Floquet Chiral Magnetic Effect

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A single Weyl fermion, which is prohibited in static lattice systems by the Nielsen-Ninomiya theorem, is shown to be realized in a periodically driven three-dimensional lattice system with a topologically nontrivial Floquet unitary operator, manifesting the chiral magnetic effect. We give a topological classification of Floquet unitary operators in the Altland-Zirnbauer symmetry classes for all dimensions, and use it to predict that all gapless surface states of topological insulators and superconductors can emerge in bulk quasienergy spectra of Floquet systems.

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In 1981, Nielsen and Ninomiya proved that a single Weyl fermion cannot be realized in lattice systems [1,2]. This theorem places a fundamental constraint on band structures due to the topology of the Brillouin zone. Weyl fermions have recently played a key role in cross-fertilizing ideas between high-energy physics and condensed-matter physics. A prime example is the prediction of Weyl semimetals [3,4], where the low-energy effective field theory of Weyl fermions predicts novel electromagnetic responses originating from the chiral anomaly [5–8]. In particular, the observations of the surface Fermi arc [9–15] and anomalous transport [16–19] have aroused considerable interest. However, if a system is defined on a lattice and thus anomaly-free, the Nielsen-Ninomiya theorem dictates that a Weyl fermion be accompanied by its partner with opposite chirality. By the same token, an anomaly induced response known as the chiral magnetic effect (CME) [20] does not occur in equilibrium [21], and numerous proposals to circumvent this difficulty have been made [22–29].

In this Letter, we demonstrate that a single Weyl fermion can be realized on a periodically driven lattice, thereby overcoming the above limitations. In periodically driven (Floquet) systems, the unitary time-evolution operator over one period defines an effective Hamiltonian and the associated quasienergies [30]. Despite the apparent similarity to static systems, Floquet systems enable the realization of exotic phases that cannot be achieved in equilibrium, such as anomalous topological insulators [31,32] and time crystals [33,34]. The key idea of our proposal is an emerging topological structure in unitary operators associated with the periodicity of quasienergies [31]. We here show that a driving protocol for a three-dimensional (3D) Thouless pump [31,35] given by a topological Floquet unitary operation realizes a single Weyl fermion in a 3D lattice system, thereby providing a platform to observe the CME. We demonstrate that chiral transport emerges under a topological 3D Floquet drive due to an applied synthetic magnetic field, leading to a Floquet realization of the CME. Our proposal can be implemented by using ultracold atomic gases, where the Thouless pump has been realized experimentally [36,37].

Furthermore, by exploiting the correspondence between anomalous gapless spectra and topological unitary operators, we provide a topological classification of Floquet unitary operators in the Altland-Zirnbauer symmetry classes. In general, a wide variety of lattice-prohibited band structures under given symmetries can be realized as gapless surface states of topological phases [38]. The impossibility of pure lattice realization of surface states is deeply connected with their symmetry-protected gaplessness via quantum anomalies [39–41]. We show that the classification of topological Floquet unitaries, which generically offer symmetry-protected gapless quasienergy spectra, coincides with that of gapless surface states of static topological insulators or superconductors (TIs/TSCs). This correspondence strongly suggests that one can realize any gapless surface states of TIs/TSCs as *bulk* quasienergy bands of Floquet systems, even though they cannot be realized in static lattice systems.

General strategy.—We first explain our strategy to obtain a lattice-prohibited band structure in a Floquet system. We consider a periodically driven system of non-interacting fermions on a lattice. A periodically driven system is characterized by a Floquet operator, which is defined as a time-evolution operator over one period [30]. Since a crystal momentum is a good quantum number, a Floquet operator defines a map from the Brillouin zone to a space of unitary matrices. To characterize topology of a Floquet operator, we assume that a Hilbert subspace of the system is mapped onto itself by the Floquet operator [31]. This condition is achieved by, e.g., (i) using generalized adiabaticity [31,42], in which a time evolution is restricted to a low-energy subspace due to a large separation between

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low and high energy bands, or (ii) some fine-tuning of a driving protocol [43]. We hereafter refer to the Hilbert subspace closed in the time evolution over one period as "lower" Floquet bands, which play a role similar to occupied bands of static insulators [31].

