Integrability-Protected Adiabatic Reversibility in Quantum Spin Chains

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We consider the out-of-equilibrium dynamics of the Heisenberg anisotropic quantum spin-1/2 chain threaded by a time-dependent magnetic flux. In the spirit of the recently developed generalized hydrodynamics (GHD), we exploit the integrability of the model for any flux values to derive an exact description of the dynamics in the limit of slowly varying flux: the state of the system is described at any time by a time-dependent generalized Gibbs ensemble. Two dynamical regimes emerge according to the value of the anisotropy Δ. For $|\Delta| > 1$, reversibility is preserved: the initial state is always recovered whenever the flux is brought back to zero. On the contrary, for $|\Delta| < 1$, instabilities of quasiparticles produce irreversible dynamics as confirmed by the dramatic growth of entanglement entropy. In this regime, the standard GHD description becomes incomplete and we complement it via a maximum entropy production principle. We test our predictions against numerical simulations finding excellent agreement.

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Understanding the nonequilibrium dynamics of isolated many-body quantum systems is currently one of the most active research areas at the boundary between condensed matter and statistical mechanics. The importance of these studies lies in its multifaceted impact, ranging from fundamental settings, such as the microscopic derivation of thermodynamical ensembles [1–3], to more applied ones such as the precise control of quantum systems [4,5] or the realization of novel out-of-equilibrium phases of matter [6]. In this context, cold-atom experiments have posed basic puzzles for theoretical understanding [7], also providing a flexible playground to test and accurately validate predictions and exact results. Quite generically, one expects that many-body systems are able to act as their own reservoirs: starting from out-of-equilibrium states $|\psi\rangle$, at long times the expectation value of a local observable $\hat{O}$ approaches the thermal equilibrium one, i.e., $\langle\psi|O(t)|\psi\rangle \to \langle O\rangle_{\text{eq}}$. This hypothesis has been thoroughly investigated in sudden quantum quenches, where a high-energy initial state $|\psi\rangle$ is evolved with a time-independent Hamiltonian $\hat{H}$ [8]. In practice, however, for generic systems, one has to resort to numerical simulations [9] which suffer by strong limitations [10,11]. For this reason, a crucial role has been played by integrable systems, for which it is possible to derive exact predictions. Several studies have clarified that integrable models which undergo a quantum quench generically exhibit relaxation [12,13]. However, in integrable models exist infinitely many conserved quantities $\hat{Q}_j = \sum_{n=1}^{N}\hat{q}_j(n)$ where $\hat{q}_j(n)$ is a (quasi-)local operator [14–19]. The presence of an extensive set of integral of motions suggests that the *generalized Gibbs ensemble* (GGE) $\langle\hat{O}\rangle_{\text{GGE}} = \text{Tr}[\hat{O}e^{-\sum_j\lambda_j\hat{Q}_j}]/Z$ has to be used in place of the standard one [20,21], where the appropriate set of charges has been accurately characterized in several studies [22–26]. The validity of the GGE conjecture has been nowadays extensively verified not only on theoretical ground [12,13,27–40], but even on the experimental side [5,41].

Beyond quantum quenches, integrability constraints can be engineered to induce exotic out-of-equilibrium properties, including superdiffusive transport [42–53], dynamical ordering [26,54], and efficient heat pumps [55,56]. In this context, *generalized hydrodynamics* [57,58] (see also Refs. [59–86]) has provided a unifying framework to accurately describe integrable systems in the quasistationary regime which emerges from inhomogeneous initial conditions.

In this Letter we consider the out-of-equilibrium dynamics of the spin-1/2 *XXZ* chain Hamiltonian in the presence of a nonvanishing magnetic flux

$$\hat{H}(\Phi) = \sum_{j=1}^{N}\frac{1}{2}(e^{\imath\Phi}\hat{s}_j^{+}\hat{s}_{j+1}^{-} + \text{H.c.}) + \Delta\hat{s}_j^{z}\hat{s}_{j+1}^{z} - B\hat{s}_j^{z}. \qquad (1)$$

Above, Φ is the flux density, $N\Phi$ being the total flux piercing the ring. $s^{x,y,z}$ are the usual spin-$\frac{1}{2}$ operators and $s^{\pm} = (s^x \pm is^y)/2$, periodic boundary conditions (PBCs) are enforced, together with the thermodynamic limit (TL) $N \to \infty$. By means of the Jordan-Wigner transformation, Eq. (1) describes spinless fermions, where Δ controls the interaction strength and B the filling. In this language, the flux density Φ is associated with a magnetic field coupled with the $U(1)$ fermionic charge. The system is initially prepared in an equilibrium state of the model at $\Phi = 0$ (a GGE) and the flux density $\Phi(t)$ is then slowly varied in

