Quantum Network Transfer and Storage with Compact Localized States Induced by Local Symmetries

M. Röntgen,[1] C. V. Morfonios,[1] I. Brouzos,[2] F. K. Diakonos,[2] and P. Schmelcher[1,3]
[1]Zentrum für optische Quantentechnologien, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany
[2]Department of Physics, University of Athens, 15771 Athens, Greece
[3]The Hamburg Centre for Ultrafast Imaging, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany

(Received 7 November 2018; published 23 August 2019)

We propose modulation protocols designed to generate, store, and transfer compact localized states in a quantum network. Induced by parameter tuning or local reflection symmetries, such states vanish outside selected domains of the complete system and are therefore ideal for information storage. Their creation and transfer is here achieved either via amplitude phase flips or via optimal temporal control of intersite couplings. We apply the concept to a decorated, locally symmetric Lieb lattice where one sublattice is dimerized, and also demonstrate it for more complex setups. The approach allows for a flexible storage and transfer of states along independent paths in lattices supporting flat energetic bands. We further demonstrate a method to equip *any* network featuring static perfect state transfer of single-site excitations with compact localized states, thus increasing the storage ability of these networks. We show that these compact localized states can likewise be perfectly transferred through the corresponding network by suitable, time-dependent modifications. The generic network and protocols proposed can be utilized in various physical setups such as atomic or molecular spin lattices, photonic waveguide arrays, and acoustic setups.

DOI: 10.1103/PhysRevLett.123.080504

Introduction.—The storage and transfer of information in quantum systems is a task of great importance for the realization of quantum computers and simulators. Storage of a quantum state implies that it is effectively decoupled from surrounding building blocks of a system and, thus, is not affected by its environment. In contrast, transfer of a state requires a successive interaction with its environment leading to its directed propagation. We here propose a quantum state that can be easily prepared and robustly stored and present protocols that transfer it through a quantum network. We thereby merge key ingredients of three different fields of research: (i) compact localized states in flatband lattices [1–14], (ii) perfect state transfer (PST) [15–23], and (iii) optimal control theory [24–28].

A compact localized state (CLS) is a Hamiltonian eigenstate defined by its strictly vanishing amplitudes outside a spatial subdomain of the system. This compact localization originates from destructive interference caused by the right combination of lattice geometry and Hamiltonian matrix elements. Such a combination is possible in a broad range of physical systems [2], and CLSs have been realized in, e.g., photonic waveguide arrays [3,6,8,13], ultracold atomic ensembles [4,5], and optomechanical setups [11]. Though typically residing in "flat" energy bands of periodic lattices with macroscopic degeneracy [3–6,8,12–14], CLSs can exist in nonperiodic setups just as well.

By their defining property, CLSs are ideally suited for storage: Because of their compactness, they can be stored using only a very small number of physical sites and, being Hamiltonian eigenstates, they can in principle be stored for an infinite amount of time. Moreover, their compactness protects CLSs against a wide range of imperfections. In particular, CLSs remain unaffected by changes of the Hamiltonian outside their localization domain. Furthermore, there is a class of CLSs which are protected by spatial *local symmetry* of the Hamiltonian against *any* perturbations preserving this local symmetry and the geometry within their localization domain [10]. As a side note, the more general study of local symmetries has recently been put on new grounds by introducing a framework of nonlocal currents [29–34] by means of which the parity and Bloch theorem are generalized to locally symmetric systems [35]. Recently, it has been shown that CLSs may result from particular local symmetries which commute with the Hamiltonian in discrete systems [10].

While favoring their storage, the compactness and consequent decoupling of CLSs from their surroundings poses the challenge of how to *transfer* them controllably across a lattice. We here demonstrate how transfer of local-symmetry-induced CLSs can be achieved using two different approaches, based on free and driven time evolution. The first approach utilizes the common perfect (i.e., with unit fidelity) quantum state transfer scenario, where static intersite couplings are tailored such that a selected state evolves freely from one location to another. Quantum

