Work Distributions on Quantum Fields

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(Received 14 February 2019; revised manuscript received 22 May 2019; published 21 June 2019)

We study the work cost of processes in quantum fields without the need of projective measurements, which are always ill defined in quantum field theory. Inspired by interferometry schemes, we propose a work distribution that generalizes the two-point measurement scheme employed in quantum thermodynamics to the case of quantum fields and avoids the use of projective measurements. The distribution is calculated for local unitary processes performed on Kubo-Martin-Schwinger (thermal) states of scalar fields. Crooks theorem and the Jarzynski equality are shown to be satisfied for a family of spatiotemporally localized unitaries, and some features of the resulting distributions are studied as functions of temperature and the degree of localization of the unitary operation. We show how the work fluctuations become much larger than the average as the process becomes more localized in both time and space.

DOI: 10.1103/PhysRevLett.122.240604

Introduction.—At microscopic scales average quantities no longer characterize completely the state of a system or the features of a thermodynamic process. There, stochastic or quantum fluctuations become relevant, being of the same order of magnitude as the expectation values [1–3]. It is therefore important to develop tools that allow us to study the properties of these fluctuations to fully understand thermodynamics at the small scales.

One of the best studied quantities in this context is work of out-of-equilibrium processes, and its associated fluctuations. The notion of work is an empirical cornerstone of macroscopic equilibrium thermodynamics. However, work in microscopic quantum scenarios is a notoriously subtle concept (e.g., it cannot be associated to an observable [4]), and although there is no single definition of work distributions and work fluctuations in quantum theory, several possibilities have been proposed (see, e.g., Ref. [5]). Perhaps the most established notion of work fluctuations is that defined through the two-point measurement (TPM) scheme [6,7], where the work distribution of a process is obtained by performing two projective measurements of the system's energy, at the beginning and at the end of the process. The TPM formalism defines a work distribution with a number of desirable properties: it is linear on the input states, it agrees with the unambiguous classical definition for states diagonal in the energy eigenbasis, and it yields a number of fluctuation theorems in different contexts [1,7,8].

An important caveat of this definition is that it cannot be straightforwardly generalized to processes involving quantum fields: projective measurements in quantum field theory (QFT) are incompatible with its relativistic nature. They cannot be localized [9], they can introduce ill defined operations due to UV divergences and, among other serious problems, they enable superluminal signaling even in the most innocent scenarios [10]. For these reasons, it has been strongly argued that projective measurements should be banished from the formalism of any relativistic field theory [10–12]. However, quantum fields are certainly subject to a wealth of thermodynamic and nonequilibrium phenomena, and as such it should be possible to define an operationally meaningful work distribution, potentially different from the standard TPM scheme. One avenue to build such a work distribution is through the ability to operate on quantum fields through locally coupling other systems, such as, e.g., atoms or particle detectors. This allows the performance of measurements on the field that are well defined [13] and physically meaningful [14]. Thus, whichever definition we construct for the work distribution, it should be based on such physically attainable localized measurements, and should not rely on projective measurements as previous works attempted (e.g., Ref. [15]).

In recent works [16,17], it was shown that the complete work distribution given by the TPM scheme for a finite dimensional system can be measured by performing measurements on an auxiliary qubit, in what is called a Ramsey interferometric scheme. This was experimentally implemented in Ref. [18]. Inspired by this idea, we propose a definition of work distributions in quantum fields based on the Ramsey scheme which is in fact well defined for a QFT despite the impossibility of projective measurements. We show that this new distribution satisfies the usual Jarzynski and Crooks theorems when the field is initially in a Kubo-Martin-Schwinger (KMS) state (the states that generalize thermal Gibbs states for quantum fields [19,20]) and evolves through a spatially localized unitary. This shows that such a work distribution is well defined for fields even though projective measurements are not. We also obtain analytical expressions for the variance and the average of the work distribution for some useful simple cases of local field operations. Finally, we discuss how, through either Crooks or Jarzynski's theorems, the proposed distribution can be used as a new way of computing ratios of partition functions between field theories, potentially yielding simpler approaches to the problem than path integral methods.