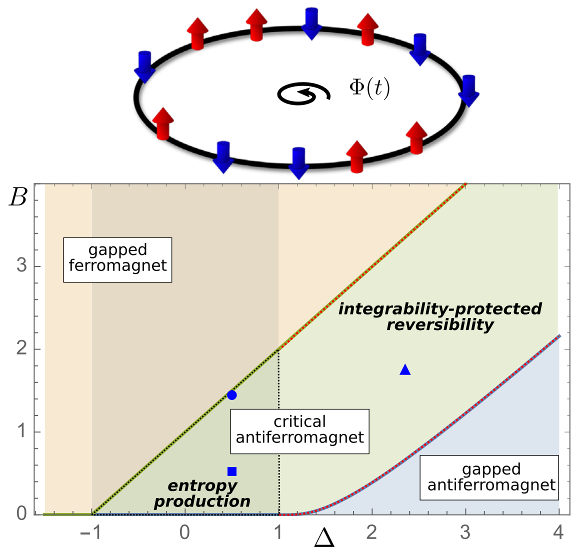

FIG. 1. Top: The Heisenberg spin-1/2 chain is threaded by a time dependent magnetic field inducing a magnetic flux density $\Phi(t)$. The instantaneous Hamiltonian Eq. (1) describing the dynamics is always integrable and in the adiabatic limit, the state of the system is always locally described by a GGE ensemble. Bottom: Phase diagram of the XXZ spin chain as a function of the anisotropy Δ and magnetic field B. In the region $|\Delta| \geq 1$, the model supports an infinite number of stable bound states, which preserve the reversibility of the dynamics. For $|\Delta| < 1$, the number of stable bound states strongly depends on Δ and their momentum support does not cover the full Brillouin zone: this instability leads to irreversibility.

time (see Fig. 1, top). Infinitesimal fluxes of $|\Phi(t)| \simeq O(1/N)$ were considered in the literature in the context of linear response [87,88] or as example of local quenches [89]. Sudden global quenches of the flux were considered in Refs. [90,91]. Here instead, we consider finite, but slowly varying fluxes, so that the system has always time to relax to a GGE and our choice for the initial state is thus not restrictive. Our forthcoming analysis can be applied to arbitrary GGEs, but for the sake of simplicity we focus on the ground state (GS) of the model for different values of the anisotropy Δ and of the magnetic field B.

In generic systems, slow modifications of the Hamiltonian are governed by the celebrated *adiabatic theorem* [92], according to which a sufficiently slow dynamics always keeps the system in the instantaneous ground state as long as a finite gap exists. This implies reversibility of the protocol: moving slowly forth and back the external parameter, the state is back to the initial condition. However, reversibility is broken if the system is gapless: in the thermodynamic limit, any finite-frequency perturbation will inevitably produce excitations in the system [93].

However, such a picture can be drastically modified by integrability: extra dynamical symmetries can prevent the production of excitations, even in the absence of an energy gap. While this can be expected for noninteracting systems, it leads to surprising behavior when interactions are turned on. In particular, we disclose a rich dynamical phase diagram (see Fig. 1). For $|\Delta| \geq 1$ the dynamics, within the validity of our assumptions, is fully reversible, despite the system being gapless. On the other hand, for $|\Delta| < 1$ and in the gapless regime, reversibility is generally broken. Note that the system does not stay in the instantaneous ground state, but still, in some sense specified later on, the Δ-dependent reversibility resembles the break down of the standard adiabatic theorem due to level crossing, but extended to the whole set of instantaneous conserved charges.

