Characterizing a Statistical Arrow of Time in Quantum Measurement Dynamics

P. M. Harrington,[1,*] D. Tan,[1,2] M. Naghiloo,[1] and K. W. Murch[1,3,†]
[1]Department of Physics, Washington University, St. Louis, Missouri 63130, USA
[2]Shenzhen Institute for Quantum Science and Engineering and Department of Physics, Southern University of Science and Technology, Shenzhen 518055, People's Republic of China
[3]Institute for Materials Science and Engineering, St. Louis, Missouri 63130, USA

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In both thermodynamics and quantum mechanics, the arrow of time is characterized by the statistical likelihood of physical processes. We characterize this arrow of time for the continuous quantum measurement dynamics of a superconducting qubit. By experimentally tracking individual weak measurement trajectories, we compare the path probabilities of forward and backward-in-time evolution to develop an arrow of time statistic associated with measurement dynamics. We compare the statistics of individual trajectories to ensemble properties showing that the measurement dynamics obeys both detailed and integral fluctuation theorems, thus establishing the consistency between microscopic and macroscopic measurement dynamics.

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The entanglement between a quantum system and its environment can be harnessed for indirect measurements, where measurements on the environment alone convey information and induce backaction on the system [1,2]. Because the outcomes of measurements on quantum systems are inherently probabilistic, the outcomes of measurements on the environment have a random character and are statistically described by the quantum state as a model parameter. As the quantum state informs a predictive model of environment fluctuations, experimental measurements on the environment can serve as a predictor for the quantum state. In the fashion of Bayesian inference, quantum state tracking then consists of estimating model parameters conditioned on experimentally detected environment fluctuations resulting in a conditional stochastic evolution of the quantum state—a quantum trajectory [2,3].

Recent experimental capabilities in cavity quantum electrodynamics have enabled high efficiency sampling of environment fluctuations and tracking of individual quantum trajectories [4–8], including statistical properties of these trajectories [9–13]. These quantum trajectories bear a conceptual similarity to classical stochastic trajectories of particles that interact with a thermal reservoir. For such classical trajectories, entropy production can be characterized by tracking the evolution of single particles and comparing the probability density for forward vs time reversed trajectories [14–18]. Experiments in classical systems [19–29] have verified that these entropy measures satisfy fundamental fluctuation theorems that relate microscopic dynamics to ensemble behavior [30–36]. More broadly, these are related to fluctuation theorems for work distributions which have been extended to quantum systems [37–43]. There have been several proposals for experimental tests [44–51] with recent experimental results in closed quantum systems [52,53]. In contrast, open quantum systems present new phenomena associated with measurement backaction [54–63]. In this Letter, we characterize the entropy production of an open quantum system with individual quantum measurement trajectories [54,57, 64–66], using information entropy measures to characterize a statistical arrow of time in quantum measurement. We show how a statistical arrow of time is revealed by path probabilities of forward vs time reversed quantum trajectories [67–70]. As in the case of classical trajectories, these probability densities satisfy a fluctuation theorem that is consistent with the correspondence between microscopic dynamics and ensemble behavior.

Our experiment focuses on a paradigmatic system of quantum measurement consisting of a pseudo spin-half system coupled to a single mode of the electromagnetic field [Fig. 1(a)] [5,10]. The two lowest levels of a transmon circuit [71,72] give a qubit transition frequency $\omega_q/2\pi = 4.01$ GHz, and coupling to a microwave cavity results in a dispersive Jaynes-Cummings interaction given by the interaction Hamiltonian $H_{\mathrm{int}} = -\chi a^\dagger a \sigma_z$, where $\chi/2\pi = -0.6$ MHz is the dispersive coupling rate, $a^\dagger a$ is the number operator for the cavity mode at frequency $\omega_c/2\pi = 6.8316$ GHz, and σ_z is the Pauli operator that commutes with the qubit Hamiltonian. Qubit measurement occurs when a microwave tone probes the cavity resonance, acquiring a qubit-state-dependent phase shift. The shift on the cavity resonance $2|\chi|$ is small compared to the cavity linewidth $\kappa/2\pi = 9.0$ MHz, endowing the measurement tone with a relatively small qubit-state-dependent phase shift. By virtue of this qubit-cavity interaction, the qubit state is correlated to a single field quadrature of the