Let us denote the Floquet operator restricted to the lower Floquet bands by $U(\boldsymbol{k})$, where $\boldsymbol{k}$ is a crystal momentum. When $U(\boldsymbol{k})$ offers a topologically nontrivial map from the Brillouin zone to a unitary group $U(N)$ (N is the number of the lower Floquet bands), the lower Floquet bands possess gapless quasienergy spectra, since a gapped Floquet operator can continuously be deformed into a trivial unitary, e.g., $U(\boldsymbol{k}) = \boldsymbol{1}_N$ [31,44]. Since the gapless quasienergy spectra cannot be gapped out by a continuous deformation of the Floquet operator, a topological Floquet operator is expected to exhibit a topologically protected gapless band structure, such as the case of Weyl fermions. In fact, it has been shown [31,45] that a single chiral fermion, which is forbidden in a static one-dimensional lattice, can be realized with a Floquet operator that has a nontrivial winding number. Our strategy for the realization of a single Weyl fermion is to construct a driving protocol that gives a Floquet operator with a nontrivial topological number in a 3D lattice system.

Model.—Let us now proceed to a construction of a model with a topological Floquet operator. We consider spin-half fermions on a cubic lattice L_C with a sublattice structure in the third direction:

$$L_C \coloneqq \left\{ \left(m_1, m_2, \frac{m_3}{2} \right) \middle| m_1, m_2, m_3 \in \mathbb{Z} \right\}, \qquad (1)$$

where the sublattice $L_{\rm ev}$ ($L_{\rm od}$) corresponds to the sites with even (odd) m_3 [purple (light blue) points in Fig. 1(a)]. The lattice constant $a_{\rm lat}$ is set to be unity: $a_{\rm lat} = 1$. The key ingredient of our model is spin-selective Thouless pumps

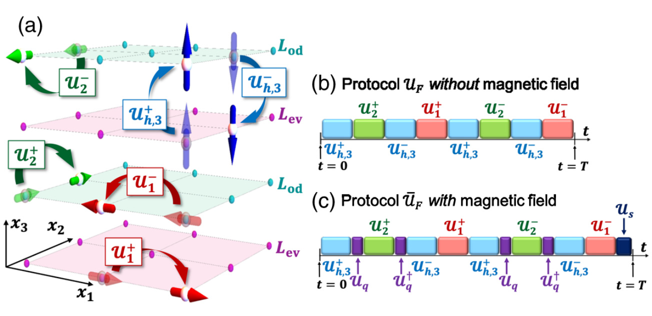

FIG. 1. (a) Schematic illustration of the fermion pump by time-evolution operators $\mathcal{U}_j^{\pm}$ ($j = 1, 2$) and $\mathcal{U}_{h,3}^{\pm}$ given in Eqs. (2) and (3) in the 3D lattice with sublattice structures $L_{\rm ev}$ and $L_{\rm od}$ in the third direction. Thick arrows show the spin directions of fermions. (b),(c) Driving protocols of the model (b) without and (c) with a magnetic field. The time-evolution operator $\mathcal{U}_q$ describes a sudden switch-on and switch-off of a quadrupole potential, and $\mathcal{U}_s$ describes hopping in the third direction with a Hamiltonian H_s.

[31,35,43] whose time-evolution operators $\mathcal{U}_j^{\pm}$ ($j = 1, 2$) are given by

$$\mathcal{U}_j^{\pm} \coloneqq \sum_{\boldsymbol{x}\alpha,\beta} [(P_j^{\pm})^{\alpha\beta} c^\dagger_{\boldsymbol{x}\pm\boldsymbol{e}_j,\alpha} c_{\boldsymbol{x},\beta} + (P_j^{\mp})^{\alpha\beta} c^\dagger_{\boldsymbol{x},\alpha} c_{\boldsymbol{x},\beta}], \qquad (2)$$

