Chiral Photons from Chiral Gravitational Waves

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We show that a parity-breaking *uniform* (averaged over all directions on the sky) circular polarization of amplitude $V_{00} \simeq 2.6 \times 10^{-17} \Delta\chi (r/0.06)$ can be induced by a chiral gravitational-wave (GW) background with a tensor-to-scalar ratio r and chirality parameter $\Delta\chi$ (which is ± 1 for a maximally chiral background). We also show, however, that a uniform circular polarization can arise from a realization of a nonchiral GW background that spontaneously breaks parity. The magnitude of this polarization is drawn from a distribution of root variance $\sqrt{\langle V_{00}^2 \rangle} \simeq 1.5 \times 10^{-18} (r/0.06)^{1/2}$, implying that the chirality parameter must be $\Delta\chi \gtrsim 0.12 (r/0.06)^{-1/2}$ to establish that the GW background is chiral. Although these values are too small to be detected by any experiment in the foreseeable future, the calculation is a proof of principle that cosmological parity breaking in the form of a chiral gravitational-wave background can be imprinted in the chirality of the photons in the cosmic microwave background. It also illustrates how a seemingly parity-breaking cosmological signal can arise from parity-conserving physics.

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The cosmic microwave background (CMB) is linearly polarized as a consequence of the anisotropic Thomson scattering of CMB photons that arises at linear order in the amplitude of cosmological perturbations [1]. This Thomson scattering does not, however, induce circular polarization. Recent work has shown that circular polarization can be generated, from primordial perturbations through postrecombination photon-photon scattering [2–7], at second order in the density-perturbation amplitude. Still, the *mean* value of the circular polarization, when averaged over the sky, is predicted to be zero. Since a nonzero circular polarization implies a particular handedness, the absence of a uniform circular polarization can be seen as a consequence of the absence of parity breaking in primordial density perturbations.

Here we show that a uniform circular polarization can arise if parity is broken in the form of a chiral primordial gravitational-wave (GW) background. Chiral GWs may arise if, for example, there is a Chern-Simons coupling of the inflaton to gravity [8], from gravity at a Lifshitz point [9], graviton self-couplings [10,11], gaugeflation and chromonatural inflation [12–15], Holst gravity [16], and in models that connect leptogenesis to primordial gravitational waves [17,18]. The circular polarization arises through interactions of linearly polarized CMB photons with CMB anisotropies along the line of sight to the surface of last scatter. The correlation between the photon anisotropy induced by the gravitational wave and the primordial linear polarization also induced by the gravitational wave leads, if the GW background is chiral, to a uniform circular polarization.

We also show, however, that a uniform circular polarization can arise as a statistical fluctuation—a cosmic variance—if the stochastic GW background is *not* chiral. Even if the expected value is zero, there will be some variation, in any given realization of the GW background, in the amplitudes of the right- and left-handed gravitational waves. The theory predicts that the uniform circular polarization V_{00} (where the 00 indices indicate the $l = 0$, $m = 0$ spherical-harmonic coefficient of the circular-polarization pattern) is selected from a distribution centered on $V_{00} = 0$ but with nonzero variance. As we show, though, such a nonzero value for the uniform circular polarization cannot arise from primordial density perturbations, and so a nonzero uniform value would indicate a GW background, even if not necessarily chiral.

Although the circular polarization induced by GWs has been discussed in terms of the photon-graviton scattering [19], we focus on the circular polarization induced through photon-photon scattering. This work builds upon a detailed analysis presented in Ref. [20] of the circular polarization induced by GWs. That work builds upon a recalculation [21], obtained with the TAM formalism [22,23], of circular polarization induced by photon-photon scattering, in Ref. [7], which itself extends several earlier analyses [2–4,6] of this effect. Note that the circular polarization induced through the photon-photon scattering is much larger than that induced through the photon-graviton scattering [20].