TPM work distributions and Ramsey scheme.—Consider a quantum field initially in an equilibrium KMS state $\hat{\rho}$ of temperature β^{-1}, which is driven out of equilibrium by a time-dependent Hamiltonian $\hat{H}(t)$, turned on during an interval $[0, T]$. The work distribution quantifies the work cost of the unitary process on the field $\hat{U}(T,0)$ generated by the Hamiltonian $\hat{H}(t)$.

As discussed above, projective measurements cannot be implemented in quantum fields because they are incompatible with relativistic causality [10–12]. Thus, the TPM scheme cannot be readily applied to processes involving quantum fields. However, the Ramsey scheme, which only involves interactions with a low-dimensional ancilla, provides an indirect way to gather the same work statistics. For completeness, let us review the TPM scheme to define a work distribution. The steps are the following.

1. A projective measurement of $\hat{H}(0)$ is done on the initial state $\hat{\rho}$. This yields the energy measured as E_i and the postmeasurement state $|E_i\rangle\langle E_i|$.

2. Unitary evolution of the postmeasurement state according to the unitary associated to the process $\hat{U}(T,0)$.

3. A projective measurement of $\hat{H}(T)$ is done on $\hat{U}(T,0)|E_i\rangle\langle E_i|\hat{U}^\dagger(T,0)$, returning the value E'_j.

The possible values of the work $w^{(ij)}$ are defined as $w^{(ij)} = E'_j - E_i$. The work probability distribution is

$$P(W) = \sum_{(ij)} \delta(W - w^{(ij)}) \langle E_i|\rho|E_i\rangle |\langle E'_j|\hat{U}(T,0)|E_i\rangle|^2, \quad (1)$$

with a corresponding characteristic function

$$\tilde{P}(\mu) = \int P(W) e^{i\mu W} dW = \langle e^{i\mu W}\rangle. \quad (2)$$

It is also important to define a "time-reversed" process, in which the driving has the opposite temporal order. That is, 1. a projective measurement is done on the basis of $\hat{H}(T)$, yielding $E'_{j,\mathrm{rev}}$, 2. the unitary evolution $\hat{U}_{\mathrm{rev}}(T,0)$ corresponding to the driven Hamiltonian $\hat{H}(T-t)$ with $t = [0,T]$ is implemented, 3. a final projective measurement in the basis of $\hat{H}(0)$ is implemented returning the value $E_{i,\mathrm{rev}}$.

The corresponding work probability distribution is

$$P_{\mathrm{rev}}(W) = \sum_{(ij)} \delta(W - w^{(ji)}_{\mathrm{rev}}) \times \langle E'_{j,\mathrm{rev}}|\rho|E'_{j,\mathrm{rev}}\rangle |\langle E_{i,\mathrm{rev}}|\hat{U}(T,0)|E'_{j,\mathrm{rev}}\rangle|^2, \quad (3)$$

where $w^{(ji)}_{\mathrm{rev}} = E_{i,\mathrm{rev}} - E'_{j,\mathrm{rev}}$. We can also define $\tilde{P}_{\mathrm{rev}}(\mu) = \int P_{\mathrm{rev}}(W) e^{i\mu W} dW$.

In the original proposals [16,17], Ramsey interferometry was employed to probe the TPM work distributions as follows: the system of interest is coupled to an auxiliary qubit, which engages the system in an evolution conditional on whether the qubit is excited or not. By preparing the qubit in a superposition of ground and excited states, this process transfers the data about the characteristic function of the TPM work distribution to the state of the qubit. This is thus a rather "noninvasive" procedure to acquire statistics which otherwise would require projective measurements. The steps are as follows.

1. The system and the auxiliary qubit are prepared in the product state $\hat{\rho} \otimes |0\rangle\langle 0|$, where $\hat{\rho}$ is the state of the quantum system at the beginning of the thermodynamic process.

2. A Hadamard gate is applied on the qubit.

3. The system and the auxiliary qubit evolve unitarily according to

$$\hat{M}_\mu = \hat{U}_S e^{-i\mu\hat{H}(0)} \otimes |0\rangle\langle 0| + e^{-i\mu\hat{H}(T)}\hat{U}_S \otimes |1\rangle\langle 1|. \quad (4)$$

Here $\hat{U}_S$ is the unitary acting on the system between times 0 and T.