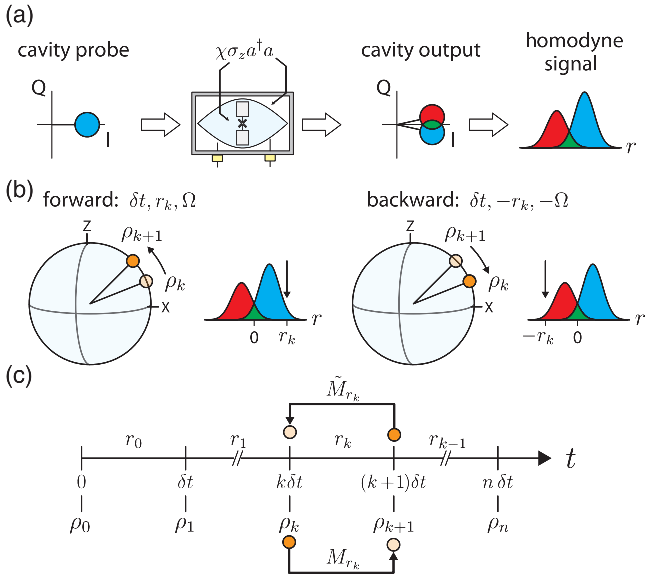

FIG. 1. Forward and reversed trajectories of a continuously monitored superconducting qubit. (a) The experiment setup consists of a superconducting qubit dispersively coupled to the fundamental mode of a waveguide cavity. The output signal reflected from the cavity acquires a qubit-state-dependent phase shift. (b) In a single update step, a measurement record r_k of duration δt from a continuous cavity probe induces backaction on the quantum state. Upon time reversal of this update step, the state responds to backaction of a measurement result of opposite sign $-r_k$ by returning to the initial state. (c) Schematic of the state and measurement labels for forward (M_{r_k}) and backward ($\tilde{M}_{r_k}$) state update procedures.

microwave probe, which is subsequently amplified by a near-quantum-limited Josephson parametric amplifier [73,74] operating in phase sensitive mode.

The amplified quadrature is down-converted to dc and digitized into time steps $\{t_k\}_{k=0}^{k=n}$ to obtain a set of measurement records $\{r_k\}_{k=0}^{k=n-1}$. From these measurement records, we reconstruct piecewise continuous trajectories, with each individual trajectory captured by a time series of density operators $\{\rho_k\}_{k=0}^{k=n}$ [75]. Informed by successive measurement records in a time series, we use an iterative update scheme to infer the qubit state and realize a single trajectory. The dynamics of the trajectory results from the impression each stochastic measurement record r_k has on our state-of-knowledge ρ_k.

The statistics of the measurement record and the dynamics imparted on the state are described by the positive operator-valued measure (POVM) [76],

$$M_{r_k} = \left(\frac{\delta t}{2\pi\tau}\right)^{1/4} \exp\left(-\frac{\delta t}{4\tau}(r_k\hat{1} - \sigma_z)^2\right), \qquad (1)$$

where the measurement strength is the product of the signal integration duration $\delta t = t_{k+1} - t_k = 16$ ns and the measurement rate $1/\tau = 1.97\ \mu\mathrm{s}^{-1}$. In addition to measurement, the evolution includes a dynamics due to a resonant Rabi drive by the Hamiltonian $H/\hbar = \Omega\sigma_y/2$ with $\Omega/2\pi = 2.16$ MHz, which is in a rotating frame of the qubit transition. The POVM provides a state update conditioned on the measurement record from the relation $\rho_{k+1} = M_{r_k}\rho_k M_{r_k}^\dagger/\mathrm{tr}[M_{r_k}\rho_k M_{r_k}^\dagger]$, where the probability density of the measurement outcome is given by $P(r_k|\rho_k)dr_k = \mathrm{tr}[M_{r_k}\rho_k M_{r_k}^\dagger]dr_k$. By this probability density, and the associated information entropy, we statistically examine time reversal in the measurement process by comparing the likelihood of quantum trajectories that are ordered forward vs backward in time.