As discussed in that earlier work, circular polarization is induced through Faraday conversion as a linearly polarized light ray propagates through a medium with an anisotropic index-of-refraction tensor. The circular polarization $V(\hat{\boldsymbol{n}})$ in direction $\hat{\boldsymbol{n}}$ is

$$V(\hat{\boldsymbol{n}}) = \epsilon_{ac} P^{ab}(\hat{\boldsymbol{n}}) \Phi_b^c(\hat{\boldsymbol{n}}). \tag{1}$$

Here,

$$P_{ab}(\hat{\boldsymbol{n}}) = \frac{1}{\sqrt{2}} \begin{pmatrix} Q(\hat{\boldsymbol{n}}) & U(\hat{\boldsymbol{n}}) \\ U(\hat{\boldsymbol{n}}) & -Q(\hat{\boldsymbol{n}}) \end{pmatrix} \tag{2}$$

is the polarization tensor, whose components are the Stokes parameters $Q(\hat{\boldsymbol{n}})$ and $U(\hat{\boldsymbol{n}})$, and

$$\Phi_{ab}(\hat{\boldsymbol{n}}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_Q(\hat{\boldsymbol{n}}) & \phi_U(\hat{\boldsymbol{n}}) \\ \phi_U(\hat{\boldsymbol{n}}) & -\phi_Q(\hat{\boldsymbol{n}}) \end{pmatrix} \tag{3}$$

is a phase-shift tensor that describes the phase shifts induced by the index-of-refraction tensor. These are obtained as line-of-sight integrals,

$$\phi_{Q,U}(\hat{\boldsymbol{n}}) = \frac{2}{c} \int_0^{\chi_{\rm LSS}} \frac{d\chi}{1+z} \omega(\chi) n_{Q,U}(\hat{\boldsymbol{n}}\chi), \tag{4}$$

where z is redshift and $\chi_{\rm LSS}$ the comoving distance to the last-scattering surface. Here, n_Q and n_U are components, in a plane transverse to the line of sight, of the index-of-refraction tensor,

$$n_{ab} = \delta_{ab} + \frac{1}{2}(\chi_{e,ab} + \chi_{m,ab}) = \begin{pmatrix} n_I + n_Q & n_U + in_V \\ n_U - in_V & n_I - n_Q \end{pmatrix}. \tag{5}$$

Finally, ϵ_{ab} is the antisymmetric tensor on the 2-sphere.

The polarization and phase-shift tensors can both be expanded,

$$P_{ab}(\hat{\boldsymbol{n}}) = \sum_{lm} [P^E_{lm} Y^{\rm TE}_{(lm)ab}(\hat{\boldsymbol{n}}) + P^B_{lm} Y^{\rm TB}_{(lm)ab}(\hat{\boldsymbol{n}})], \tag{6}$$

$$\Phi_{ab}(\hat{\boldsymbol{n}}) = \sum_{lm} [\Phi^E_{lm} Y^{\rm TE}_{(lm)ab}(\hat{\boldsymbol{n}}) + \Phi^B_{lm} Y^{\rm TB}_{(lm)ab}(\hat{\boldsymbol{n}})], \tag{7}$$

in terms of tensor spherical harmonics $Y^{\rm TE}_{(lm)ab}(\hat{\boldsymbol{n}})$ and $Y^{\rm TB}_{(lm)ab}(\hat{\boldsymbol{n}})$ [24–26]. The expansion coefficients in Eqs. (6) and (7) are random variables chosen from a distribution of zero mean and variances $C_l^{XX} = \langle |X_{lm}|^2 \rangle$ for $X = P^E, P^B, \Phi^E, \Phi^B$. In the absence of parity breaking, there are, moreover, cross-correlations $C_l^{P^E\Phi^E}$ and $C_l^{P^B\Phi^B}$. If there is no parity breaking, then there is no cross-correlation between the E modes and the B modes—the two have opposite parity and any cross-correlation would imply a preferred handedness. Expressions for all the power spectra C_l^{XY} are given as integrals over the primordial gravitational-wave power spectra in Ref. [20].