4. A second Hadamard is applied to the qubit.

At the end of this procedure, we obtain that the reduced state of the auxiliary qubit is $\hat{\rho}_\mu = \frac{1}{2}\{\mathbb{1} + \mathrm{Re}[\tilde{P}(\mu)]\hat{\sigma}_z + \mathrm{Im}[\tilde{P}(\mu)]\hat{\sigma}_y\}$. By iterating this process over many values of μ and performing state tomography, the work distribution of any unitary process on a system of interest can then be constructed without projective measurements.

Work distributions for thermal states of quantum fields.—We will now design a version of the Ramsey scheme to obtain a characteristic function that defines the work distribution of a process, which will be a localized unitary on a scalar field. Consider a scalar quantum field $\hat{\phi}(t, \boldsymbol{x})$ written in terms of plane-wave modes as

$$\hat{\phi}(t,\boldsymbol{x}) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2\omega_{\boldsymbol{k}}}} (\hat{a}_{\boldsymbol{k}} e^{i\mathsf{k}\cdot\mathsf{x}} + \hat{a}^\dagger_{\boldsymbol{k}} e^{-i\mathsf{k}\cdot\mathsf{x}}), \quad (5)$$

where $\mathsf{k}\cdot\mathsf{x} \coloneqq \boldsymbol{k}\cdot\boldsymbol{x} - \omega_{\boldsymbol{k}} t$, $\omega_{\boldsymbol{k}} = \sqrt{m^2 + \boldsymbol{k}^2}$, and $[\hat{a}_{\boldsymbol{p}}, \hat{a}^\dagger_{\boldsymbol{q}}] = \delta^{(3)}(\boldsymbol{p} - \boldsymbol{q})$. We take the field to be in a KMS state [19,20] of inverse temperature β, $\hat{\rho}_\beta$. KMS thermality generalizes Gibbs's notion of thermality to cases where, due to the dimensionality of the Hilbert space, Gibbs thermal states are not well defined. This is the case of QFTs, where usually the partition function is ill defined. More formally, for a KMS state $\hat{\rho}_\beta$ (with inverse KMS temperature β) with respect to time translations generated by a Hamiltonian $\hat{H}$, the two-point correlator $\mathcal{W}_{\hat{\rho}}(\tau,\tau') \coloneqq \mathrm{Tr}\{\hat{\rho}\,\hat{\phi}[t(\tau)\boldsymbol{x}(\tau)]\hat{\phi}[t(\tau')\boldsymbol{x}(\tau')]\}$ satisfies the following two conditions (see, among many others, Refs. [21,22]): 1. $\mathcal{W}_\rho(\tau,\tau') = \mathcal{W}_\rho(\Delta\tau)$ (stationarity), 2. $\mathcal{W}_\rho(\Delta\tau + i\beta) = \mathcal{W}_\rho(-\Delta\tau)$ ($\mathbb{C}$ antiperiodicity).

Notice that the vacuum state is a KMS state with $\beta \to \infty$, that is, zero temperature.

We proceed to characterize the localized unitary we apply on the field. For a free scalar field, any local observable is a linear combination of the field amplitude $\hat{\phi}$ and its canonical momentum $\hat{\pi}$. For concreteness, in this Letter, we focus on unitaries acting on the field that are generated by Hamiltonians of the form

$$\hat{H}_\phi(t) = \hat{H}_0 + \lambda\chi(t)\int_{\mathbb{R}^3} d^3\boldsymbol{x} F(\boldsymbol{x})\hat{\phi}(t,\boldsymbol{x}) = \hat{H}_0 + \hat{H}_I(t), \tag{6}$$

in the interaction picture, where $\hat{H}_0$ is the free Hamiltonian of the field, and $\chi(t)$ and $F(\boldsymbol{x})$ are the switching and smearing functions, respectively. We assume that the switching function has strong support in a finite region [23] and, without loss of generality, we take the strong support of the switching function to be in the interval $[0,T]$, where 0 and T are the starting and ending times of the process under study. In other words, the field evolves freely (or very approximately freely if the switching function is not strictly compact) except for the interval $[0,T]$, where we perform a spatiotemporally localized unitary operation on the support of $F(\boldsymbol{x})$. By doing this, we obtain that $\hat{H}_\phi(0) = \hat{H}_\phi(T) = \hat{H}_0$, which simplifies our analysis. This is a particular unitary operation on a localized field observable (representing a multimode displacement operation [24]). Considering localized unitaries generated by a smeared $\hat{\pi}$ is completely analogous, so this particular case is easily generalizable to all localized unitaries on a free field.