where $\boldsymbol{x} = (x_1, x_2, x_3) \in L_C$ denotes the lattice site, $\boldsymbol{e}_j$ is a unit vector in the x_j direction, and $c_{\boldsymbol{x}} = (c_{\boldsymbol{x},\uparrow}, c_{\boldsymbol{x},\downarrow})$ is the annihilation operator of a fermion with spin α ($\uparrow$ or $\downarrow$) at site $\boldsymbol{x}$. The matrix $P_j^{\pm} \coloneqq (\sigma_0 \pm \sigma_j)/2$ is a projection operator on a spin state $\sigma_j = \pm 1$, with σ_0 and σ_j ($j = 1, 2, 3$) being the 2×2 identity matrix and the Pauli matrices. From the projective nature of $P_j^{\pm}$, under the pump $\mathcal{U}_j^{+}$ ($\mathcal{U}_j^{-}$), fermions in a spin state $\sigma_j = +1$ (-1) are displaced by one lattice site in the positive (negative) x_j direction, while fermions in a spin state $\sigma_j = -1$ ($+1$) are not, thereby achieving spin-selective transport [see red and green arrows in Fig. 1(a)]. We also introduce spin-selective Thouless pumps $\mathcal{U}_{h,3}^{\pm}$ which displace fermions by a half lattice site in the x_3 direction:

$$\mathcal{U}_{h,3}^{\pm} \coloneqq \sum_{\boldsymbol{x},\alpha,\beta} [(P_j^{\pm})^{\alpha\beta} c^\dagger_{\boldsymbol{x}\pm(\boldsymbol{e}_3/2),\alpha} c_{\boldsymbol{x},\beta} + (P_j^{\mp})^{\alpha\beta} c^\dagger_{\boldsymbol{x},\alpha} c_{\boldsymbol{x},\beta}]. \qquad (3)$$

We note that fermions can be displaced between the unit cells by $\mathcal{U}_{h,3}^{\pm}$ [see blue arrows in Fig. 1(a)].

The driving protocol of our topological pump is constituted from eight successive applications of $\mathcal{U}_1^{\pm}$, $\mathcal{U}_2^{\pm}$, and $\mathcal{U}_{h,3}^{\pm}$ as shown in Fig. 1(b), where the total time-evolution operator $\mathcal{U}_F^{\rm wh}$ for the whole four bands over one cycle is given as follows [45]:

$$\mathcal{U}_F^{\rm wh} \coloneqq \mathcal{U}_1^{-}\mathcal{U}_{h,3}^{-}\mathcal{U}_2^{-}\mathcal{U}_{h,3}^{+}\mathcal{U}_1^{+}\mathcal{U}_{h,3}^{-}\mathcal{U}_2^{+}\mathcal{U}_{h,3}^{+} = \sum_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}} V^{\rm wh}(\boldsymbol{k}) c_{\boldsymbol{k}}, \qquad (4)$$

where $\boldsymbol{k} = (k_1, k_2, k_3)$ is the crystal momentum. Then, the Floquet operator $V^{\rm wh}(\boldsymbol{k})$ is decomposed into two 2×2 matrices: $V^{\rm wh}(\boldsymbol{k}) = U(\boldsymbol{k}) \oplus U^H(\boldsymbol{k})$, where

$$U(\boldsymbol{k}) \coloneqq U_1^{-}(k_1) U_{h,3}^{-}(k_3) U_2^{-}(k_2) U_{h,3}^{+}(k_3) \times U_1^{+}(k_1) U_{h,3}^{-}(k_3) U_2^{+}(k_2) U_{h,3}^{+}(k_3), \qquad (5)$$

and $U^H(\boldsymbol{k}) \coloneqq U(k_1, k_2, k_3 - 2\pi)$, $U_j^{\pm}(k) \coloneqq P_j^{\pm} e^{-ik} + P_j^{\mp}$ and $U_{h,3}(k) \coloneqq U_3(k/2)$ represent the Floquet operators of the spin-selective Thouless pumps. Here we focus on $U(\boldsymbol{k})$ as a Floquet operator of lower Floquet bands. A straightforward calculation shows that $U(\boldsymbol{k})$ stays the constant value $-\sigma_0$ if $\boldsymbol{k}$ belongs to the boundary of the Brillouin zone $\mathbb{T}^3 \coloneqq [-\pi, \pi]^3$ and hence satisfies the periodic boundary condition on $\mathbb{T}^3$.