The coefficients $V_{lm} = \int d\hat{\boldsymbol{n}} V(\hat{\boldsymbol{n}}) Y^*_{lm}(\hat{\boldsymbol{n}})$ in the spherical-harmonic expansion of the circular polarization can be expressed in terms of P_{lm} and Φ_{lm} as [20]

$$\begin{aligned} V_{lm} = & \sum_{l_1 m_1 l_2 m_2 ({\rm odd})} (P^E_{l_1 m_1} \Phi^E_{l_2 m_2} (iG^{l-m}_{l_1 m_1 l_2 m_2}) \\ & + P^B_{l_1 m_1} \Phi^B_{l_2 m_2} (iG^{l-m}_{l_1 m_1 l_2 m_2})) \\ & + \sum_{l_1 m_1 l_2 m_2 ({\rm even})} (P^E_{l_1 m_1} \Phi^B_{l_2 m_2} (-G^{l-m}_{l_1 m_1 l_2 m_2}) \\ & - P^B_{l_1 m_1} \Phi^E_{l_2 m_2} (-G^{l-m}_{l_1 m_1 l_2 m_2})), \end{aligned} \tag{8}$$

where the subscripts (odd) and (even) indicate summations over $l_1 + l_2 + l = \text{odd}$ and $l_1 + l_2 + l = \text{even}$, respectively. Here $G^{lm}_{l_1 m_1 l_2 m_2} = -\xi^{lm}_{l_1 -m_1, l_2 m_2} H^l_{l_1 l_2}$, where $\xi^{lm}_{l_1 m_1, l_2 m_2}$ and $H^l_{l_1 l_2}$ are defined in terms of Wigner 3-j symbols as

$$\xi^{lm}_{l_1 m_1 l_2 m_2} \equiv (-1)^m \sqrt{\frac{(2l_1+1)(2l+1)(2l_2+1)}{4\pi}} \times \begin{pmatrix} l_1 & l & l_2 \\ -m_1 & m & m_2 \end{pmatrix}, \tag{9}$$

$$H^l_{l_1 l_2} \equiv \begin{pmatrix} l_1 & l & l_2 \\ 2 & 0 & -2 \end{pmatrix}. \tag{10}$$

In Ref. [20] we considered anisotropies in the circular polarization and found that the circular polarizations induced by density (scalar metric) perturbations are much larger than those induced by tensor perturbations. However, scalar perturbations induce no *uniform* circular polarization V_{00}. We therefore focus here on V_{00}, as it provides a clean signature of tensor perturbations. Setting $l = m = 0$ and taking $G^{00}_{l_1 m_1 l_2 m_2} = \delta_{l_1 l_2} \delta_{-m_1 m_2} / \sqrt{4\pi}$, we express the uniform circular polarization as

$$V_{00} = \frac{1}{\sqrt{4\pi}} \sum_{lm} (P^E_{lm} \Phi^{B*}_{lm} - P^B_{lm} \Phi^E_{lm}). \tag{11}$$

Taking the expectation value, over all realizations of metric perturbations, we find,

$$\langle V_{00} \rangle = \frac{1}{\sqrt{4\pi}} \sum_{lm} (C_l^{P^E \Phi^B} - C_l^{P^B \Phi^E}). \tag{12}$$

Through calculations that parallel those in Ref. [20] and lead to the power spectrum $C_l^{P^E\Phi^B}$ induced by photon-photon scattering, we infer power spectra,

$$\begin{aligned} C_l^{P^E \Phi^B} = & 4\pi A \int \frac{{\rm d}k}{k} 2 \left(\frac{k^3}{2\pi^2} P^{({\rm TE,TB})}(k) \right) \\ & \times \int_0^{\eta_0} {\rm d}\eta g(\eta) (-\sqrt{6} \mathcal{P}^{(2)}(k,\eta)) \epsilon_l^{(2)} [k(\eta_0 - \eta)] \\ & \times \int_{\eta_{\rm lss}}^{\eta_0} {\rm d}\eta (1+z)^4 (\bar{a}^E_{2,2}(k,\eta)) \beta_l^{(2)} [k(\eta_0 - \eta)]. \end{aligned} \tag{13}$$