At the beginning of the Ramsey scheme, the state of the field-qubit system is $\hat{\rho} = \hat{\rho}_\beta \otimes |0\rangle\langle 0|$. Applying the Hadamard on the qubit results in $\hat{\rho}_0 = \hat{\rho}_\beta \otimes |+\rangle\langle +|$. We apply the controlled unitary evolution

$$\hat{M}_\mu = \hat{U}_\phi(T)e^{-i\mu\hat{H}_0} \otimes |0\rangle\langle 0| + e^{-i\mu\hat{H}_0}\hat{U}_\phi(T) \otimes |1\rangle\langle 1|, \tag{7}$$

where $\hat{U}_\phi(T)$ is the unitary on the field generated by the Hamiltonian Eq. (6), given by

$$\hat{U}_\phi(T) = \mathcal{T}\exp\left(-i\lambda\int_{\mathbb{R}} dt\chi(t)\int_{\mathbb{R}^3} d^3\boldsymbol{x}F(\boldsymbol{x})\hat{\phi}(t,\boldsymbol{x})\right), \tag{8}$$

where $\mathcal{T}$ represents time ordering. Assuming that the coupling λ is small enough, we can obtain an approximate expression for $\hat{U}_\phi(T)$ through a Dyson expansion: $\hat{U}_\phi(T) = \mathbb{1} + \hat{U}^{(1)} + \hat{U}^{(2)} + \mathcal{O}(\lambda^3)$, where in the interaction picture

$$\hat{U}^{(1)} = -i\lambda\int_{\mathbb{R}} dt\hat{H}_I(t),$$
$$\hat{U}^{(2)} = -\lambda^2\int_{-\infty}^{\infty} dt\int_{-\infty}^{t} dt'\hat{H}_I(t)\hat{H}_I(t'). \tag{9}$$

The reduced state of the qubit at time T can be written as $\hat{\rho}_T = \hat{\rho}_T^{(0)} + \hat{\rho}_T^{(1)} + \hat{\rho}_T^{(2)} + \mathcal{O}(\lambda^3)$, where $\hat{\rho}_T^{(i)}$ is proportional to λ^i (see Ref. [25] for details).

$\mathrm{Tr}[\hat{\sigma}_z\hat{\rho}_\mu]$ and $\mathrm{Tr}[\hat{\sigma}_y\hat{\rho}_\mu]$ give the real and imaginary parts, respectively, of the characteristic function Eq. (2). Using the KMS two-point correlator (see, e.g., Ref. [26]), we can write the characteristic function for this process as

$$\begin{aligned}\tilde{P}(\mu) \coloneqq 1 + \lambda^2\int\frac{d^3\boldsymbol{k}}{(2\pi)^3 2\omega_{\boldsymbol{k}}(e^{\beta\omega_k}-1)}|\tilde{\chi}(\omega_{\boldsymbol{k}})|^2|\tilde{F}(\boldsymbol{k})|^2 \\ \times(e^{\beta\omega_k}+1)[\cos(\mu\omega_{\boldsymbol{k}})-1] \\ + i\lambda^2\int\frac{d^3\boldsymbol{k}}{(2\pi)^3 2\omega_{\boldsymbol{k}}}|\tilde{\chi}(\omega_{\boldsymbol{k}})|^2|\tilde{F}(\boldsymbol{k})|^2\sin(\mu\omega_{\boldsymbol{k}}).\end{aligned} \tag{10}$$

By taking the inverse Fourier transform of this characteristic function, the work probability distribution can be obtained. When the smearing function is spherically symmetric and the field is massless, it is

$$\begin{aligned}P(W) = (1-p)\delta(W) + \frac{\lambda^2}{2\pi}|\tilde{\chi}(W)|^2|\tilde{F}(W)|^2 W \\ \times\left(\frac{e^{\beta W}}{e^{\beta W}-1}\Theta(W) + \frac{1}{1-e^{-\beta W}}\Theta(-W)\right),\end{aligned} \tag{11}$$

where $p \coloneqq \int_{W\neq 0} dWP(W)$ and $\Theta(W)$ is the Heaviside function. Note that the case of the vacuum state of the field can be obtained by taking the well-defined limit $\beta\to\infty$ on Eq. (11).