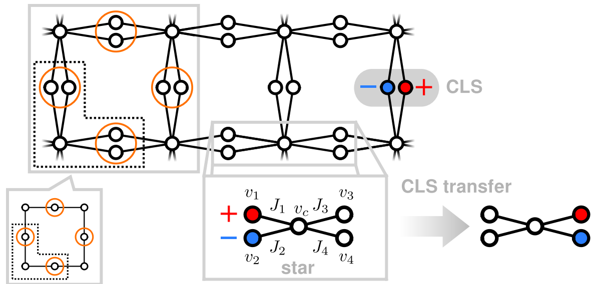

FIG. 1. Decorated Lieb lattice (DLL), constructed from the original Lieb lattice (whose plaquette is shown in the lower left-hand inset) by replacing the encircled sites with dimers. Each such dimer can host one CLS with opposite amplitudes on the two dimer sites. The lower right-hand inset shows the isolated "star" subsystem functioning as a unit for the CLS transfer.

state transfer techniques are especially explored in connection with entanglement transfer [17,18], while engineered coupling conditions for perfect transfer are also applied to network setups [15,16]. As we show here, a CLS can be perfectly transferred under free evolution after a suitable local phase flip in its amplitude or in selected intersite hoppings. The second approach uses optimal control theory [24–28], where the system is dynamically driven to the target state. For the tight-binding systems treated here, it aims at maximizing the fidelity of the transfer of a CLS across the system by designing smooth time-dependent modulations of the couplings. The main advantage is its applicability in cases where instantaneous changes, like the phase flips above, are not feasible in practice. A special representative of optimal control is, e.g., the celebrated stimulated adiabatic Raman passage (STIRAP) [36] in three level systems. Recently [37], CLSs in Lieb lattices have been used, in the form of "dark states" [38], as a transfer channel between two local states during a spatial STIRAP process. We here take an orthogonal viewpoint, since our aim is to transfer the dark state (CLS) itself to other dark states through the network.

For definiteness, we shall apply the proposed concept of storage and transfer of CLSs to a decorated Lieb lattice (DLL); see Fig. 1. It is derived from the original Lieb lattice [39] by replacing the sites of one sublattice with dimers. The resulting network can be extended to more complex geometries and to higher dimensions, and different CLSs can be routed independently across the network.

Star subsystem.—The basic building block for CLS transfer in the DLL is its isolated unit cell. It represents a five-point star graph and is shown in the lower inset of Fig. 1. It is governed by the Hamiltonian

$$H = v_c|c\rangle\langle c| + \sum_{n=1}^{4} v_n|n\rangle\langle n| + J_n(|n\rangle\langle c| + |c\rangle\langle n|), \quad (1)$$

with on-site potentials v_n and outer nodes $n = \{1, \ldots, 4\}$ coupled to the central node c by real hoppings J_n. The above Hamiltonian can be physically realized in various contexts. One possibility is a coupled waveguide array [40,41], with each node representing a waveguide cross section and neighboring waveguides evanescently coupled through the overlap of their fundamental modes. By tuning the distance between neighboring waveguides, their coupling can be individually controlled [42,43] and also made negative [44]. The system is then effectively described by a discrete Schrödinger equation in terms of single-site excitations $|n\rangle$, with time t replaced by the coordinate along the waveguide axis [41]. Another possible physical realization of H is in terms of spins, with each node representing a spin-$1/2$ qubit (measured up or down). The Heisenberg XX interaction Hamiltonian reduces to this simple description within the subspace of one excitation (1 spin up and all others down) [17].

In the presence of local symmetry under permutation only of sites 1 and 2, that is, $J_1 = J_2 \equiv J$ and $v_1 = v_2 \equiv v$, the star Hamiltonian H hosts the eigenstate $|I\rangle = [(|1\rangle - |2\rangle)/\sqrt{2}]$. This is a CLS with opposite amplitudes on sites 1 and 2 and vanishing amplitudes on all others. Its hopping rate $J_1 I_1 + J_2 I_2 = J_1 - J_2$ (with $I_n = \langle n|I\rangle$) to the central site vanishes. It is thus decoupled by local symmetry from the rest of the star system, and, being an eigenstate, "stored" for an arbitrary time interval until the local symmetry condition is violated. Crucially, this local-symmetry-induced decoupling persists even if $J(t)$ and $v(t)$ are time dependent, allowing for a gradual modulation without perturbing the CLS [up to a global phase, $|I(t)\rangle = e^{-i\phi(t)/\hbar}|I\rangle$ with $\phi(t) = \int_{t_0}^{t} v(t')dt'$]. Moreover, $|I\rangle$ is unaffected by any change of the remainder of the Hamiltonian, i.e., of v_c, v_3, v_4, J_3, or J_4. In complete similarity, for $J_3 = J_4$ and $v_3 = v_4$ the star hosts a second compact localized eigenstate $|F\rangle = [(|3\rangle - |4\rangle)/\sqrt{2}]$. Both $|I\rangle$ and $|F\rangle$ are also dark states since they are the only eigenstates of the Hamiltonian Eq. (1) which have nonzero coefficients only in two sites. An important fact is that $|I\rangle$ and $|F\rangle$ are also eigenstates of the full Hamiltonian when the star subsystem is repeatedly connected to form the DLL (see Fig. 1). In fact, *different* locally symmetric star subsystems, each with its own parameters v_n, J_n, can be connected in a suitable way to form a network hosting multiple independent CLSs, each stored on only two sites with opposite amplitude.

