

Unveiling Electronic Correlation and the Ferromagnetic Superexchange Mechanism in the van der Waals Crystal CrSiTe₃

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The recent discovery of intrinsic ferromagnetic order in the atomically thin van der Waals crystal CrXTe₃ ($X = \text{Si}, \text{Ge}$) stimulates intensive studies on the nature of low-dimensional magnetism because the presence of long-range magnetic order in two-dimensional systems with continuous symmetry is strictly prohibited by thermal fluctuations. By combining advanced many-body calculations with angle-resolved photoemission spectroscopy we investigate CrSiTe₃ single crystals and unveil the pivotal role played by the strong electronic correlations at both high- and low-temperature regimes. Above the Curie temperature (T_c), Coulomb repulsion (U) drives the system into a charge transfer insulating phase. In contrast, below T_c the crystal field arranges the Cr-3d orbitals such that the ferromagnetic superexchange profits, giving rise to the bulk ferromagnetic ground state with which the electronic correlations compete. The excellent agreement between theory and experiment establishes CrSiTe₃ as a prototype low-dimensional crystal with the cooperation and interplay of electronic correlation and ferromagnetism.

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Introduction.—The manipulation of long-range magnetic order in low-dimensional systems is an intriguing challenge holding great promise for future spintronic applications. Experimental progress over the past few years has made it possible to test the fundamental concept of magnetic correlations related to the motion of electrons in low dimensions. Recently, several van der Waals (vdW) materials (e.g., CrGeTe₃/CrSiTe₃, CrI₃/CrBr₃) have been found to host ferromagnetism (FM) down to the monolayer limit and to demonstrate a thickness dependence of their transition temperatures [1–6]. As spontaneous magnetization of uncompensated electron spins is hard to achieve particularly in the low dimension limit due to the Mermin-Wagner theorem [7], the experimental discovery of ferromagnetism in these vdW materials stimulates a renewed interest in low-dimensional magnetism.

An intriguing fact about these transition-metal compounds is that they are natural hosts of both electronic correlations and magnetism. Their interplay is often the source of many exotic phenomena, such as a Mott insulator [8,9], high temperature superconductivity [10–16] and colossal magnetoresistance [17,18]. After the successful demonstration of ferromagnetism, the nature of the charge

insulating state and the FM coupling mechanism of these vdW materials were urged to be clarified. Furthermore, at low-temperature the strength of the spin-orbit coupling (SOC) inherited from Te- p bands becomes comparable to the spin couplings. Thus, the presence of different competing degrees of freedom (d.o.f.), i.e., electronic correlation, magnetic superexchange and SOC, and their relations with the robustness of low-dimensional ferromagnetism poses a fundamental quest to be resolved.

In order to address this question, we grew high-quality CrSiTe₃ single crystal and performed high-resolution angle-resolved photoemission spectroscopy (ARPES). By performing systematic photon-energy and polarization dependence measurements we were able to identify the full set of electronic structure near the Fermi level. After comparing to the advanced many-body calculations, we reveal the correlated nature of CrSiTe₃ at both high and low-temperature regimes with the electronic structure further renormalized by the SOC. The excellent agreement between theory and experiment establishes CrSiTe₃ as a Mott-type FM insulator.

Results.—A. *Sample preparation and magnetic properties.*—CrSiTe₃ single crystals were synthesized using the flux method (see Supplemental Material for details [19]).