The notion that the quantum measurement process can be reversed stems from studies of "measurement undoing" [77], where weak measurements can essentially erase information from previous measurements. As such, time reversal of the measurement process is established by reversing dynamics for a single measurement update step, where time reversed measurement "undoes" the backaction from forward measurement in a physically realizable way [Fig. 1(b)]. This measurement reversal has been observed in a variety of experimental platforms [5,78–80] and analyzed in the context of POVMs as we employ here [81]. For each measurement by POVM M_{r_k}, there is a corresponding measurement $\tilde{M}_{r_k} = M_{\tilde{r}_k}$, where $\tilde{r}_k = -r_k$ is the time reversed measurement record which restores the initial state of knowledge, albeit with a statistical weight

$$\tilde{M}_{r_k} M_{r_k}\rho_k M_{r_k}^\dagger \tilde{M}_{r_k}^\dagger = \frac{\delta t}{2\pi\tau} e^{-\frac{\delta t}{\tau}(r_k^2+1)}\rho_k.$$

In addition, at each step the unitary evolution of the Rabi drive is reversed ($\Omega \to -\Omega$). To explore the statistical cost of time-reversed dynamics along a quantum trajectory with many time steps, we use the path probability densities of forward and backward evolution to define a quantity $\mathcal{Q}$ that characterizes the length of time's arrow

$$\mathcal{Q} = \sum_k \ln\frac{P(r_k|\rho_k)}{P(\tilde{r}_k|\rho_{k+1})} = \sum_k \ln\frac{\mathrm{tr}[M_{r_k}\rho_k M_{r_k}^\dagger]dr_k}{\mathrm{tr}[\tilde{M}_{r_k}\rho_{k+1}\tilde{M}_{r_k}^\dagger]d\tilde{r}_k}. \qquad (2)$$

Since $\mathcal{Q}$ is the sum of relative entropies between forward and reversed time steps, a trajectory with positive $\mathcal{Q}$ indicates a forward pointing arrow of time, for its measurement record has a greater probability density when considered forward, opposed to backward, in time.

To gain an intuition into this arrow of time quantity $\mathcal{Q}$ we examine the measurement record in the continuous limit, $r(t) \propto z(t) + \sqrt{\tau}d\xi$, where $r(t)$ and $z(t)$ are, respectively, the measurement record and qubit expectation value $\langle\sigma_z\rangle$, and $d\xi$ is a zero-mean Gaussian random variable. Consequently, we calculate Eq. (2) as an integral and find $\dot{\mathcal{Q}}_k \simeq 2r(t)z(t)/\tau$ [68,82]. Hence, it is clear that the contributions to the forward arrow of time occur when the record and state are correlated.

In our experiment, the measurements take place with a finite quantum efficiency $\eta = 0.4$, determined by comparing the measurement rate to the total dephasing rate $\Gamma = 1/(2\eta\tau)$ [84]. Measurement with finite efficiency can be modeled by the measurement dynamics of multiple measurement channels, where our experimental measurement record is but one of these channels [76]. In the case of multiple channels, the dynamics is described by a POVM characterizing simultaneous measurement from every channel. An observer who has access to only one channel then describes system dynamics by averaging over all unknown measurement outcomes. Averaging over the unknown measurement outcomes results in dephasing of the qubit state, which breaks the time reversibility of the measurement dynamics. To restore reversibility, we estimate the quantum trajectories that would be obtained for an observer with access to all measurement channels. These trajectories serve as the model that governs the probability density for forward and reversed measurement sequences.

As shown in Fig. 2, the finite quantum efficiency in our experiment arises predominantly from attenuation of the cavity probe between the cavity and Josephson parametric amplifier, and this attenuation can be modeled as a beam splitter where the cavity probe is split between two observers which we denote as "Alice" (our experimental record) and "Bob" (an unmonitored channel). A third observer, "Charlie" has access to both Alice's and Bob's measurement records and can therefore track quantum trajectories that are reversible as previously discussed.