In the following, we first present different protocols transferring a CLS in the star subsytem. These can easily be extended to the full DLL, as we will show afterwards. In the Supplemental Material (SM) [45], we demonstrate the robustness of the considered protocols to perturbations. For the state transfer within the time T, the two CLSs $|I\rangle$ and $|F\rangle$ will serve as the initial and final state. Throughout the rest of this work, we set $v_n = v$, but J_n are not necessarily equal to each other and may also be time dependent during the pulse. However, we impose the symmetry condition $J_3 = J_4$ at the end of the transfer to ensure that $|F\rangle$ is an energy eigenstate. The initially stored CLS is thus transferred to the target

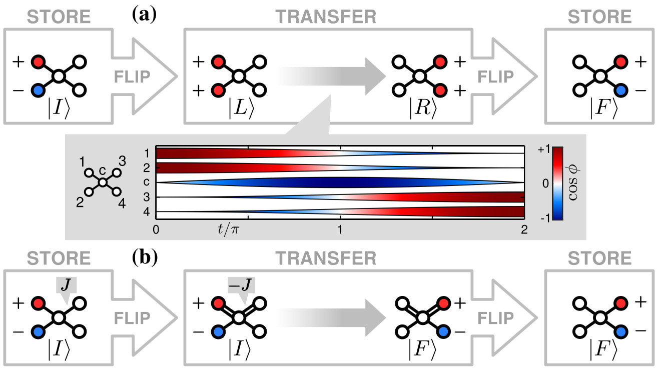

FIG. 2. Transfer of a stored dimer-localized CLS $|I\rangle$ to a final CLS $|F\rangle$ within a star subsystem via free time evolution between sign flips of **(a)** one component of the initial and final states and **(b)** one of the couplings (double line) of the central site to each of the initial and final CLS dimers. The inset in (a) shows the evolution of the state $|\psi\rangle$ over time $T = 2\pi$ with $|\psi(0)\rangle = |L\rangle$ and $|\psi(T)\rangle = |R\rangle$ for $v_n = 2J_n = 1/4 \ \forall \ n$. The phase of $\psi_n(t) = \langle n|\psi(t)\rangle$ at site n is color coded and its amplitude is indicated by the width of the corresponding stripe. The evolution for (b) is identical except for opposite signs on $\psi_2(t)$, $\psi_c(t)$, and $\psi_4(t)$.

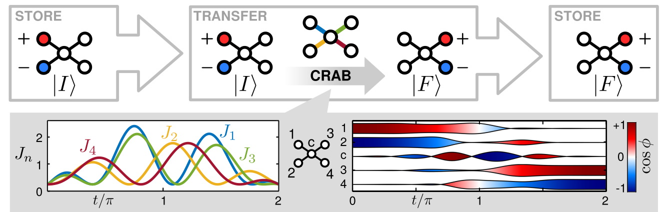

FIG. 3. CLS transfer via optimal control using the CRAB method for the couplings J_n of the form in Eq. (2). At the end of the procedure, all couplings $J_n = J$ again. The inset shows the temporal profile of the $J_n(t)$ and the evolution of the state over time $T = 2\pi$.

location and can be stored again indefinitely. In a spin network setting, the initial state $|I\rangle$ is a maximally entangled state between the spins at sites 1 and 2 of a dimer.