Let $h_{\rm eff}(\boldsymbol{k})$ be an effective Hamiltonian defined by $U(\boldsymbol{k}) \eqqcolon \exp[-ih_{\rm eff}(\boldsymbol{k})]$, where the driving period T is

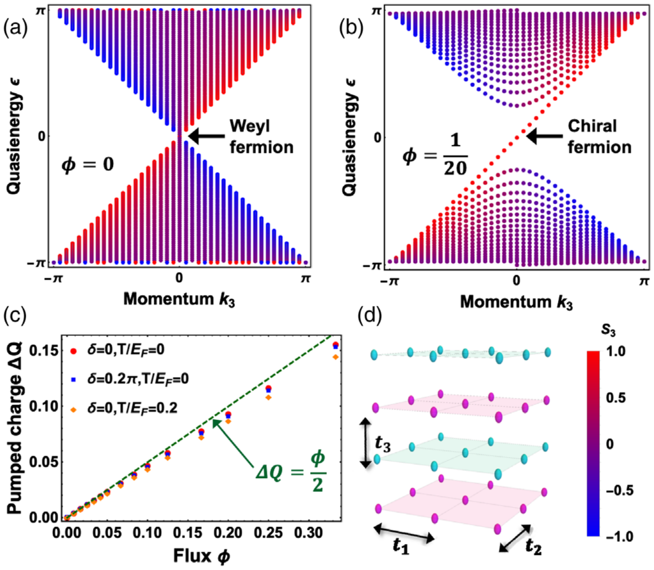

FIG. 2. (a),(b) Quasienergy spectra of $\bar{U}(0,k_3)$ with flux (a) $\phi=0$ and (b) $\phi=1/20$. The color represents the expectation value S_3 of σ_3 for each eigenstate according to the gauge shown on the right. (c) Pumped charge ΔQ as a function of flux ϕ for the initial state (8) at half-filling and zero temperature without (red) and with (blue) spin-mixing perturbation. The orange points are the results at finite temperature $T=0.1E_F$ [E_F is the Fermi energy]. (d) Hopping amplitudes t_i for preparing the initial state.

set to unity. Since the quasienergies $\epsilon(\boldsymbol{k})$ [eigenvalues of $h_{\rm eff}(\boldsymbol{k})$] satisfy $\cos[\epsilon(\boldsymbol{k})]=\mathrm{Tr}[U(\boldsymbol{k})]/2$, $\epsilon(\boldsymbol{k})=\pm\cos^{-1}[2\cos^2(k_1/2)\cos^2(k_2/2)\cos^2(k_3/2)-1]$ follows from Eq. (5). Therefore, $\epsilon(\boldsymbol{k})$ has only one gapless point at $\boldsymbol{k}=0$. Since $U(\boldsymbol{k})$ is expanded around $\boldsymbol{k}=0$ as $U(\boldsymbol{k})\approx\sigma_0-i\boldsymbol{k}\cdot\boldsymbol{\sigma}$ from $U_j^{\pm}(k)\approx\sigma_0\mp iP_j^{\pm}k$ for $k\approx 0$, we have $h_{\rm eff}(\boldsymbol{k})\approx\boldsymbol{k}\cdot\boldsymbol{\sigma}$, which clearly indicates the presence of a single left-handed Weyl fermion. The presence and stability of this single Weyl fermion is protected by a nontrivial topology in $U(\boldsymbol{k})$. In fact, $U(\boldsymbol{k})$ achieves a topologically nontrivial map from $\mathbb{T}^3$ to $\mathrm{SU}(2)\cong S^3$ (3D sphere) with a unit winding number [69]:

$$W\coloneqq-\int\frac{d\boldsymbol{k}}{24\pi^2}\sum_{i,j,k=1}^{3}\epsilon^{ijk}\mathrm{Tr}[R_iR_jR_k]=1, \qquad (6)$$

where $R_i\coloneqq U(\boldsymbol{k})^\dagger\partial_{k_i}U(\boldsymbol{k})$. We note that our model should be distinguished in topology from the previous proposals for realizing Floquet Weyl semimetals [70–79], where the Weyl nodes always appear in pairs in accordance with the Nielsen-Ninomiya theorem.

