Comment on "Is a Trineutron Resonance Lower in Energy than a Tetraneutron Resonance?"

The quantum Monte Carlo study [1] of few-neutron resonant states provided results incompatible with rigorous few-body calculations [2–4]. In this Comment, we point out serious shortcomings in the framework of Ref. [1], leading to misinterpretation of unbound few-body systems.

The study of unbound few-neutron systems [1] followed a quite popular strategy consisting of two steps: (i) make the system bound with additional attractive potential, controlled by strength parameter V_0; (ii) extrapolate the resulting binding energy to the physical limit in continuum at $V_0 = 0$. Two different ways for step (i) have been employed in Ref. [1]: (1) adding an external trap potential and fixing center-of-mass (c.m.) of the system; (2) enhancing the nn interaction by factor $\alpha = 1 + V_0$. Such a procedure is sound if (a) the calculated bound state is physical and it evolves into resonance, and (b) the analytic continuation to a different Riemann sheet with resonance is performed correctly, taking into account threshold effects.

We argue that both these conditions are not satisfied in Ref. [1]. For definiteness, we consider the four-neutron ($4n$) system. Additional attraction may generate a bound dineutron with energy $E_d < 0$, which, then, defines the stability threshold for tetraneutrons: only those with $E_{4n} \leq E_d$ in the trap (or those with $E_{4n} \leq 2E_d$ for the enhanced force) are stable. Otherwise, even in the case $E_{4n} < 0$, they can decay into dineutron plus two infinitesimally slow neutrons moving around the common mass center (trap) or into two dineutrons (produced by enhanced force).

Our study reveals that a bound $^1S_0$ dineutron emerges in Woods-Saxon (WS) trap with radius $R_{WS} = 6$ fm and potential depth $V_0 \approx -0.09$ MeV only, or when the enhancement factor α in the $^1S_0$ wave exceeds ≈ 1.1 (these values slightly depend on the underlying nn potential). Further examples for $R_{WS} = 4.5$ and 7.5 fm are presented in Fig. 1(a). However, $4n$ states declared to be bound tetraneutrons with $E_{4n} \to 0$ in Ref. [1] were found only at significantly larger absolute values of $V_0 \approx -1.2$ MeV and $\alpha \approx 1.3$. For such Hamiltonians, the dineutrons are already well bound, thus, the lowest-energy state of the system is not a true bound state [5], but a continuum state that asymptotically looks like a dineutron in a trap plus two slow peripheral neutrons (two uncorrelated dineutrons cannot be placed in the same trap). It appears that Ref. [1] ignored this effect in the presumed $E_{4n} \approx 0$ region, which is decisive for the extrapolation. The tetraneutron states of Ref. [1] are above the stability threshold and, therefore, are not true bound states but most probably represent some discretized continuum states that do not evolve into a resonance. Extrapolation of their energies does not lead to proper resonance energy.

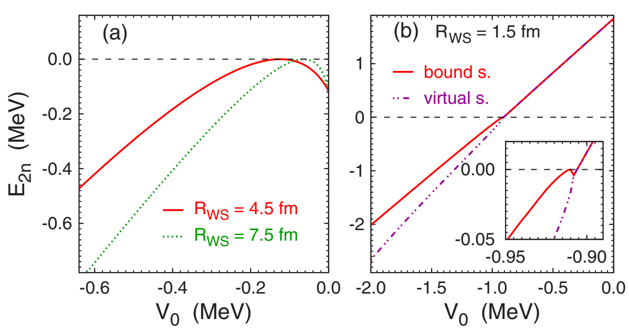

FIG. 1. $^1S_0$ dineutron pole trajectories in Wood-Saxon traps with given range parameters for realistic (a) and two-Gaussian (b) potentials.

Furthermore, a caution is needed in the extrapolation procedure itself if real bound states are calculated, since trajectory of a bound state evolving into continuum state involves branching at each threshold with discontinuity in the second derivative of energy with respect to a strength parameter [6]. Polynomial extrapolations [1] neglect this discontinuity and, therefore, are conceptually incorrect.

We show two examples in Fig. 1 corresponding to the $^1S_0$ virtual state for a realistic potential and to the resonance of the two-Gaussian potential [1]. Obtained $^1S_0$ pole trajectories have a typical bending shape, resulting in -0.12 MeV virtual state energy, in sharp contrast with the positive 0.1 MeV value of Ref. [1]. The latter is obtained by a polynomial extrapolation neglecting the near-threshold bending region. The resonance of the two-Gaussian potential does not necessarily evolve from the ground state in the trap. In a favorable case, a linear extrapolation, avoiding the input from the near-threshold region, may give a reasonable estimation for the energy of a narrow resonance. However, the presence of a branching point at the threshold, as shown in Fig. 1 (inset), produces highly nonlinear effects rendering naive extrapolation procedures mathematically unjustified.

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