Transfer by phase flips.—For the transfer protocol visualized in Fig. 2(a), we set $J_n = J$ and consider the possibility to instantaneously imprint a phase flip by π (that is, a sign change) on one of the components of $|I\rangle$ at $t = 0$, turning it into $|L\rangle \equiv [(|1\rangle + |2\rangle)/\sqrt{2}]$. This new state $|L\rangle$ is no longer an eigenstate of H, and will evolve freely and with unit fidelity within time T to the state $|R\rangle \equiv [(|3\rangle + |4\rangle)/\sqrt{2}]$ for suitable chosen on-site potentials and couplings. Exactly at $t = T$ another sign flip is applied to one of the components of $|R\rangle$ in order to turn it into the desired target CLS $|F\rangle$. For the choice $J = 1/4$ energy units and $v = 2J$, the transfer time is $T = (\hbar\pi/2J) = 2\pi$ (setting $\hbar = 1$). General analytical derivations for the evolution $|\psi(t)\rangle = e^{-iHt}|I\rangle$ are given in the SM [45], exploiting the so-called "equitable partition theorem" [10,50,51]. As an alternative version of this transfer protocol, we can apply the instantaneous sign flips at $t = 0$ and T to the hoppings J_1, J_3 (or J_2, J_4) instead, as depicted in Fig. 2(b). The free time evolution is then essentially equivalent to the previous one in Fig. 2(a).

Transfer by optimal control.—Now we turn to optimal control solutions in order to design smooth pulses to avoid instantaneous operations. Taking into account that the initial state $|I\rangle$ is an eigenstate of $H(t = 0)$, we need to smoothly drive it out of stationarity in order to end up with the final state $|F\rangle$ as an eigenstate of the final Hamiltonian $H(t = T)$. To find smooth optimal driving pulses for the couplings J_n, we apply the chopped random-basis (CRAB) [52,53] optimal control method to the functional form:

$$J_n(t) = J\left(1 + \sin\left(\frac{t}{2}\right)[x_n \sin(\omega_n t) + x'_n \cos(\omega_n t)]^2\right). \quad (2)$$

This determines the optimal parameters x_n, x'_n, ω_n ($n = 1, 2, 3, 4$) for transferring $|I\rangle$ to $|F\rangle$ in time T, with the same initial and final conditions as previously (that is, $J_n = J$). More information on the CRAB procedure and the specific optimizations performed here can be found in the SM [45]. For this particular case, we further impose $J_n(t) \geq J = 1/4$, so that sign changes in the J_n are avoided. The resulting optimal J_n-driving pulses are presented in Fig. 3 together with the state evolution. The infidelity $1 - |\langle F|\Psi(T_f)\rangle|^2$ of these pulses is approximately 10^{-12}.

Transfer across a network.—By exploiting the robustness of CLSs, the state transfer schemes established above can now be used in the full DLL, as shown in Fig. 4(a). Starting from the upper left, the transfer process consists of (1) separating the star subsystem hosting the initial CLS $|I\rangle$ from the remainder of the lattice, (2) performing the transfer to the final CLS $|F\rangle$ within the star, and

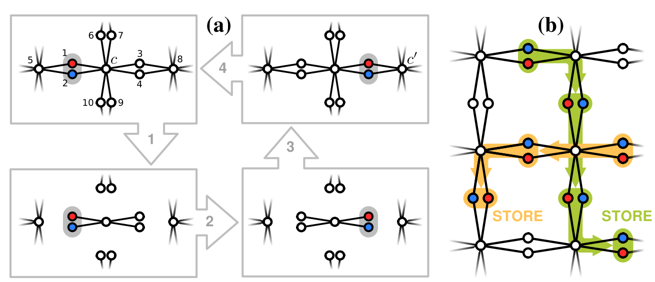

FIG. 4. **(a)** The "dimer-jump" process reducing state transfer within the decorated Lieb lattice to a star subsystem. It transfers a CLS on sites 1,2 to another CLS on sites 3,4 by (1) decoupling the corresponding star from the remainder of the system, (2) performing the transfer, and subsequently (3) recoupling the star, whence the process can (4) start anew, as indicated by $c \rightarrow c'$. **(b)** The robustness of CLSs with respect to perturbations outside their localization domain allows for the transfer of multiple CLSs along different routes in the network (orange and green directed paths), as long as these routes do not simultaneously use the same star subsystem.

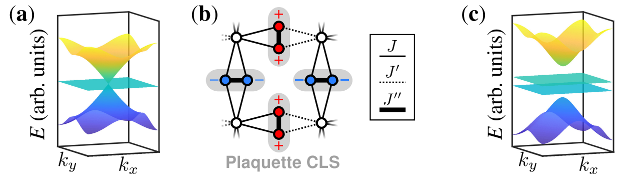

FIG. 5. **(a)** Band structure of the DLL. **(b)** Plaquette CLS in the modified DLL. **(c)** Band structure of the modified lattice for $0 < |J''| < |J - J'|$. See text for details.