As the flux changes, the Hamiltonians Eq. (1) are connected to the $\Phi = 0$ case through a gauge transformation $\hat{H}(\Phi) \sim W_\Phi^\dagger H(0) W_\Phi$, where boundary terms were neglected and $W_\Phi = e^{-i\Phi \sum_{j=1}^N j \hat{s}_j^z}$. This gauge symmetry not only guarantees the instantaneous Hamiltonian to be integrable, but also connects the whole set of conserved charges for different fluxes $\hat{Q}_j(\Phi) = W_\Phi^\dagger \hat{Q}_j(0) W_\Phi$. The total magnetization $\hat{S}^z = \sum_{j=1}^N \hat{s}_j^z$ is flux independent and a conserved charge for any $\hat{H}(\Phi)$; thus it is constant along the time evolution. The fact that the flux density Φ can be locally (but not globally) gauged away indicates that it will not affect the microscopic scattering of quasiparticles in the model. In the spirit of the recently introduced generalized hydrodynamics (GHD) [57,58], we assume a separation of time scales: the system quickly relaxes to a local GGE which slowly evolves due to the flux variation. Quantifying the precise regime of validity of hydrodynamics is an open issue [82,83]; here, we pragmatically assume the existence of a microscopic relaxation time scale against which the change of the flux must be compared, then benchmark our findings against numerical simulations.

XXZ chain and generalized thermodynamics.—The XXZ spin chain Eq. (1) is among the best known interacting integrable models. Here we provide a basic summary, leaving to the Supplemental Material (SM) [94] and Ref. [95] a more exhaustive description. The Hamiltonian Eq. (1) at Δ is unitarily equivalent to $-\hat{H}$ at $-\Delta$; thus without loss of generality we analyze the spectrum for $\Delta > 0$. Similarly to free systems, the Hilbert space of integrable models can be understood in terms of quasiparticles which undergo elastic scattering. In the presence of interactions these excitations can form bound states (also known as strings), which behave as stable quasiparticles of different species and constitute the particle content of the model. In the thermodynamic limit, one can introduce the root densities $\rho_j(\lambda)$ which count, on average, how many quasiparticles of the species j at a given rapidity λ are present in the state. An exponentially large number $\sim e^{N\mathcal{S}}$ of eigenstates, *microstates*, with $\mathcal{S}$ the Yang-Yang entropy [95,96], correspond to the same macrostate identified by the root densities $\{\rho_j(\lambda)\}_j$ and have identical local properties [97]. For example, the expectation value of local charges in the TL is

$$\lim_{\mathrm{TL}} N^{-1}\langle\hat{Q}_i\rangle = \sum_i \int d\lambda q_i^j(\lambda)\rho_j(\lambda). \tag{2}$$

Above, the ρ_j-independent functions $q_i^j(\lambda)$ are commonly known as single-particle eigenvalues relative to the ith charge and jth bound state type. Relevant examples are energy, momentum, and number single-particle eigenvalue, which we indicate as $h_j(\lambda)$, $p_j(\lambda)$ and m_j, respectively. The number eigenvalue m_j is independent of the rapidity and counts the number of fundamental particles in the bound state and equivalently the number of spinflips. For a complete set of charges Eq. (2) can be inverted and in the TL, a GGE is in one-to-one correspondence with a macrostate [16,27].

Beyond leading to bound states, interactions induce collective behaviors which have a net effect (*dressing*) on the low-lying excitations over a GGE [57,98]. More formally, for an arbitrary function $\tau_j(\lambda)$, we define the dressing operation as the solution of the integral linear equation

$$\tau_j^{\mathrm{dr}}(\lambda) = \tau_j(\lambda) - \sum_{j'} \int d\mu T_{j,k}(\lambda-\mu)\sigma_k\vartheta_k(\mu)\tau_k^{\mathrm{dr}}(\mu), \tag{3}$$

where the parities $\sigma_j \in \{-1, 1\}$ and the kernel $T_{j,k}(\lambda)$ depend on the value of Δ. Then, adding an excitation over GGEs modifies the charge in Eq. (2) by a state-dependent amount governed by $(q_i^j(\lambda))^{\mathrm{dr}}$.

Consistently, the density of modes available for each quasiparticle $\rho_j^t(\lambda) > \rho_j(\lambda)$ satisfies $2\pi\rho_j^t(\lambda) = \sigma_j(\partial_\lambda p_j(\lambda))^{\mathrm{dr}}$. From the filling $\vartheta_j(\lambda) = \rho_j(\lambda)/\rho_j^t(\lambda)$, one can express the Yang-Yang entropy functional $\mathcal{S}$ as

$$\mathcal{S} = \sum_j \int d\lambda \rho_j^t \eta(\vartheta_j(\lambda)), \tag{4}$$

and $\eta(x) = -x\log x - (1-x)\log(1-x)$. Equation (4) describes the extensive part of the entanglement entropy in a GGE state [79,99–107]. Consistently, for ground states $\mathcal{S} = 0$, as the fillings behave as Fermi seas, i.e., $\vartheta_j(\lambda) = \delta_{j1}\Theta(\Lambda - |\lambda|)$, with $\Theta(x)$ the Heaviside function and Λ the Fermi-point which depends on the magnetic field B.