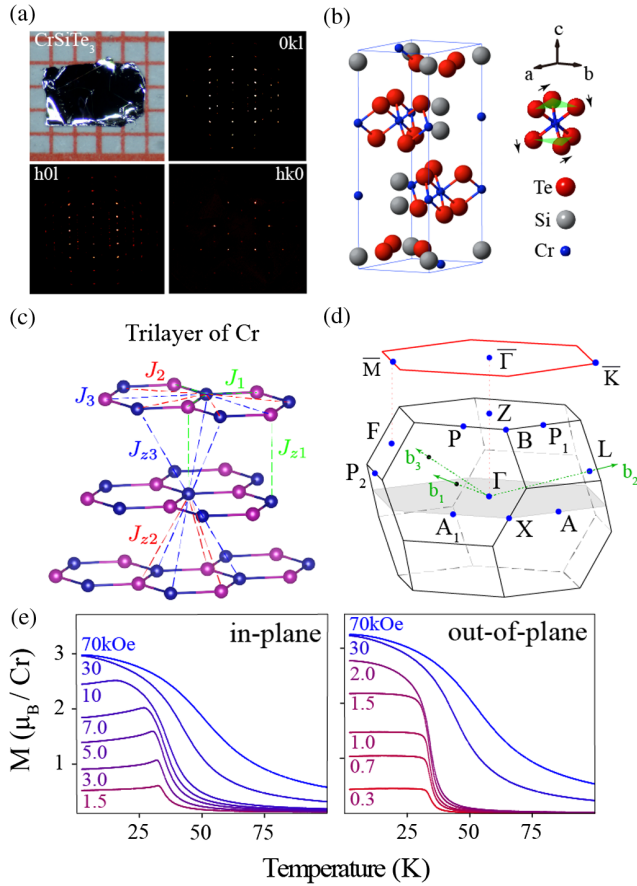


FIG. 1. Crystal structure and physical property of CrSiTe_3 . (a) Photo of CrSiTe_3 single crystal and x-ray diffraction patterns, with indices of lattice planes labeled. (b) Unit cell of the crystal structure. Each Cr atom is surrounded by six Te atoms distorted along the directions indicated by the arrows. (c) In each unit cell, inequivalent Cr atoms form three layers of effective honeycomb stacking along the c direction. The intralayer and interlayer spin couplings used in the effective Heisenberg model are illustrated as J_1, J_2, J_3 , and J_{z1}, J_{z2}, J_{z3} , respectively. (d) The Brillouin zone (BZ) and the high symmetry points. The $k_z = 0$ plane intersects with the BZ boundary at arbitrary k points. (e) Temperature dependent in-plane (left panel) and out-of-plane (right panel) magnetic moment per Cr atom under different magnetic fields.

The high quality of the crystal is verified by the x-ray diffraction (XRD) measurements shown in Fig. 1(a). The rhombohedral crystal structure of CrSiTe_3 is presented in Fig. 1(b). A Cr-Te form edge shared octahedron layer-stacking along the c direction. Inside each layer, connecting nearest-neighbor Cr atoms gives rise to an effective honeycomb lattice which displaces its center from those of the layers above and below it as shown in Fig. 1(c). The six Te atoms around the central Cr atom form an octahedron which is slightly distorted along the direction indicated by the arrows in Fig. 1(b), making the local point symmetry, known as D_{3d} , slightly lower than cubic symmetry.

The magnetic nature is characterized by the temperature dependent magnetic susceptibility of bulk CrSiTe_3 under

parallel (in-plane) and perpendicular (out-of-plane) magnetic field (with respect to the a - b plane of the sample) with the field varying from 1.5 to 70 kOe [Fig. 1(e)]. As clearly visualized, a magnetic phase transition occurs at $T_c \sim 33$ K, which is consistent with other reports [30–32]. We notice the λ shape in the susceptibility curve below T_c with the in-plane field coming from the magnetic anisotropy and suggesting the FM points out of plane [33]. The layered CrSiTe_3 crystal could be easily cleaved along its (111) directions. To prepare clean CrSiTe_3 surfaces for surface sensitive x-ray photoelectron spectroscopy (XPS) and ARPES measurements, we cleaved the single crystal CrSiTe_3 inside the vacuum chamber prior to the measurements. After cleaving, XPS measurement shows the high quality of the sample by seeing the characteristic core level peaks from Cr, Si, and Te (see Fig. S1a in the Supplemental Material [19]) while the broad Fermi surface mapping (see Fig. S1c [19]) covering multiple BZs confirmed the (111) cleavage plane, which allows us to carry out the study below.