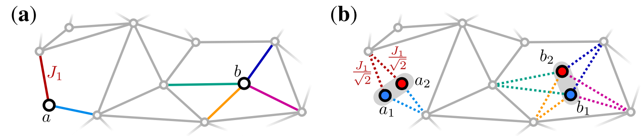

FIG. 6. Equipping a network capable of PST between sites a, b with CLS storage. **(a)** Original network. **(b)** Modified network, along with CLSs (see text).

(3) reconnecting the star to the remainder. The process can then start anew (4) to transport the state over longer distances.

To separate the star while keeping $|I\rangle$ unaffected, we ramp-down its outer couplings to surrounding sites to zero within time δt such that the local symmetry protecting $|I\rangle$ is *preserved*. With the site indexing in Fig. 4(a), starting at $t = 0$ this means that $J_{1,5}(t) = J_{2,5}(t)$ for $0 \leq t \leq \delta t$ with $J_{1,5}(\delta t) = 0$. This modulation does not perturb $|I\rangle$, even for $\delta t \to 0$. The other couplings of the star to its surroundings ($J_{3,8}$, $J_{4,8}$ and $J_{n,c}$ with $n = 6$, 7, 9, 10) can be ramped down in an *arbitrary* way, since they do not connect to the localization domain of $|I\rangle$. Within the separated star, the actual state transfer can be performed according to one of the above protocols over time T. Afterwards, we ramp-up the outer couplings again, now preserving the local symmetry which protects $|F\rangle$, that is, increasing $J_{3,8}$ and $J_{4,8}$ symmetrically, and the other couplings arbitrarily, from zero to their final values.

Transferring a CLS to a distant dimer in another star subsystem is achieved via consecutive "dimer jumps" like the one just described, as depicted in Fig. 4(b). This procedure can be employed simultaneously along different paths in the network. These may also intersect in space, as long as in each instant in time the different paths use different star subsystems; see orange and green paths in Fig. 4(b). Once a CLS has reached its final destination, it is stored for arbitrary time. In this sense, the proposed network becomes a simple model for a hybrid setup functioning simultaneously as a directional transfer device and as a multiple quantum memory unit. Crucially, this functionality is retained with high fidelity in the presence of various perturbations [45], even when the transfer paths remain connected to the surrounding lattice during the dimer jumps.

In practice, an actual realization of an equivalent network will depend on the limitations of the underlying physical platform. We emphasize that the DLL operated on here is a very basic lattice geometry enabling the proposed CLS transfer concept, but can be generalized to other geometries in a straightforward way. For instance, if intradimer coupling is non-negligible due to the small spatial separation, an alternative CLS transfer unit can be used, with essentially the same procedure applied [45].

Robustness.—As we demonstrate in the SM [45], the proposed protocols are quite robust against imperfections. For example, when considering finite-time linear ramps of duration δt instead of instantaneous hopping flips, a single dimer jump can still be performed with a fidelity above 0.998 even for slow ramps of $\delta t = T/4$. One can easily increase the robustness of the protocols further, as we now show by optimizing the storage fidelity of the DLL. In this network, the dimer CLSs correspond to two flatbands, which are degenerate to an additional flatband related to so-called "plaquette" CLSs [shown in Fig. 5(b)]. Additionally, two Dirac cones touch these three flatbands, as shown in Fig. 5(a). Under (small) perturbations, the dimer CLSs mix—due to their small energy separation—with other states. As a result, they spread across the lattice, which decreases the storage fidelity. However, this issue can be solved [45] by modifying the DLL as shown in Fig. 5(b). For $0 < |J''| < |J - J'|$, the two degenerate dimer CLS flatbands are gapped from all others, as demonstrated in Fig. 5(c), thus increasing the storage performance. In this modified DLL, dimer jumps are achieved by (i) decoupling the corresponding star subsystem, (ii) modifying the couplings within the star so as to recover the values of the original DLL, (iii) transferring the CLS, and (iv) reestablishing the coupling values of the modified DLL before recoupling the star.

Equipping static transfer protocols with storage.—Lastly, we demonstrate the versatility of our approach by showing how to equip *any* static (i.e., relying on simple time evolution) PST network with CLSs, thus greatly enhancing its storage capabilities. Figure 6 sketches the procedure. One replaces each of the sites a, b by dimers $a_{1,2}, b_{1,2}$, and all couplings between a, b and the remainder of the network by symmetric, renormalized couplings to these dimers. The modified network then features one CLS per dimer, which can be perfectly transferred by a procedure similar to the hopping flip presented above [45]. Interestingly, the transfer fidelity of the modified network can *exceed* that of the original network, as we demonstrate for a one-dimensional chain in the SM [45].