Floquet chiral magnetic effect.—When a magnetic field is applied, a Weyl fermion shows chiral transport antiparallel to the applied magnetic field, a phenomenon known as the CME [20]. A magnetic field can be introduced in our model through the replacement of $\mathcal{U}_2^{\pm}$ in Eq. (5) with $\mathcal{U}_q^\dagger\mathcal{U}_2^{\pm}\mathcal{U}_q$, where $\mathcal{U}_q\coloneqq\exp(-i2\pi\phi x_1x_2)$ is the time evolution operator induced by a sudden switch-on and -off of a quadrupole potential [80,81]. Since $\mathcal{U}_q^\dagger U_2^{\pm}(k_2)\mathcal{U}_q=U_2^{\pm}(k_2-2\pi\phi x_1)$, the effective Hamiltonian near $\boldsymbol{k}=0$ is given by $h_{\rm eff}=(\boldsymbol{k}+\boldsymbol{A})\cdot\boldsymbol{\sigma}$ with $\boldsymbol{A}=(0,-2\pi\phi x_1,0)$, which describes a Weyl fermion under a magnetic field $\boldsymbol{B}=(0,0,-2\pi\phi)$. However, $\mathcal{U}_q$ couples the lower and higher Floquet bands. Therefore, to decouple them, we need an additional time evolution $\mathcal{U}_s$ with duration τ_s under a Hamiltonian $H_s=J_s\sum_{\boldsymbol{x}}(ic_{\boldsymbol{x}+(\boldsymbol{e}_3/2)}c_{\boldsymbol{x}}+\mathrm{H.c.})$ with $4J_s\tau_s=\pi\phi$ at the end of the cycle [see Fig. 1(c)]. Since k_2 and k_3 remain as good quantum numbers, the Floquet operator $\bar{U}_F$ acting on the lower Floquet band is decomposed into a set of one-dimensional lattice models as $\bar{U}_F=\sum_{k_2,k_3}\bar{U}(k_2,k_3)$ [45]. Figures 2(a) and 2(b) show the quasienergy spectra of $\bar{U}(0,k_3)$ without ($\phi=0$) and with ($\phi=1/20$) a magnetic field, respectively, where the color of each point represents the spin polarization $S_3\coloneqq\langle u_a(k_3)|\sigma_3|u_a(k_3)\rangle$ of the eigenstate $|u_a(k_3)\rangle$. Because of the flux ϕ, the Landau gap with size $2\omega_L\coloneqq2\sqrt{2B_s}\approx1.6$ opens near the Weyl point at $k_3=0$, and a spin-polarized chiral fermion emerges inside the gap.

The chiral dispersion emerging under a magnetic field produces a current parallel to the field, leading to a Floquet realization of the CME. To see this, we calculate the amount of charge ΔQ pumped during one period using $\Delta Q\coloneqq\int_0^T dtJ_3(t)$, where $J_3(t)\coloneqq\int_{-\pi}^{\pi}(dk_3/2\pi)\times\mathrm{Tr}[\rho_t\partial_{k_3}H(k_3,t)]$ is a current parallel to the magnetic field, ρ_t is the density matrix at time t, and $H(k_3,t)$ is the time-dependent Hamiltonian. As shown in Ref. [31], the pumped charge can be rewritten in terms of the quasienergy $\epsilon_{k_2,b}(k_3)$ of $\bar{U}(k_2,k_3)$ [b specifies a Landau level] as

$$\Delta Q=\sum_{k_2,b}\int_{-\pi}^{\pi}\frac{dk}{2\pi}f_{k_2,b}(k_3)\frac{\partial\epsilon_{k_2,b}(k_3)}{\partial k_3}, \qquad (7)$$

where $f_{k_2,b}(k_3)\coloneqq\langle k_3,k_2,b|\rho_0|k_3,k_2,b\rangle$ is a distribution function of the Floquet eigenstates $|k_3,k_2,b\rangle$ in the initial state. As an initial state, we here take an equilibrium state under a Hamiltonian $H_0=\sum_{\boldsymbol{q},\alpha}\epsilon_0(\boldsymbol{q})c_{\boldsymbol{q},\alpha}^\dagger c_{\boldsymbol{q},\alpha}$ on the lattice L_C with dispersion relation $\epsilon_0(\boldsymbol{q})\coloneqq-t_3\cos(q_3/2)-t_1\cos q_1-t_2\cos q_2$ [$t_i>0$ is the hopping amplitude along the x_i direction, see Fig. 2(d)],