B. High-temperature charge transfer insulating phase.— Figure 2(a) (right) presents the electronic structure measured by ARPES in the paramagnetic (PM) phase. From the broad range dispersion along the A_1 - Γ - A direction [Fig. 2(a)(ii)] we could identify both dispersive feature (near the valence band maximum, or VBM and down to 6 eV below) and rather flat bands (~ 1.5 eV below the VBM). The dispersive features are found to be the p orbitals of Te and Si while the flat features are coming from the Cr- d bands. By performing photon-energy and polarization dependent measurement, we obtained the full 3D electronic structure of CrSiTe_3 shown in Figs. 2(b) and 2(c), where several holelike bands near the VBM were detected and one of them shifts 400 meV down when moving from the Γ to the Z point (More details of the k_z evolution could be found in SI.II [19]). The absence of spectral weight above the observed valence band top [as indicated by the integrated energy distribution curve plot by the side of Figs. 2(a)(ii) and 2(c)(i),(iv)] suggests the semiconducting or insulating nature of CrSiTe_3 in the PM phase. Further optical absorption measurement provides an indirect gap size ~ 0.4 eV and direct gap size ~ 1.2 eV [31,34]. Both experimental shreds of evidence suggest that, before the onset of long-range FM, electrons have been fully localized, which is in sharp contrast to first-principles calculations that will be discussed below.

In the absence of FM, CrSiTe_3 is predicted, by *ab initio* calculations, to be metal as evidenced by the bands crossing the Fermi level [Fig. S3(a)]. Inspecting the nonmagnetic electronic band structure where the energy bands are projected to Cr- d orbitals under D_{3d} crystal environment, we find the d_{z^2} and $d_{x^2-y^2}$ distribute mainly at the Fermi level, while $d_{zx} + d_{yz}$ occupy also at the conduction bands between 1 and 2 eV. The Cr- d bands, at the Fermi level, are confined in a narrow energy range within 1 eV, which strongly favors the interactions between electrons. Further,