Conclusion.—We have demonstrated how local symmetries in a decorated Lieb lattice can be exploited to store and transfer compact localized states via different, easily realizable, modulation protocols. This provides a powerful prototype for a quantum device which simultaneously performs flexible transfer and robust storage of information within the same physical platform. We also have demonstrated how *any* network capable of static perfect state transfer of single-site excitations can be equipped with CLSs, such that these can be both perfectly transferred and stored. The transfer protocols utilize either instantaneous phase flips (with unit fidelity) or optimal temporal control of intersite couplings with near-unit fidelity. They can thus be adapted to the needs of different potential realizations in, e.g., electronics, atom optics, photonics, or acoustics. Under the very weak requirement of local symmetries protecting the CLSs, extension of the proposed concept to alternative network geometries and different dimensionality is straightforward. Based on multiple intersecting CLS transfer paths as proposed here, a future vision would be the design of a dynamical network with switchable quantum gates and embedded quantum memories.

M. R. gratefully acknowledges financial support by the "Stiftung der deutschen Wirtschaft" in the framework of a scholarship. I. B. acknowledges financial support by Greece and the European Union (European Social Fund–ESF) through the Operational Programme "Human Resources Development, Education and Lifelong Learning" in the context of the project "Reinforcement of Postdoctoral Researchers" (MIS-5001552), implemented by the State Scholarships Foundation (IKY).