The coefficient A is inferred from the Euler-Heisenberg Lagrangian and provided in Ref. [7]. The second and third lines in Eq. (13) represent the transfer functions of P^E and Φ^B, respectively. In particular, $g(\eta)$ is the visibility function, $\mathcal{P}^{(2)}$ is the function defined in Ref. [27], $\bar{a}^E_{2,2}$ is the transfer function of the local E-mode moment induced by primordial perturbations, and $\epsilon_l^{(2)}$ and $\beta_l^{(2)}$ are the radial functions coming from the nature of the E mode and B mode, respectively (see Ref. [28] for detail). Here, $P^{(\mathrm{TE,TB})}(k)$ is defined as the power spectrum for the cross-correlation between the amplitudes $h^{k,\mathrm{TE}}_{(lm)}$ and $h^{k,\mathrm{TB}}_{(lm)}$,

$$\langle h^{k,\mathrm{TE}}_{(lm)}(h^{k',\mathrm{TB}}_{(lm)})^*\rangle = i\frac{(2\pi)^3}{k^2}\delta(k-k')P^{(\mathrm{TE,TB})}, \tag{14}$$

where $h^{k,\mathrm{TE}}_{(lm)}$ and $h^{k,\mathrm{TB}}_{(lm)}$ are the TE and TB modes in the TAM decomposition for the transverse-traceless metric perturbation. The i in Eq. (14) is canceled out by the i in front of $\beta_l^{(2)}$ in Eq. (41) in Ref. [20]. $C_l^{P^B\Phi^E}$ are also given by the same equation except for $\epsilon_l^{(2)} \leftrightarrow \beta_l^{(2)}$. The TE and TB modes have opposite parity, and so any nonzero correlation between them indicates parity breaking. Such a cross-correlation arises if parity is broken by a disparity in the amplitudes of the right- and left-circularly polarized gravitational waves. This can be seen by writing the amplitudes,

$$h^{k,\pm}_{(lm)} = \frac{1}{\sqrt{2}}(h^{k,\mathrm{TE}}_{(lm)} \mp i h^{k,\mathrm{TB}}_{(lm)}), \tag{15}$$

for the $\pm$ (R and L) helicity-basis TAM waves [cf., Eq. (14) in Ref. [20]]. Following Ref. [29], we define the chirality parameter $\Delta\chi$ through $P_\pm(k) = (1 \mp \Delta\chi)P_T(k)$, where $P_T(k)$ is the primordial GW power spectrum, and P_+ and P_- correspond to P_R and P_L in Ref [29], respectively. From this and Eq. (15), it follows that

$$P^{(\mathrm{TE,TB})}(k) = \Delta\chi P_T(k). \tag{16}$$

As a result, we find by numerical evaluation the uniform circular polarization, normalized by the physical units $T_{\mathrm{CMB}} = 2.7255$ K [30], to be

$$\begin{aligned}\langle V_{00}\rangle &= \sum_l \frac{2l+1}{\sqrt{4\pi}}(C_l^{P^E\Phi^B} - C_l^{P^B\Phi^E}) \\ &\simeq 2.6\times 10^{-17}\Delta\chi\left(\frac{r}{0.06}\right),\end{aligned} \tag{17}$$

for a gravitational-wave background with tensor-to-scalar ratio r ($\lesssim$0.06 [31,32]) and chirality parameter $\Delta\chi$. This numerical result is obtained assuming a scale-invariant power spectrum and chirality. Although the analysis of a non-scale-invariant spectrum is beyond the scope of this Letter, we do not expect any qualitative difference for non-scale-invariant spectra consistent, at the relevant length scales, with observational constraints.