The hydrodynamic approach to the flux dynamics.—Let us now consider the out-of-equilibrium protocol: in particular, we imagine an infinitesimal change of the flux density $\Phi \to \Phi + d\Phi$ and wait long enough to attain local equilibration to the new GGE. From the charge conservation an infinite number of constraints is obtained,

$$\langle\hat{Q}_j(\Phi + d\Phi)\rangle_{\Phi+d\Phi} = \langle\hat{Q}_j(\Phi + d\Phi)\rangle_{\Phi}. \tag{5}$$

Above, with $\langle\cdots\rangle_\Phi$ we mean the expectation value with respect to the GGE describing the state at flux density Φ. The lhs of the above condition is readily computable [Eq. (2)], but accessing the rhs is not trivial. In this respect, the gauge transformation provides the missing information: the rhs can be computed at first order in $d\Phi$ and, invoking the completeness of the charges, an evolution equation for $\vartheta_j(\lambda)$ can be obtained. We leave the technical details to the SM [94], while here we report and comment the result: an infinitesimal increment of the flux is translated into a rapidity shift of the fillings, i.e.,

$$\vartheta_j(\lambda, \Phi + d\Phi) = \vartheta_j\left(\lambda - d\Phi\frac{m_j^{\mathrm{dr}}(\lambda)}{(\partial_\lambda p_j(\lambda))^{\mathrm{dr}}}, \Phi\right). \tag{6}$$

The semiclassical soliton-gas interpretation [60] of Eq. (6) is clear: for any Φ, $\vartheta_j(\lambda)$ describes a set of homogeneously distributed particles with momentum $p_j(\lambda)$, which undergo a collective acceleration $p_j(\lambda) \to p_j(\lambda) + m_j d\Phi$ due to Lenz's law, i.e., the force caused by the variation of the magnetic field. Because of interactions, the effective force and momentum must be suitably dressed.

We point out that Eq. (6) is the time-dependent analogue of the already-known GHD equations in the presence of small spatial inhomogeneities [59]. However, its meaning is quite different: in Ref. [59] the integrable model described by the root densities is constant in time, while in our case it is flux dependent. We can therefore describe arbitrarily large values of Φ, provided they are reached slowly enough.

Reversible hydrodynamics for $\Delta \geq 1$.—In this case there are infinitely many strings $\{\rho_j(\lambda)\}_{j=1}^{\infty}$, the parity $\sigma_j = 1$ in Eq. (3) and $m_j = j$. The rapidities cover the compact domain $\lambda \in [-\pi/2, \pi/2]$ and the momenta belong to a Brillouin zone satisfying $p_j(\pi/2) = p_j(-\pi/2) \bmod 2\pi$. The kernel $T_{j,k}(\lambda)$ and single-particle eigenvalues of quasilocal charges $q_i^j(\lambda)$ are π periodic (expressions can be found in the SM [94]). Periodic boundary conditions must be imposed on the fillings $\vartheta_j(\lambda)$ and thus Eq. (6) guarantees the reversibility of the adiabatic protocol. Consistently, the entropy [Eq. (4)] does not change $\partial_\phi\mathcal{S} = 0$ [94]. These conclusions hold true for arbitrary states, in Fig. 2, panels (a) and (b), we test Eq. (6) against numerical simulations [108] initializing the system on the ground state. We find perfect agreement.

Hydrodynamics for $\Delta < 1$: Irreversibility and entropy production.—In this case the structure of the root densities is far richer and more complicated than before [95]. The coupling is parametrized as $\Delta = \cos(\gamma)$, then the particle spectrum is finite for rational values of γ/π and any value of Δ is obtained from rational approximations (see SM [94] for details). In contrast with the previous case, the rapidities live on the whole real axis $\lambda \in (-\infty, \infty)$. In this case, $p_j(\lambda = +\infty) \neq p_j(\lambda = -\infty) \bmod 2\pi$; i.e., momenta do not belong to a Brillouin zone any longer, leaving out the problem of fixing the correct boundary conditions. To clarify the issue in physical terms we resort to the