[1] W. Maimaiti, A. Andreanov, H. C. Park, O. Gendelman, and S. Flach, Compact localized states and flat-band generators in one dimension, Phys. Rev. B **95**, 115135 (2017).
[2] D. Leykam, A. Andreanov, and S. Flach, Artificial flat band systems: From lattice models to experiments, Adv. Phys. **3**, 1473052 (2018).
[3] S. Mukherjee, A. Spracklen, D. Choudhury, N. Goldman, P. Öhberg, E. Andersson, and R. R. Thomson, Observation of a Localized Flat-Band State in a Photonic Lieb Lattice, Phys. Rev. Lett. **114**, 245504 (2015).
[4] S. Taie, H. Ozawa, T. Ichinose, T. Nishio, S. Nakajima, and Y. Takahashi, Coherent driving and freezing of bosonic matter wave in an optical Lieb lattice, Sci. Adv. **1**, e1500854 (2015).
[5] V. Apaja, M. Hyrkäs, and M. Manninen, Flat bands, Dirac cones, and atom dynamics in an optical lattice, Phys. Rev. A **82**, 041402(R) (2010).
[6] S. Mukherjee and R. R. Thomson, Observation of localized flat-band modes in a quasi-one-dimensional photonic rhombic lattice, Opt. Lett. **40**, 5443 (2015).
[7] O. Derzhko, J. Richter, and M. Maksymenko, Strongly correlated flat-band systems: The route from Heisenberg spins to Hubbard electrons, Int. J. Mod. Phys. B **29**, 1530007 (2015).
[8] R. A. Vicencio, C. Cantillano, L. Morales-Inostroza, B. Real, C. Mejía-Cortés, S. Weimann, A. Szameit, and M. I. Molina, Observation of Localized States in Lieb Photonic Lattices, Phys. Rev. Lett. **114**, 245503 (2015).
[9] L. Morales-Inostroza and R. A. Vicencio, Simple method to construct flat-band lattices, Phys. Rev. A **94**, 043831 (2016).
[10] M. Röntgen, C. V. Morfonios, and P. Schmelcher, Compact localized states and flat bands from local symmetry partitioning, Phys. Rev. B **97**, 035161 (2018).
[11] L.-L. Wan, X.-Y. Lü, J.-H. Gao, and Y. Wu, Hybrid interference induced flat band localization in bipartite optomechanical lattices, Sci. Rep. **7**, 15188 (2017).
[12] R. Shen, L. B. Shao, B. Wang, and D. Y. Xing, Single Dirac cone with a flat band touching on line-centered-square optical lattices, Phys. Rev. B **81**, 041410(R) (2010).
[13] B. Real, C. Cantillano, D. López-González, A. Szameit, M. Aono, M. Naruse, S.-J. Kim, K. Wang, and R. A. Vicencio, Flat-band light dynamics in Stub photonic lattices, Sci. Rep. **7**, 15085 (2017).
[14] S. Rojas-Rojas, L. Morales-Inostroza, R. A. Vicencio, and A. Delgado, Quantum localized states in photonic flat-band lattices, Phys. Rev. A **96**, 043803 (2017).
[15] M. Christandl, N. Datta, A. Ekert, and A. J. Landahl, Perfect State Transfer in Quantum Spin Networks, Phys. Rev. Lett. **92**, 187902 (2004).
[16] M. Christandl, N. Datta, T. C. Dorlas, A. Ekert, A. Kay, and A. J. Landahl, Perfect transfer of arbitrary states in quantum spin networks, Phys. Rev. A **71**, 032312 (2005).
[17] S. Bose, Quantum communication through spin chain dynamics: An introductory overview, Contemp. Phys. **48**, 13 (2007).
[18] A. Kay, Perfect, efficient, state transfer and its application as a constructive tool, Int. J. Quantum. Inform. **08**, 641 (2010).
[19] J. I. Cirac, P. Zoller, H. J. Kimble, and H. Mabuchi, Quantum State Transfer and Entanglement Distribution among Distant Nodes in a Quantum Network, Phys. Rev. Lett. **78**, 3221 (1997).
[20] M.-H. Yung and S. Bose, Perfect state transfer, effective gates, and entanglement generation in engineered bosonic and fermionic networks, Phys. Rev. A **71**, 032310 (2005).
[21] A. Wójcik, T. Łuczak, P. Kurzyński, A. Grudka, T. Gdala, and M. Bednarska, Unmodulated spin chains as universal quantum wires, Phys. Rev. A **72**, 034303 (2005).
[22] T. J. Osborne and N. Linden, Propagation of quantum information through a spin system, Phys. Rev. A **69**, 052315 (2004).
[23] R. J. Chapman, M. Santandrea, Z. Huang, G. Corrielli, A. Crespi, M.-H. Yung, R. Osellame, and A. Peruzzo, Experimental perfect state transfer of an entangled photonic qubit, Nat. Commun. **7**, 11339 (2016).
[24] C. Brif, R. Chakrabarti, and H. Rabitz, Control of quantum phenomena: Past, present and future, New J. Phys. **12**, 075008 (2010).
[25] S. J. Glaser, U. Boscain, T. Calarco, C. P. Koch, W. Köckenberger, R. Kosloff, I. Kuprov, B. Luy, S. Schirmer, T. Schulte-Herbrüggen, D. Sugny, and F. K. Wilhelm, Training Schrödinger's cat: Quantum optimal control, Eur. Phys. J. D **69**, 279 (2015).
[26] M. Murphy, S. Montangero, V. Giovannetti, and T. Calarco, Communication at the quantum speed limit along a spin chain, Phys. Rev. A **82**, 022318 (2010).
[27] X.-P. Zhang, B. Shao, S. Hu, J. Zou, and L.-A. Wu, Optimal control of fast and high-fidelity quantum state transfer in spin-1/2 chains, Ann. Phys. (Amsterdam) **375**, 435 (2016).