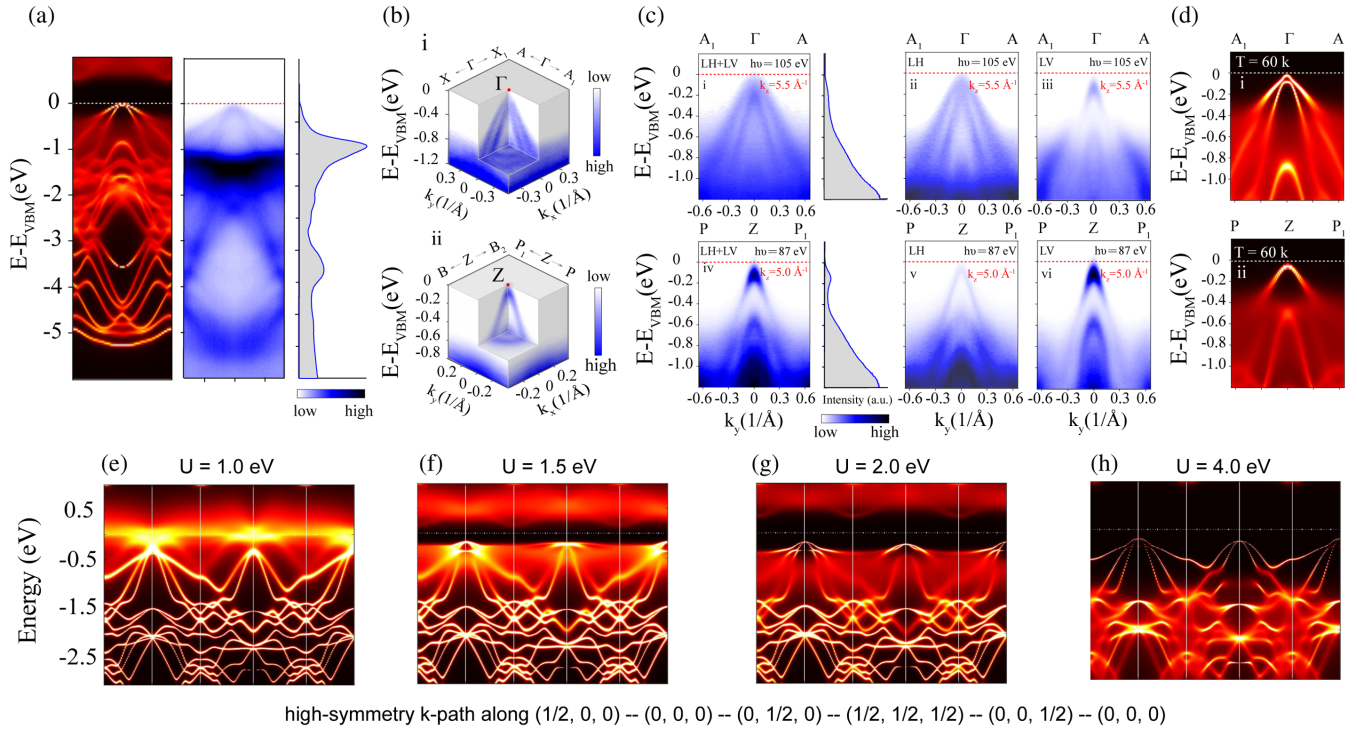


FIG. 2. Electronic band structure of paramagnetic CrSiTe₃. (a) The dispersion calculated with eDMFT + SOC at 60 K with $U = 4.0$ eV along A_1 - Γ - A k path (i) and the corresponding ARPES measurement at 200 K (ii). (b) Three-dimensional electronic structure measured at the Γ (i) and Z (ii) points. (c) High symmetry cuts along the A_1 - Γ - A direction measured with linear horizontal (ii) and linear vertical (iii) polarized photons, and their superposition (i). (iv)–(vi) Same as (i)–(iii), but along the P - Z - P_1 direction. Data of (b) and (c) are measured at room temperature. (d) The calculated band structure from eDMFT + SOC around Γ and Z in the energy range, same as in (c). (e)–(h). Correlated spectra of CrSiTe₃ calculated by eDMFT with different U value at 230 K, i.e., (e) 1.0, (f) 1.5, (g) 2.0, and (h) 4.0 eV. In these calculations SOC is neglected for simplicity.

given the fact that the valence electrons of Cr-ion are in their d^3 configuration and there are 3 bands/Cr-ion occupied [we note the saturated magnetic moment is close to $3 \mu_B/\text{Cr}$, see Fig. 1(e)], the two necessary ingredients for the Mott transition are both satisfied, i.e., the strong Coulomb interactions from the narrow d bands and the half-filled condition. The insulating states at high temperature are, thus, likely to be a Mott state.

To investigate this hypothesis and to characterize the nature of the high-temperature charge gap, we employed the DFT + dynamical mean-field theory (DMFT) as implemented in the embedded-DMFT package [35] to account for the local Coulomb interactions between the Cr- d electrons (see details of the eDMFT calculations in SI.III [19]). As shown in Figs. 2(e)–2(h), as a function of U , the correlated spectra of CrSiTe₃ gradually loses its weight at the Fermi level and a charge gap opens at the U value between 1.0 and 1.5 eV. Upon further increasing U values, the correlated Cr- d bands pass through the dispersive valence p bands and move further to high energies. The low-energy p bands are less influenced by the renormalization of Cr- d bands as the p - d hybridization is not strong in this system evidenced from the Wannier orbital plot shown in the Supplemental Material, Fig. S4, where the projected orbitals nicely preserve their atomic symmetries [19].

We note that, though immune to electronic correlations, the low-energy p bands can be strongly modified by SOC. The cooperation of interaction and SOC included in an eDMFT + SOC calculation significantly changes the upper valence bands, resulting in a nice agreement with the ARPES measured electronic structure [see Figs. 2(a), 2(c), and 2(d) for comparison].

Based on the experiment-theory agreement, we conclude that the high-temperature CrSiTe₃ can be driven into a gapped phase solely by electronic correlations; i.e., no assistance from the long-range magnetic order is needed. However, the possible influence of short-range magnetic correlations cannot be excluded by our calculations. They are found to persist to 150 K in CrSiTe₃ [5,36], i.e., a temperature much higher than Curie temperature T_c . The charge gap is likely to result from the interplay of short-range magnetic correlations and electronic interactions, while our many-body calculations show that the nature of the charge gap is essentially determined by the correlation.