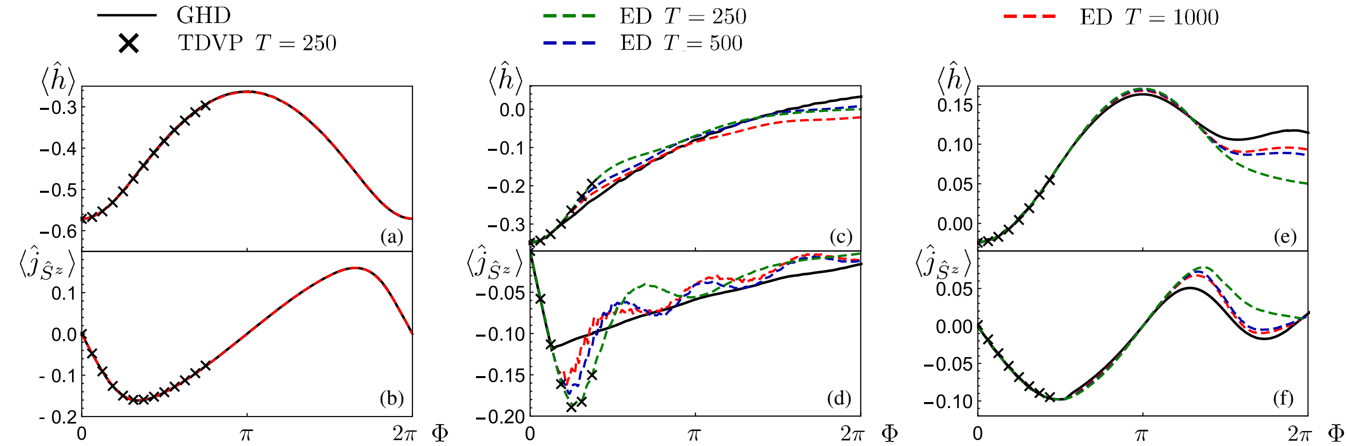

FIG. 2. Expectation value of the instantaneous energy and spin current vs flux density. The GHD prediction is compared against exact diagonalization (ED) and time-dependent variational principle (TDVP) [108], the flux density being changed as $\Phi(t) = 2\pi t/T$. Choices of larger T result in a more faithful hydrodynamic description. Panels (a) and (b), we choose $\Delta = \cosh(1.5) > 1 \langle \hat{s}_j^z \rangle = 0.1$ (ED with 25 sites, blue triangle in Fig. 1). In this case the evolution is perfectly 2π periodic [94]. In (c) and (d) we rather consider $\Delta = 0.5$ and $\langle \hat{s}_j^z \rangle = 0.1$ (ED with 25 sites, blue square in Fig. 1), while in (e) and (f) $\langle \hat{s}_j^z \rangle = 0.4$ (ED with 50 sites, blue circle in Fig. 1). While for $\Delta > 1$ we testified an excellent convergence even for relatively fast flux changes and small system sizes, significantly longer time scales and larger systems are needed for $\Delta < 1$. This can be understood looking at the spin current: the GHD for $\Delta < 1$ displays nonanalyticity points, which are located at those values of the flux density such that the Fermi sea hits the boundaries $\lambda = \pm\infty$. In order for the smooth time evolution to well approximate such a behavior, very long times are needed.

semiclassic interpretation given above: increasing Φ, a quasiparticle is accelerated, but for a finite increase of the flux density, it reaches a momentum corresponding to infinite rapidity. What happens when the flux density is further increased? An appealing physical insight can be gained looking at single-particle eigenvalues: since it holds true $\lim_{\lambda\to\pm\infty} q_i^j(\lambda) = m_j \times \lim_{\lambda\to\pm\infty} q_i^1(\lambda)$ [94], from the point of view of any quasilocal charge, at infinite rapidity $\lambda = \pm\infty$, it is not possible to distinguish between a bound state of type j and m_j unbounded excitations [94]. In the absence of dynamical constraints, bound states break and merge when $|\lambda| = \infty$: fillings shifting towards a boundary $\lambda = \pm\infty$ will recombine into fillings which emerge from the same boundary. Since the charges are unable to fix the recombination rates, we revert to the other pillar of GGE: entropy maximization.