[28] H. Nakao and N. Yamamoto, Optimal control for perfect state transfer in linear quantum memory, J. Phys. B **50**, 065501 (2017).
[29] P. A. Kalozoumis, C. Morfonios, F. K. Diakonos, and P. Schmelcher, Local symmetries in one-dimensional quantum scattering, Phys. Rev. A **87**, 032113 (2013).
[30] P. A. Kalozoumis, C. V. Morfonios, F. K. Diakonos, and P. Schmelcher, Invariant currents and scattering off locally symmetric potential landscapes, Ann. Phys. (Amsterdam) **362**, 684 (2015).
[31] A. G. B. Spourdalakis, G. Pappas, C. V. Morfonios, P. A. Kalozoumis, F. K. Diakonos, and P. Schmelcher, Generalized continuity equations from two-field Schrödinger Lagrangians, Phys. Rev. A **94**, 052122 (2016).
[32] C. V. Morfonios, P. A. Kalozoumis, F. K. Diakonos, and P. Schmelcher, Nonlocal discrete continuity and invariant currents in locally symmetric effective Schrödinger arrays, Ann. Phys. (Amsterdam) **385**, 623 (2017).
[33] M. Röntgen, C. Morfonios, F. Diakonos, and P. Schmelcher, Non-local currents and the structure of eigenstates in planar discrete systems with local symmetries, Ann. Phys. (Amsterdam) **380**, 135 (2017).
[34] V. E. Zampetakis, M. K. Diakonou, C. V. Morfonios, P. A. Kalozoumis, F. K. Diakonos, and P. Schmelcher, Invariant current approach to wave propagation in locally symmetric structures, J. Phys. A **49**, 195304 (2016).
[35] P. A. Kalozoumis, C. Morfonios, F. K. Diakonos, and P. Schmelcher, Invariants of Broken Discrete Symmetries, Phys. Rev. Lett. **113**, 050403 (2014).
[36] N. V. Vitanov, A. A. Rangelov, B. W. Shore, and K. Bergmann, Stimulated Raman adiabatic passage in physics, chemistry, and beyond, Rev. Mod. Phys. **89**, 015006 (2017).
[37] S. Taie, T. Ichinose, H. Ozawa, and Y. Takahashi, Spatial adiabatic passage of massive quantum particles, arXiv:1708.01100.
[38] M. O. Scully, *Quantum Optics* (Cambridge University Press, Cambridge, England, 1997).
[39] E. H. Lieb, Two Theorems on the Hubbard Model, Phys. Rev. Lett. **62**, 1201 (1989).
[40] I. L. Garanovich, S. Longhi, A. A. Sukhorukov, and Y. S. Kivshar, Light propagation and localization in modulated photonic lattices and waveguides, Phys. Rep. **518**, 1 (2012).
[41] A. Szameit, F. Dreisow, and S. Nolte, *Discrete Optics in Femtosecond Laser Written Waveguide Arrays*, Topics in Applied Physics (Springer, Berlin, 2012).
[42] A. Perez-Leija, R. Keil, A. Kay, H. Moya-Cessa, S. Nolte, L.-C. Kwek, B. M. Rodríguez-Lara, A. Szameit, and D. N. Christodoulides, Coherent quantum transport in photonic lattices, Phys. Rev. A **87**, 012309 (2013).
[43] L. J. Maczewsky, J. M. Zeuner, S. Nolte, and A. Szameit, Observation of photonic anomalous Floquet topological insulators, Nat. Commun. **8**, 13756 (2017).
[44] R. Keil, C. Poli, M. Heinrich, J. Arkinstall, G. Weihs, H. Schomerus, and A. Szameit, Universal Sign Control of Coupling in Tight-Binding Lattices, Phys. Rev. Lett. **116**, 213901 (2016).
[45] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.123.080504 for calculations regarding the state transfer protocols, a method to generate CLS, the treatment of a modified DLL for evanescently coupled waveguides, an exhaustive analysis of the behavior of the proposed protocols under perturbations providing several improvements increasing their robustness, as well as technical details for equipping any static network with CLSs, and Refs. [46–49].
[46] E. Fritscher and V. Trevisan, Exploring symmetries to decompose matrices and graphs preserving the spectrum, SIAM J. Matrix Anal. Appl. **37**, 260 (2016).
[47] S. Yang, A. Bayat, and S. Bose, Spin-state transfer in laterally coupled quantum-dot chains with disorders, Phys. Rev. A **82**, 022336 (2010).
[48] S. Xia, A. Ramachandran, S. Xia, D. Li, X. Liu, L. Tang, Y. Hu, D. Song, J. Xu, D. Leykam, S. Flach, and Z. Chen, Unconventional Flatband Line States in Photonic Lieb Lattices, Phys. Rev. Lett. **121**, 263902 (2018).
[49] D. Moldovan, M. Anđelković, and F. Peeters, PYBINDING v0.9.4: A PYTHON package for tight-binding calculations, Zenodo (2017).
[50] W. Barrett, A. Francis, and B. Webb, Equitable decompositions of graphs with symmetries, Linear Algebra Appl. **513**, 409 (2017).
[51] A. Francis, D. Smith, D. Sorensen, and B. Webb, Extensions and applications of equitable decompositions for graphs with symmetries, Linear Algebra Appl. **532**, 432 (2017).
[52] P. Doria, T. Calarco, and S. Montangero, Optimal Control Technique for Many-Body Quantum Dynamics, Phys. Rev. Lett. **106**, 190501 (2011).
[53] T. Caneva, T. Calarco, and S. Montangero, Chopped random-basis quantum optimization, Phys. Rev. A **84**, 022326 (2011).