C. Mechanism of low-temperature ferromagnetic coupling.—We further elaborate on the low-temperature FM phase by studying the nature of the magnetic coupling. With the decreasing of temperature and the localization of the electrons, the charge d.o.f. become less active due to the formation of the energy gap. Meanwhile, the spin d.o.f.

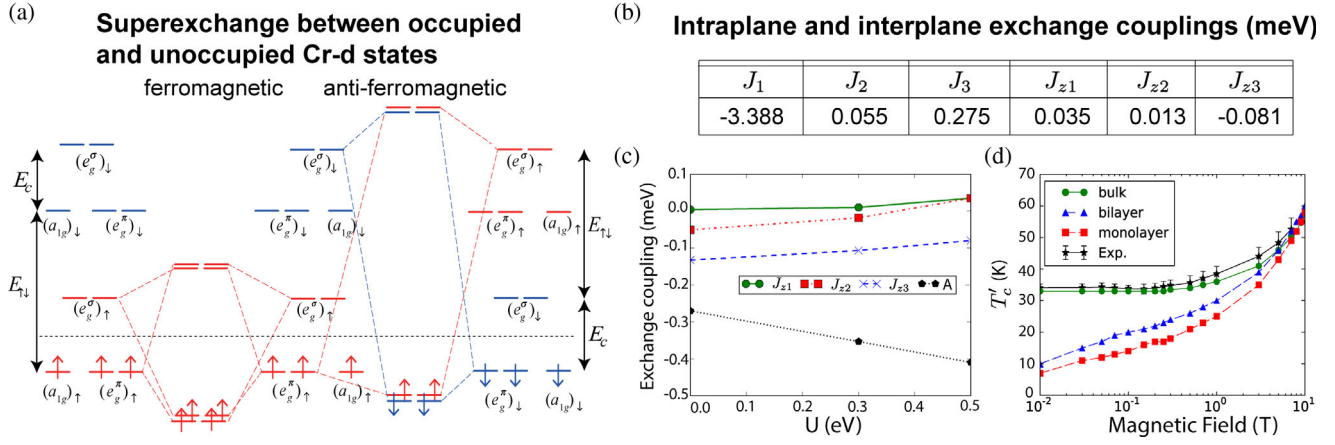


FIG. 3. (a) Superexchange process favouring the ferromagnetic coupling of the Cr local moments. (b) The value of exchange couplings determined for bulk CrSiTe₃. (c) The interlayer exchange couplings J_{z1} , J_{z2} , J_{z3} , and magnetic anisotropy strength A are sensitive to the choice of U parameters, which become antiferromagnetic with the increase of U value. However, the magnetic anisotropy A is enhanced with larger interactions. (d) The predicted transition temperatures T'_c under external magnetic field. Here the bulk exchange coupling parameters are used in monolayer and bilayer calculations; the obtained transition temperature decreases with the reduction of layer thickness.

come into play. First, it is clear that the FM coupling between Cr local magnetic moments is indirect, because T_c is much larger in CrGeTe₃ (~ 67 K) than in CrSiTe₃ (~ 33 K) despite, in the former, the distance between two neighboring Cr atoms is larger. Second, we identify two different superexchange processes, i.e., the exchange of $a_{1g} + e_g^\pi$ states with $a_{1g} + e_g^\pi$ and e_g^σ states, respectively. Here a_{1g} and e_g^π are the half-filled singlet and doublet states, while e_g^σ is the unoccupied doublet state. Only the exchange of $a_{1g} + e_g^\pi$ states with e_g^σ states leads to a ferromagnetic coupling between the Cr neighboring local moments. The energy levels of $a_{1g} + e_g^\pi$ and e_g^σ states, after taking hybridization into account, are plotted in Fig. 3(a). If the two neighboring local moments are FM coupled, the lowest energy levels are given by the bonding states between the e_g^π and e_g^σ states with the same spin projections. Given the hybridization strength Δ , four of the six electrons at two neighboring Cr atoms will stay at lower energy levels $-6\Delta^2/E_c$ with E_c being the strength of crystal field splitting. The two remaining a_{1g} electrons do not participate in this superexchange process. Otherwise, if the local moments are AFM coupled, the bonding states between neighboring e_g^π and e_g^σ states only gain smaller energy reduction $-6\Delta^2/(E_c + E_{\uparrow\downarrow})$, where $E_{\uparrow\downarrow}$ is the spin polarization amplitude. Consistent with the Goodenough-Kanamori rule [37,38], we conclude that, in addition to the magnetic anisotropy, the long-range FM in CrSiTe₃ is mainly derived from the superexchange between the e_g^π and e_g^σ states, which is strongly competed with by the AFM coupling resulting from e_g^π and e_g^π exchange (see Supplemental Material, Fig. S6 for more details [19]).