For each experimentally sampled quantum trajectory we create an ensemble of pure state trajectories that are estimates for the pure state quantum trajectory determined by Charlie, as depicted schematically in Fig. 2. This ensemble of possible trajectories for Charlie corresponds to an unraveling of the Lindblad master equation that describes Alice's quantum trajectory.

This unraveling, however, depends on what type of homodyne measurements Bob makes on his channel, which we will analyze in the two extremal cases. Alice uses a parametric amplifier to measure the quadrature of the microwave probe that is correlated with the qubit populations in the σ_z basis, which we denote as z measurement. In one case, Bob performs homodyne measurement of the cavity probe in the same quadrature, revealing further information about the qubit populations. In the other case, Bob measures the cavity probe in an orthogonal quadrature, which does not reveal information about the qubit populations but rather about phase shifts on the qubit imparted by an ac Stark shift due to photon fluctuations of the cavity probe, which we refer to as ϕ measurement [5,85,86]. The resulting backaction from z and ϕ measurement has been studied in previous work [5]. The corresponding quantum trajectories for Charlie are markedly different in both cases, resulting in different possible arrows of time for Alice.

We model the finite efficiency beam splitter [Fig. 2(a)] in a time segmented fashion, such that Alice makes perfectly efficient measurements for a fraction η of her measurement records, and records noise upon the remaining $1-\eta$ of measurement records. A possible trajectory for Charlie is constructed by updating the state with Alice's measurement record with probability η, otherwise with Bob's measurement record at each time step. Both state updates are implemented with unit efficiency ($1/\tau \to 1/\eta\tau$). In the two limiting cases for Bob's measurement basis, we construct Bob's measurement by statistically sampling measurement values distributed according to Charlie's state: Bob's z measurements are characterized by Eq. (1) and ϕ measurements are drawn from a zero-mean Gaussian of variance $1/(2\Gamma\delta t) = \eta\tau/\delta t$ [82]. From a single sequence of experimentally obtained measurement records we create an ensemble of unraveled trajectories which has an average evolution consistent with the single finite efficiency experiment trajectory. Several unraveled quantum trajectories are shown for these two limiting cases in Figs. 2(c) and 2(d).

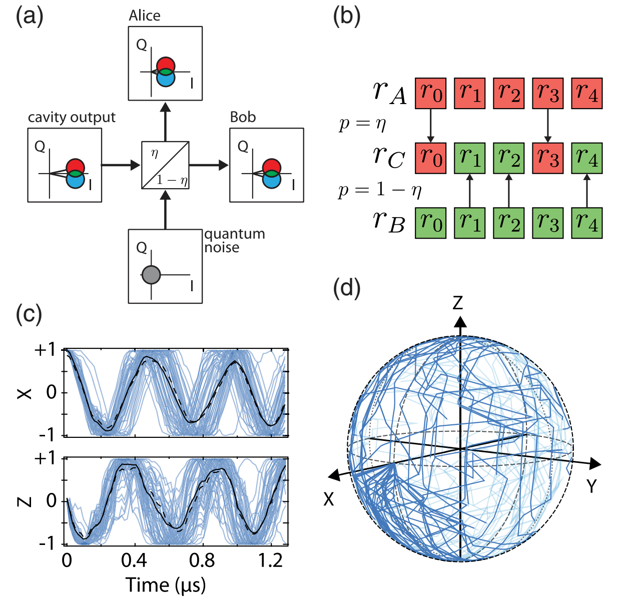

FIG. 2. Measurement inefficiency as multiple observers. (a) Finite quantum efficiency can be modeled as a beam splitter, where the cavity probe is split between two observers, Alice and Bob. (b) We model the beam splitter as a time segmented splitter, which directs the signal to Alice or Bob at each time step with probabilities η and $1-\eta$, respectively. The measurement record of a third observer, Charlie, who has access to both Alice's and Bob's records, can be constructed by taking either Alice's record or Bob's record at each time step. (c) By sampling several possible measurement records for Bob, where Bob performs measurements on his signal in the same quadrature as Alice, we construct an ensemble of possible pure state trajectories for Charlie. The average of these unraveled trajectories (black solid line) corresponds to the finite quantum efficiency trajectory based only on Alice's record (dashed line). (d) If Bob instead measures the cavity probe in an orthogonal quadrature to Alice's measurement, the resulting backaction on the qubit causes state evolution outside the X–Z plane.