Using the hydrodynamic equation, it can be shown that the change in entropy is due to boundary terms [94] $\partial_\Phi \mathcal{S} = \partial_\Phi \mathcal{S}^+ + \partial_\Phi \mathcal{S}^-$, where

$$\partial_\Phi \mathcal{S}^\pm = \mp \lim_{\lambda\to\pm\infty} \left(\sum_j \sigma_j m_j^{\mathrm{dr}}(\lambda) \eta(\vartheta_j(\lambda)) \right). \qquad (7)$$

Therefore, we set as the desired boundary conditions the choice of the outgoing fillings that maximizes $\partial_\Phi \mathcal{S}^\pm$, together with the particle-flux conservation $\lim_{\lambda\to\pm\infty}[\sum_j \sigma_j m_j m_j^{\mathrm{dr}}(\lambda)\vartheta_j(\lambda)] = 0$. This last condition is needed to enforce Eq. (5) and naturally arises in the derivation of Eq. (6) [94]. In practice, when starting from the ground state, the Fermi sea in $\vartheta_1(\lambda)$ is shifted while the flux density is increased, up to a value where the Fermi point reaches the boundary $\lambda = \infty$; then, the other fillings start to be populated according to the maximum-entropy principle. Our prediction is tested against numerics in Fig. 2, considering the GS as the initial state. We find good agreement although convergence to adiabaticity $\dot{\Phi}(t) \to 0$ is much slower than for $|\Delta| > 1$. Consistently with our interpretation, the entanglement-entropy production (see Fig. 3) remains suppressed up to a critical value of Φ where bound states start recombining. Then, entropy starts growing, demonstrating the irreversibility of the process (see Fig. 4).

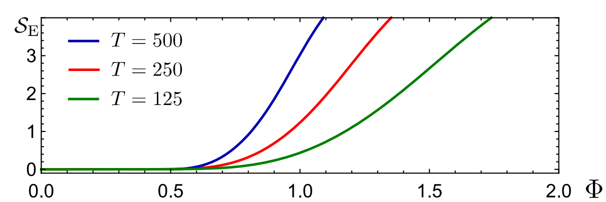

FIG. 3. Entanglement entropy relative to the initial state vs flux density for $\Delta = 0.5$ and $\langle \hat{s}_j^z \rangle = 0.1$. The infinitely large system is divided in two halves and different velocities of the flux density are considered $\Phi(t) = 2\pi t/T$. We interpret the plot as follows: the initial filling is a Fermi sea in the first string and a finite change in flux density is needed for translating it up to the boundaries $\lambda \pm \infty$. As long as the boundaries are not involved, no entropy is produced. As soon $\lambda = \pm\infty$ is reached, the thermodynamic entropy of the GGE starts to increase, accordingly with Eq. (7). GHD predicts an infinite entanglement entropy of the infinite half as soon as Φ overcomes the critical value: consistently, the slopes in the plot increase with T.

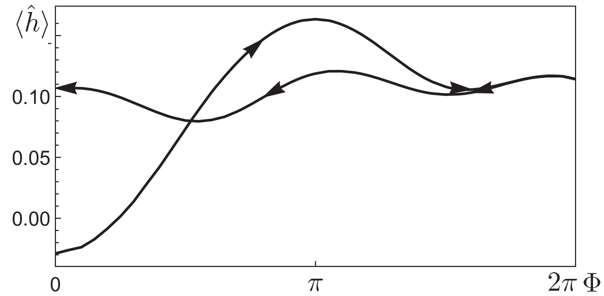

FIG. 4. The irreversibility of the GHD equations is displayed for the density of energy for $\Delta = 0.5$ and $\langle \hat{s}^z_j \rangle = 0.4$. The flux density is changed from 0 to 2π, then back.

Conclusions.—In this Letter we investigated the effects of integrability on slow out-of-equilibrium protocols, focusing on the experimentally relevant case of magnetic flux in the XXZ spin chain. We unveiled the possibility of having fully reversible dynamics in a truly interacting model, despite the absence of any energy gap, as usually required by the adiabatic theorem.

However, the reversibility of the process is deeply rooted into the thermodynamic description of the system: for $|\Delta| < 1$ bound states can be recombined in an irreversible manner. We provide a hydrodynamic description of both regimes, finding good agreement with numerical simulations. We show that GHD in the presence of force fields [59] can be incomplete when lattice systems are considered due to the instability of bound states: we complement it via the maximum-entropy principle.

Finally, our numerical simulations show that, for $|\Delta| < 1$, the breaking of reversibility is associated with a very slow convergence to the hydrodynamic description. We suspect that this phenomenon is associated with a divergent relaxation time scale, similar to what happens when quantum phase transitions are dynamically crossed. In this case, deviations to GHD could be universal and analogous to the Kibble-Zurek mechanism [93]. We postpone the analysis of this intriguing possibility to future studies.

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