. We set $\epsilon = 0$ and prepare the full system in its ground state using DMRG [52,53]. We then switch $\epsilon > 0$, at time $\tau = 0$, and evolve the system using time-dependent DMRG (TDMRG) [53] and the ITensor library [54]. We set $L_{\mathrm{sys}} = 2$, for simplicity [55], and compute the Hall response P_y^{LB}/J_x in the middle of the system at times $1 \lesssim \tau/t_x \lesssim L_{\mathrm{res}}$ [56], averaging over a time window where J_x is approximately stationary. Figure 3 shows typical results for $U_\parallel = U_\perp = t_x/2$. For comparison, we simulate transport in the AB setup by quenching, instead, a small linear potential $-(\epsilon/N_x)\sum_{x,y} x n_{x,y}$. While J_x increases linearly in time in that case [Fig. 3(b)], the ratio P_y^{AB}/J_x oscillates around a constant value corresponding to the stationary Hall response [41].

The results shown in Fig. 3 are consistent with our theoretical analysis: LB and AB Hall responses are identical (with time averaging, within errorbars) when the initial ground state is characterized by a central charge $c = 1$ [33,57]. More importantly, they strongly differ when $c = 2 = N_y$ [Fig. 3(c)], with large oscillations of P_y^{LB}/J_x around an average consistent with $P_y^{\mathrm{LB}}/J_x = 0$, and a finite P_y^{AB}/J_x. Our results (including additional data presented in the Supplemental Material [33]) fully support our theoretical result that P_y^{LB}/J_x vanishes when $c = 2 = N_y$.

Discussion.— The conservation law found in this work exemplifies the rich and sometimes counterintuitive phenomena that can occur in ballistic coherent systems. Solid-state and synthetic-matter experiments would be well suited to observe it [8,15,16]. In fact, a platform for realizing the LB setup has recently been proposed [58]. We emphasize that our results extend to bosons: a vanishing transverse polarization would be observed in photonic systems [59], e.g., by selectively populating the $c = N_y$ states in Eq. (6) [60].

Our results provide additional clues to better understand the Hall response of strongly correlated (non-Fermi-liquid) systems, for which low-energy quasiparticle descriptions of quantum transport inexorably fail. Presently, they raise intriguing questions regarding the behavior of the transverse polarization P_y of interacting systems at finite temperatures: Although a transition to dissipative or metallic regimes is expected, explicit calculations of P_y remain challenging [25]. Recent studies have shown the persistence of ballistic and superdiffusive behavior in specific cases [19,20]. It will be interesting to investigate analogs in quasi-1D lattice systems.

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