In Fig. 1, we plot the work distribution for the unitary Eq. (8) (omitting the deltas at the origin) acting on initial KMS states with $\beta = 1$ and the vacuum state ($\beta\to\infty$), for a particular choice of the switching and smearing functions. As shown in Fig. 1, for the nonzero temperature states, there is a nonzero probability of the field doing work against the performer of the unitary, $W < 0$. However, the probability of $W > 0$ is larger than the probability of $W < 0$, as granted by the second law. For the vacuum case the performer of the unitary always has to work. As the duration of the process goes to infinity, the probability distribution gets concentrated around zero and the negative part of the distribution vanishes, as expected in the quasistatic limit.

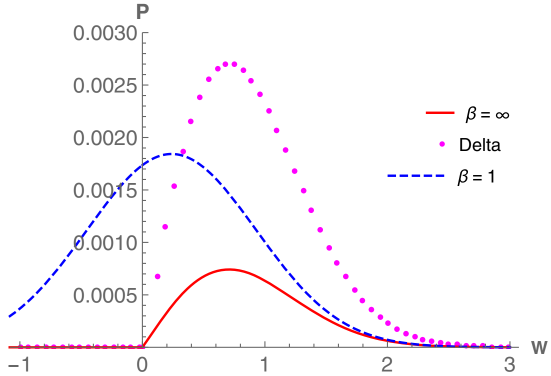

FIG. 1. Work distribution for three cases (a) a localized unitary acting on the vacuum, (b) the same localized unitary acting on KMS states of finite temperature, and (c) delta-coupling unitary acting on the vacuum. For (a) and (b), the switching function is of the form $\chi(t) = \exp\{-[t-(T/2)]^2(T^2/72)^{-1}\}$, with $T=1$, and $F(\boldsymbol{x})$ is a normalized Gaussian distribution with $\sigma = 1$. Note that the length of the interval $[0, T]$ is 12 times the standard deviation of the switching function. For (c), $F(\boldsymbol{x})$ is a normalized Gaussian distribution with $\sigma = 1$.

From Eq. (10), we can now calculate the moments of $P(W)$ to gain some insight about the energy cost of applying a localized unitary to a quantum field. Since $\tilde{P}(\mu) = \langle e^{i\mu W}\rangle$, the kth moment is

$$\langle W^k\rangle = i^{-k}\frac{d^k}{d^k\mu}\tilde{P}(\mu)|_{\mu=0}. \tag{12}$$

From Eqs. (12) and (10), we obtain that the first and second moments of the work distribution for the vacuum are

$$\langle W\rangle = \lambda^2\int\frac{d^3\boldsymbol{k}}{(2\pi)^3 2}|\tilde{\chi}(\omega_{\boldsymbol{k}})|^2|\tilde{F}(\boldsymbol{k})|^2, \tag{13}$$

$$\langle W^2\rangle = \lambda^2\int\frac{d^3\boldsymbol{k}}{(2\pi)^3 2}|\tilde{\chi}(\omega_{\boldsymbol{k}})|^2|\tilde{F}(\boldsymbol{k})|^2\omega_{\boldsymbol{k}}, \tag{14}$$

obtaining $\sigma_W^2 = \langle W^2\rangle - \langle W\rangle^2 = \langle W^2\rangle + \mathcal{O}(\lambda^4)$ for the work variance.

An interesting observation is that, for the vacuum, if we consider unitaries that are very localized in time and space, both $\tilde{\chi}(\omega_{\boldsymbol{k}})$ and $\tilde{F}(\boldsymbol{k})$ will be wide in frequency space, which means that the work variance will become larger than the expectation value, making the variance of the work increasingly significant as the operation on the field becomes increasingly localized in both time and space.