So far, we have calculated the *expectation* value for V_{00}. However, the *observed* value of the circular polarization arises as the result of a single realization of a random field. The prediction is thus that V_{00} is selected from a random distribution, with some nonzero variance $\langle(\Delta V_{00})^2\rangle = \langle V_{00}^2\rangle - \langle V_{00}\rangle^2$, with an expectation value $\langle V_{00}\rangle$. Note that, in the absence of the parity breaking, $\langle(\Delta V_{00})^2\rangle = \langle V_{00}^2\rangle$ is satisfied. From Eq. (11), we find,

$$\begin{aligned}\langle(\Delta V_{00})^2\rangle = \sum_l \frac{2l+1}{4\pi}(&C_l^{P^EP^E}C_l^{\Phi^B\Phi^B} + C_l^{P^BP^B}C_l^{\Phi^E\Phi^E} \\ &- 2C_l^{P^E\Phi^E}C_l^{P^B\Phi^B} + (C_l^{P^E\Phi^B})^2 + (C_l^{P^B\Phi^E})^2 \\ &- 2C_l^{P^EP^B}C_l^{\Phi^E\Phi^B}).\end{aligned} \tag{18}$$

There are contributions—those in the first and second lines—that arise even in the absence of parity breaking (while those that arise from parity-breaking physics are listed in the third line). Thus, the observed Universe can have a uniform circular polarization as a consequence of a parity-breaking realization of scalar and tensor metric perturbations, even in the absence of parity breaking in the underlying physics (in analogy to spontaneous symmetry breaking in particle theory). Such a possibility does not arise if there are only density perturbations (and thus only E-mode polarization and index-of-refraction tensor); a uniform circular polarization requires tensor perturbations (and thus B modes). As a result, we find numerically,

$$\langle(\Delta V_{00})^2\rangle^{1/2} \simeq 1.5\times 10^{-18}\left(\frac{r}{0.06}\right)^{1/2}. \tag{19}$$

Comparing Eq. (19) with Eq. (17), we see that the chirality parameter must be $\Delta\chi \gtrsim 0.12(r/0.06)^{-1/2}$ if detection of a uniform circular polarization can be attributed, at the 2σ level, to a chiral GW background. Although detection of a nonzero uniform circular polarization with $V_{00} \lesssim 3\times 10^{-18}(r/0.06)^{1/2}$ would not necessarily indicate a chiral GW background, it would still indicate the presence of tensor (or perhaps vector) perturbations. It is interesting to understand why the uniform circular polarization $\langle V_{00}\rangle$ for maximal chirality ($\Delta\chi = 1$) is roughly 20 times the root variance $\langle(\Delta V_{00})^2\rangle^{1/2}$ in the absence of any chirality. To do so, we first note that the numerical result in Eq. (17) arises from Faraday conversion by GWs of a primordial linear polarization that is also induced by GWs, while the variance in Eq. (19) arises primarily from Faraday conversion by GWs of primordial linear polarization induced by density perturbations (and vice versa). The contribution to the variance from Faraday conversion of GW-induced linear polarization by GWs turns out to be $\langle(\Delta V_{00})^2\rangle^{1/2} \simeq 1.5\times 10^{-19}(r/0.06)$ (smaller by $\sim$10 given that the relevant primordial linear polarization induced by GWs is $\sim$0.1 times that induced by density perturbations for $r\sim 0.1$). If $\sim N_{\mathrm{GW}}$ GW modes are contributing to the signal, then we expect, on average, $N_{\mathrm{GW}}/2$ to be right handed and a similar number left handed. Still, there will be root-N

fluctuations in both signals from right-handed and left-handed GWs in any given realization of a nonchiral GW background, implying a variance $\sim N_{\mathrm{GW}}^{-1/2}$ times the expectation value in the maximally chiral case. From the numbers reported above, this suggests $N_{\mathrm{GW}} \sim 10^4$, or that the uniform circular polarization is (given that there are $2l + 1$ modes for each l) dominated by GW modes with multipole moments $l \lesssim 100$. We have verified numerically that this is the case.

In practice, these values of the circular polarization are too small to be detected in the foreseeable future. Still, the result is a proof of principle that gravitational-wave chirality can be imprinted in the chirality of the cosmic microwave background. Measurement of the circular polarization would, in the event of detection of a nonzero E- and B-mode correlation in the CMB polarization, help distinguish a chiral-GW explanation [33] for such an effect [8,29,34] from cosmic birefringence. It would also complement probes of the GW chirality at nanoHertz frequencies [35] and at LIGO/LISA frequencies [36,37].

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