After clearly demonstrating the high-temperature correlated phase and the low-temperature FM superexchange

mechanism, we want to further discuss the competition of electronic correlations with the long-range FM order. Figure 3(b) lists the calculated spin coupling strengths by employing the four-states method [39] within *Dudarev's* DFT + U scheme [40]. Compared to the values reported for CrGeTe₃ [1], the absolute values of all couplings are reduced in CrSiTe₃, with only the nearest neighbor coupling J_1 remaining ferromagnetic. Others become essentially antiferromagnetic. Furthermore, we find that these couplings are sensitive to the value of Coulomb interactions. All three interlayer couplings increase towards AFM with the increase of U as shown in Fig. 3(c). Further increasing U CrSiTe₃ transforms from FM to a new magnetic state with FM intralayer coupling and AFM interlayer coupling. The strong competition between electronic Coulomb interaction and magnetic superexchange coupling, in particular at low-dimensions with an enhanced electronic repulsion, may generate larger magnetic anisotropy, leading to the emergence of new magnetic states and increased transition temperatures [41].

With the obtained bulk exchange coupling strength, following Ref. [1] we further calculate the transition temperature T'_c of CrSiTe₃ by solving the Heisenberg model with linear spin wave theory under the out-of-plane magnetic field. With finite magnetic field, the transition temperature T'_c is no longer the Curie temperature defined for the ferromagnetic phase transition. Nevertheless, it similarly defines the temperature above which the calculated magnetic moment drops to zero. T'_c for the bulk, bilayer, and monolayer CrSiTe₃ are shown in Fig. 3(d), which monotonically decreases with the decrease of magnetic field strength. As for bulk CrSiTe₃, a constant value at low field and a linear dependence on B of T'_c at high field are observed, which is highly consistent with the

measurement and, on the other hand, validates the theoretically obtained exchange coupling parameters shown in Fig. 3(b). In sharp contrast to the bulk case, bilayer and monolayer CrSiTe₃ display a magnetic field dependence of T'_c even at the low field limit. Thus, the low-dimensional CrSiTe₃ is more tunable by an external magnetic field.

Discussions.—The interplay of charge and spin d.o.f. is one of the fascinating mechanisms that has triggered many novel phenomena in many-body systems. Though both electronic correlations and magnetic ordering can drive MIT individually, they often coexist in transition-metal compounds, which makes the nature of the insulating gap hard to interpret. It is then highly interesting to see in CrSiTe₃, especially at temperatures above T_c , the two mechanisms are nicely separated, which allows us to characterize the nature of the charge gap. The narrow Cr- d bands around the Fermi level are strongly correlated leading to the Mott insulating phase above the Curie temperature. At the low-temperature regime, the charge d.o.f. give their way to the spin d.o.f. due to the formation of charge gap. The specific electronic structure is found to strongly favor the superexchange coupling between the e_g^π and e_g^σ states, leading to the ferromagnetic ground states of bulk CrSiTe₃. Compared to CrGeTe₃, the strong competition from electronic correlations significantly weakens the interlayer ferromagnetic coupling, making CrSiTe₃ a system residing at the border of FM and A-type AFM (i.e., intralayer ferromagnetic and interlayer antiferromagnetic) that waits for further experimental verification. The discovered correlated nature of CrSiTe₃ greatly enriches our understanding of low-dimensional magnetism and may provide a new engineering parameter to achieve high- T_c 2D magnetism.

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