For an arbitrary KMS state of inverse temperature β, the value for $\langle W\rangle$ coincides with that of the vacuum (and $\langle W\rangle \geq 0$ as expected from the passivity of KMS states). In fact, since the imaginary part of the characteristic function does not depend on β, none of the odd-numbered moments will depend on temperature. For the variance, we have

$$\sigma_\beta^2 = \frac{\lambda^2}{2}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{e^{\beta\omega_{\boldsymbol{k}}}+1}{e^{\beta\omega_{\boldsymbol{k}}}-1}|\tilde{\chi}(\omega_{\boldsymbol{k}})|^2|\tilde{F}(\boldsymbol{k})|^2\omega_{\boldsymbol{k}} + \mathcal{O}(\lambda^4), \tag{15}$$

showing that it monotonically increases with temperature.

We can also check that the Crooks theorem [27] is satisfied for these localized unitaries. The theorem states that for a process in which the Hamiltonian evolves from $\hat{H}(0) = \hat{H}_1$ to $H(T) = \hat{H}_2$, together with its time-reversed process, we have that

$$\frac{P(W)}{P_{\text{rev}}(-W)} = e^{\beta W}\frac{Z_2}{Z_1}, \tag{16}$$

where Z_1, Z_2 are the partition functions of the thermal states of $\hat{H}(t_1)$ and $\hat{H}(t_2)$ and the initial state must be thermal in both processes, with the corresponding Hamiltonian.

In our example, $\tilde{P}(\mu) = \tilde{P}_{\text{rev}}(-\mu + i\beta)$ from Eq. (10), and since $\hat{H}(0) = \hat{H}(T) = \hat{H}_0$, $Z_2/Z_1 = 1$. Thus, by taking the inverse Fourier transform we recover Eq. (16). Finally, the Jarzynski equality $\langle e^{-\beta W}\rangle = 1$, which is implied from the Crooks theorem, is satisfied. This can be seen just by evaluating the characteristic function at $\mu = i\beta$.

A nonperturbative example.—The examples in the previous section used small perturbations acting on thermal states only for calculational convenience. However, the work distribution we introduced is not limited to perturbative scenarios. Indeed, one of the main aims of fluctuation theorems is precisely to go beyond the regime of small perturbations by providing relations that hold for states arbitrarily far from equilibrium.

To illustrate this, we consider an intense unitary applied on the field very fast on a spatial distribution given by $F(\boldsymbol{x})$. In this case, $\chi(t) = \delta(t)$, and the unitary in Eq. (8) becomes $\hat{U}_\phi(T) = \exp[-i\lambda\int d^3\mathbf{x}F(\mathbf{x})\hat{\phi}(\mathbf{x})]$. Following the Ramsey scheme protocol, and using the nonperturbative techniques detailed in Ref. [28], it is possible to obtain closed forms for the characteristic function of the work distribution, which is [25]

$$\tilde{P}(\mu) = \exp\left(\lambda^2\int\frac{d^3\mathbf{k}}{(2\pi)^3 2\omega_{\mathbf{k}}}|\tilde{F}(\mathbf{k})|^2(e^{i\mu\omega_{\mathbf{k}}}-1)\right). \tag{17}$$

Choosing a normalized Gaussian centered at zero as smearing, and changing to polar coordinates (since the smearing is spherically symmetrical), yields for the characteristic function (see Ref. [25] for details)

$$\tilde{P}(\mu) = e^{-\lambda^2/8\pi^2\sigma^2} e^{-\{\lambda^2 e^{-\mu^2/4\sigma^2}[2e^{\mu^2/4\sigma^2}\mu\sigma\mathcal{D}(\mu/2\sigma)-2e^{\mu^2/4\sigma^2}\sigma^2-i\sqrt{\pi}\mu\sigma]\}/(4\pi^2)4\sigma^4}, \tag{18}$$

where $\mathcal{D}(x)$ is the Dawson integral [29], defined as $\mathcal{D}(x) \coloneqq \exp(-x^2)\int_0^x \exp(y^2)dy$. By taking the inverse Fourier transform numerically, we see how the Crooks theorem Eq. (16) is also satisfied in this case, as $P(W) = 0$ for $W < 0$ (note that this is the Crooks theorem for $\beta \to \infty$ when the forward and reverse processes are identical), as we show in Fig. 1.