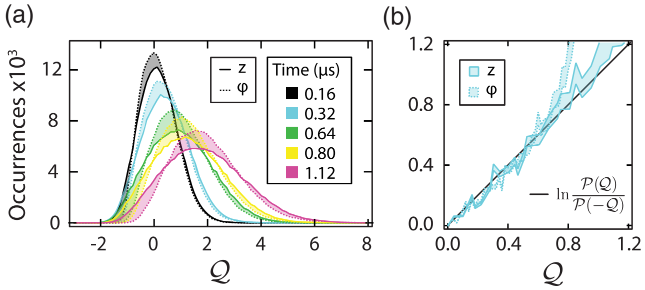

FIG. 3. Statistical arrow of time for quantum measurement evolution. (a) The distributions $\mathcal{P}(\mathcal{Q})$ for different propagation times which are obtained from 2.8×10^5 runs of experiments. Here the solid curves correspond to Alice's arrow of time when Bob measures in the same quadrature as Alice, and the dashed curve indicates the case where Bob measures in the orthogonal quadrature, constraining a range of possible values for Alice's arrow of time. (b) Calculation of the detailed fluctuation theorem. The distributions of $\mathcal{Q}$ at time $t = 0.32$ μs (blue curves) is used to calculate the quantity $\ln[\mathcal{P}(\mathcal{Q})/\mathcal{P}(-\mathcal{Q})]$. The detailed fluctuation theorem in Eq. (3) for both of Bob's measurement bases agrees well with the theory prediction (black line). Error bars indicate the statistical uncertainty in quantity $\ln[\mathcal{P}(\mathcal{Q})/\mathcal{P}(-\mathcal{Q})]$.

We now examine the arrow of time for an ensemble of experimentally sampled quantum trajectories. Figure 3(a) displays distributions of $\mathcal{Q}$ based on 2.8×10^5 trajectories for different evolution times. Each evolution time is associated with two different distributions for $\mathcal{Q}$, corresponding to Alice's arrow of time, given the limiting cases of Bob's measurement basis. Here we see the role of measurement backaction in the choice of Bob's measurement, where z measurement leads to a greater occurrence of both forward-likely and backward-likely trajectories as indicated by the broad $\mathcal{Q}$ distribution compared to the counterpart $\mathcal{Q}$ distribution for ϕ measurements. When Bob measures in the same quadrature as Alice, the effectively stronger measurement causes the trajectory to take on more extremal values of z resulting in stronger correlation and anticorrelation to the observed measurement record.

Notably, negative values of $\mathcal{Q}$ occur for Alice's arrow of time for both Bob's z and ϕ measurements, corresponding to trajectories where the time reverse process is more likely. This phenomenon of negative entropy production is well known in microscopic stochastic systems and is typically characterized by a fluctuation theorem [15–18,31–35,87]. In Fig. 3(b), we show that the data are in agreement with a detailed fluctuation theorem

$$\frac{\mathcal{P}(\mathcal{Q})}{\mathcal{P}(-\mathcal{Q})} = e^{\mathcal{Q}}, \tag{3}$$

which quantifies the relative probability of obtaining a forward pointing arrow of time with length $\mathcal{Q}$ to the probability of a backward arrow of the same length. For small values of $\mathcal{Q}$, the agreement with the detailed fluctuation theorem indicates that the experimentally sampled relative occurrence of $\mathcal{Q}$, as given by the left-hand side of Eq. (3), is in agreement with the definition of $\mathcal{Q}$ on the right-hand side. The self-consistency of such a fluctuation arises because the microscopic dynamics and the macroscopic statistics are consistent with the definition of $\mathcal{Q}$. However, for larger values of $\mathcal{Q}$, the fluctuation relation is clearly nonlinear, a feature that is related to the presence of absolute irreversibility which we now explore in detail.