$$\rho_0=\sum_{\boldsymbol{q},\alpha=\uparrow,\downarrow}f_{\rm FD}(\boldsymbol{q})|\boldsymbol{q},\alpha\rangle_0\langle\boldsymbol{q},\alpha|_0, \qquad (8)$$

where $f_{\rm FD}(\boldsymbol{q})$ is the Fermi-Dirac distribution function with Fermi energy E_F, and $|\boldsymbol{q},\alpha\rangle_0$ is the Bloch state with momentum $\boldsymbol{q}(\in[-\pi,\pi]^2\times[-2\pi,2\pi])$ and spin α. Note that this thermal Fermi gas was used for detecting a static topological phase [82–84]. By tuning the lattice parameters to be $t_1,\ t_2\ll t_3$, we can make the distribution $f_{k_2,b}(k_3)$

TABLE I. Tenfold-way topological classification of Floquet operators for spatial dimension $d = 0, 1, 2, 3$. The Floquet single Weyl fermion in Eq. (5) corresponds to class A in $d = 3$.

Class	$d = 0$	$d = 1$	$d = 2$	$d = 3$
A	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	0	0	0	$2\mathbb{Z}$
BDI	$\mathbb{Z}$	0	0	0
D	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
DIII	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
AII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
CII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
C	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
CI	0	0	$2\mathbb{Z}$	0

populated almost only on the lower Floquet bands, as shown below.

In Fig. 2(c), we show the calculated pumped charge ΔQ for a half-filling and zero-temperature initial state with $t_1 = t_2 = t_3/3$. The obtained value close to $\phi/2$ [green dashed line in Fig. 2(c)] actually has a topological origin. In the limit t_1, $t_2 \ll t_3$, only the lower Floquet bands are occupied, i.e., $f_{\mathrm{FD}}(\boldsymbol{k}) = 1$, and hence

$$f_{k_2,b}(k_3) = \sum_{\boldsymbol{q},\alpha} |\langle k_3, k_2, b|\boldsymbol{q}, \alpha\rangle|^2 = 1. \tag{9}$$

Then, ΔQ in Eq. (7) reduces to the sum of the one-dimensional winding number divided by the number of Landau levels $N_L = 2L_1$ [L_1 is the number of sites along the x_1 direction]. Since we have the (ϕL_1)-chiral bands due to the flux ϕ, each of which has the winding number $+1$, we obtain $\Delta Q = \phi L_1/N_L = \phi/2$. We emphasize that this quantized chiral current cannot arise in usual Floquet-Weyl semimetals with topologically trivial $U(\boldsymbol{k})$ because left- and right-handed Weyl fermions appear with equal numbers in accordance with the Nielsen-Ninomiya theorem. The quasienergy band in our setup, in contrast, hosts a single-chirality Weyl fermion without a partner of opposite chirality within a single band, enabling us to realize the maximally imbalanced population with only one chiral component being occupied. Although the CME is a many-body phenomenon originating from the chiral imbalance, a similar effect can be observed in the single-particle dynamics [45].

The chiral current in the Floquet CME is robust against perturbations of the model due to the topological stability of the single Weyl point protected by the 3D winding number W. The blue points in Fig. 2(c) show the pumped charge under a modified protocol with imperfect spin-selective Thouless pumps, where $\mathcal{U}_j^{\pm}$ $(j = 1,\ 2,\ 3)$ is replaced by $\mathcal{U}_j^{\pm} e^{-i\delta\sigma_l}$ $(l = j+1 \bmod 3)$. Although $\mathcal{U}_j^{\pm}$ is neither spin preserving nor spin selective due to the spin-mixing term $e^{-i\delta\sigma_l}$, ΔQ is almost unaffected. Furthermore, we confirm that ΔQ persists even at finite temperature as shown by the orange points in Fig. 2(c).

Classification of gapless Floquet states.—In static topological insulators, symmetries dramatically enrich the classification of insulators as well as that of their gapless surface states, which cannot be realized in bulk lattices under the symmetry constraint [38]. This fact naturally motivates us to topologically classify Floquet unitary operators under various symmetries. As shown below, the symmetry-protected Floquet unitaries offer a wide range of lattice-prohibited band structures under symmetries that include the single Weyl fermion with the topological Floquet unitary as an example.