Conclusion.—The notion of work distributions for localized operations on quantum fields is challenging because (a) energy eigenstates are not localized and (b) projective measurements cannot be allowed in a relativistic quantum theory [10–12]. The TPM scheme employed in the literature [5] is hence ill defined in QFT, but we have shown that one can still make sense of it via the Ramsey scheme that was designed to measure TPM work distributions [16,17]. We propose a well-defined work distribution in QFT that, unlike Ref. [15], does not require the existence of projective measurements and does not inherit any complications from the fact that energy eigenstates are nonlocal. We have shown that this work distribution satisfies both the Jarzynski equality and the Crooks theorem for KMS states for a general class of perturbative unitary actions arbitrarily localized in space and time. We also explicitly showed how the Crooks theorem is satisfied for a general class of fast nonperturbative actions on the field vacuum generated by localized observables. These are limited cases. Showing that the Crooks and Jarzynski theorems are satisfied in the most general case is a nontrivial problem. It is known that for nonrelativistic quantum systems, unitary operations acting on Gibbs thermal states implies satisfaction of these theorems [1,30], but showing whether this is true for all unitaries on all KMS states of fully relativistic field theories will require advanced tools from algebraic quantum field theory [13,31]. This is an interesting question that should be addressed elsewhere but is out of the scope of this Letter.

The proposed work distribution also suggests experiments where it can be measured. A potential setup would be a quantum field in a superconducting transmission line to which we couple superconducting qubits. The control in time that is required for an experiment implementing the example that we present in the Letter can be achieved with the switchable coupling that has been experimentally realized in Ref. [32]. The fact that the Ramsey scheme can be implemented in superconducting circuits was shown in, e.g., Ref. [33], and the fact that a fully relativistic QFT setup is implementable in superconducting circuits in those regimes can be seen, e.g., in Refs. [34–36].

An interesting observation is that the work distribution that we define can be used to compute ratios of partition functions of field theories. Indeed, we can invert the relationship Eq. (16) and write

$$\frac{Z_2}{Z_1} = e^{-\beta W}\frac{P(W)}{P_{\mathrm{rev}}(-W)}. \tag{19}$$

This can, in fact, be more simply obtained from Jarzynski's equality:

$$\frac{Z_2}{Z_1} = \langle e^{-\beta W}\rangle. \tag{20}$$

This potentially provides a new way to compute these ratios, analytically, numerically, or even experimentally, by measuring the work distribution through a Ramsey scheme. These ratios are remarkably difficult to calculate in QFT through path integrals, which makes new methods to access it a research avenue that merits exploration. The idea of calculating the ratio of partition functions from a nonequilibrium process has been used in very different contexts (see, e.g., Refs. [37–39]).

With our framework, we have been able to obtain expressions for the work fluctuations associated to a process generated by a local Hamiltonian on a scalar field. We observe that the work fluctuations increase with temperature, and that they dominate the average work cost as the process becomes increasingly localized in both time and space. Also, we find that for KMS states of finite temperature, there is a nonzero probability of the field doing work when the process is of finite duration. It should be interesting to see how the work distribution relates to the variation of internal energy in the field in adiabatic and nonadiabatic processes. The internal energy of the field is given by the renormalized stress-energy density, and exploring the connection between the stress-energy density deposited (or extracted) from the field and the work distributions of the processes where the energy is deposited can shed some light into the thermodynamics of local processes in quantum field theory, a particularly relevant aspect of phenomena ranging from entanglement harvesting [40–43], quantum energy teleportation [44], or the Unruh and Hawking effects [45].

E. M.-M. is supported by an Ontario Early Researcher Award and the NSERC Discovery program. Research at Perimeter Institute is supported by the Government of Canada through the Department of Innovation, Science and Economic Development and by the Province of Ontario through the Ministry of Research, Innovation and Science. A. O. acknowledges the support of Fundacio Privada Cellex, through a Mobility Research Award.

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