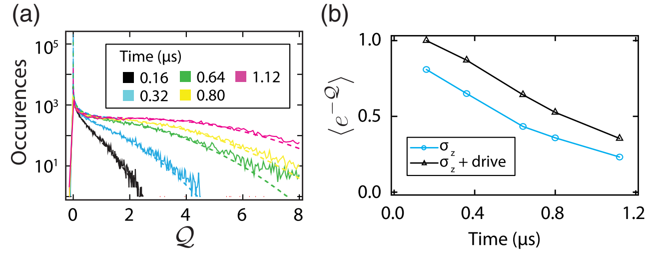

FIG. 4. Distribution of $\mathcal{Q}$ and absolute irreversibility. (a) The distribution of $\mathcal{Q}$ for different propagation times obtained from 2.8×10^5 experimental trajectories. As the time increases, the distribution is more biased to large positive values of $\mathcal{Q}$. (b) Calculation of the absolute irreversibility in the case Bob measures in the same quadrature as Alice. The integral fluctuations theorem gives a quantity less than unity as a consequence of the initial state of the trajectory. Here, the dynamics of state projection favor measurement records correlated with the qubit state, resulting in a surplus of state updates when $\mathcal{Q} > 0$, causing an overall longer arrow of time.

We now focus on a special measurement condition, where the Rabi frequency of the drive $\Omega = 0$. In this case, the measurement operators commute with the qubit Hamiltonian, resulting in a quantum nondemolition measurement. We consider the case where the qubit is prepared such that $\langle \sigma_x \rangle \simeq 1$ and measurements project the system toward the stationary points $\langle \sigma_z \rangle \to \pm 1$. Figure 4 displays the distributions for $\mathcal{Q}$ for several evolution times. Note that Bob's measurement does not affect Alice's arrow of time in this case.

For the simple dynamics of this semiclassical measurement, the probability density of $\mathcal{Q}$ is found analytically by performing a change of variables in the measurement record probability density [68]

$$\mathcal{P}(\mathcal{Q}) = \sqrt{\frac{T}{2\pi\tau} \frac{e^{\mathcal{Q}}}{e^{\mathcal{Q}} - 1}} \exp\left(-\frac{T}{2\tau} - \frac{\tau}{2T}[\cosh^{-1}(e^{\mathcal{Q}/2})]^2\right). \tag{4}$$

Histograms of $\mathcal{Q}$ from experiment are plotted for a selection of final times T with their corresponding theoretical probability density in dashed lines.

Clearly, the relative probabilities for forward and backward arrows of time in this measurement case do not satisfy the detailed fluctuation theorem [Eq. (3)]. This is because the detailed fluctuation relation is only satisfied for the total statistical entropy change during a process [16]. In the presented case, the statistical arrow of time quantity $\mathcal{Q}$ does not capture the contributing influence of the initial state of the trajectory; quantum measurement is, in general, a nonequilibrium, irreversible process. Here, the initial state imposes a lower bound on the possible values of $\mathcal{Q}$ [68]. This sensitivity to initial conditions results from the "un"-likelihood of a particular initial state, quantified by an absolute irreversibility [88–91]. As presented in Fig. 4(b), the absolute irreversibility is quantified by the integral fluctuation theorem, which gives a deviation from unity resulting from the ensemble of trajectories containing a surplus of state updates that have a positive statistical arrow of time. This is due to the favoring of correlations between the qubit state and measure record from the measurement projection process. This contribution to the entropy is physically analogous to the entropy increase associated with irreversible expansion of gas. The semiclassical measurement case discussed here clearly illustrates absolute irreversibility due to initial conditions since the initial state is far from the fixed points of the measurement dynamics.

Continuous quantum measurement leads to a probabilistic dynamics of the quantum state. Under conditions where this dynamics is time reversible, we can consider the probabilities associated with both forward and reversed dynamics. These probabilities allow us to infer a statistical arrow of time by considering the information entropy associated to measurement sequences. Using experimental data we have shown that a statistical arrow of time emerges fundamentally in quantum measurement, where information and backaction arise from entanglement with a fluctuating environment.

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P. M. H. and D. T. contributed equally to this work.

[*]patrick.harrington@wustl.edu
[†]murch@wustl.edu

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