Let us take a Floquet operator $U(\boldsymbol{k}) \in U(N)$ given by some unitary matrix. We here consider three symmetries in the Altland-Zirnbauer classes [85,86]: time-reversal symmetry $\Theta H(\boldsymbol{k}, t)\Theta^{-1} = H(-\boldsymbol{k}, T - t)$, particle-hole symmetry $CH(\boldsymbol{k}, t)C^{-1} = -H(-\boldsymbol{k}, t)$, and chiral symmetry $\Gamma H(\boldsymbol{k}, t)\Gamma^{-1} = -H(\boldsymbol{k}, T - t)$. In terms of the Floquet operators, these symmetries are expressed as $\Theta U(\boldsymbol{k})\Theta^{-1} = U^{\dagger}(-\boldsymbol{k})$, $CU(\boldsymbol{k})C^{-1} = U(-\boldsymbol{k})$, and $\Gamma U(\boldsymbol{k})\Gamma^{-1} = U^{\dagger}(\boldsymbol{k})$ [31]. We allow any continuous deformation of Floquet operators which respect the symmetry of the system, and classify their stable equivalence classes according to the K theory [85,87]. Note that we do *not* assume energy gaps of the quasienergy band. Then, the classification of the unitary matrices can be performed in a manner similar to the classification of "unitary loops" for Floquet TIs/TSCs [88,89]. Since the derivation is parallel to Refs. [88,89], we here outline the general idea and give the full derivation in the Supplemental Material [45]. We define a Hermitian matrix $H_U(\boldsymbol{k})$ by

$$H_U(\boldsymbol{k}) = \begin{pmatrix} 0 & U(\boldsymbol{k}) \\ U^{\dagger}(\boldsymbol{k}) & 0 \end{pmatrix}, \tag{10}$$

which satisfies $H_U(\boldsymbol{k})^2 = \boldsymbol{I}_{2N}$ and thus has eigenvalues ± 1. Note that in the case of the classification of Floquet TIs/TSCs, the unitary matrix is taken as $U(\boldsymbol{k}, t) \coloneqq \mathcal{T}\exp[-i\int_0^t dt' H(\boldsymbol{k}, t')]$ instead of $U(\boldsymbol{k})$ [88]. Regarding $H_U(\boldsymbol{k})$ as a "Hamiltonian" and identifying its symmetry class, we can show that the classification of $U(\boldsymbol{k})$ is equivalent to that of $H_U(\boldsymbol{k})$ given by some K groups of static TIs/TSCs. Using the K-group isomorphism between different spatial dimensions [85,87], we find that the K group of Floquet operators of a symmetry class in d dimensions is given by that of static TIs/TSCs of the same symmetry class in $(d+1)$ dimensions. Since the latter is equivalent to the classification of d-dimensional gapless surface states through the bulk-boundary correspondence, we arrive at the conclusion that the classification of d-dimensional gapless Floquet states is equivalent to that of d-dimensional gapless surface states of TIs/TSCs. The final result is summarized in Table I. This result strongly suggests that the gapless surface states of TIs/TSCs, which cannot have any pure lattice realization without bulk, can be realized in bulk quasienergy spectra of periodically driven lattice systems. In fact, a single Weyl fermion presented in this Letter corresponds to a surface state of a four-dimensional topological insulator [90] and to class A in $d = 3$ in Table I. It merits further study to explicitly construct examples of Floquet operators in other symmetry classes.

Summary.—We have presented a periodically driven 3D lattice system that exhibits a single Weyl fermion in a quasienergy spectrum, thereby demonstrating a Floquet version of the CME. Our proposal utilizes the topology of the Floquet unitary. While the mathematical formula of the 3D winding number (6) was presented in a seminal work [31], its physical consequence and concrete realization had remained elusive. We have resolved this problem and provided a generalization to a topological classification of Floquet operators in the Altland-Zirnbauer symmetry classes. We expect that the unique topological structure arising from unitary operators will serve as a useful guideline for designing nonequilibrium systems free from the limitations of static phases of matter.

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