Mixed-Order Symmetry-Breaking Quantum Phase Transition Far from Equilibrium

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We study the current-carrying steady state of a transverse field Ising chain coupled to magnetic thermal reservoirs and obtain the nonequilibrium phase diagram as a function of the magnetization potential of the reservoirs. Upon increasing the magnetization bias we observe a discontinuous jump of the magnetic order parameter that coincides with a divergence of the correlation length. For steady states with a nonvanishing conductance, the entanglement entropy at zero temperature displays a bias dependent logarithmic correction that violates the area law and differs from the well-known equilibrium case. Our findings show that out-of-equilibrium conditions allow for novel critical phenomena not possible at equilibrium.

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Introduction.—Nonequilibrium phases of quantum matter in open systems is a topical issue of immediate experimental relevance [1–6]. However, a theoretical framework for the description of out-of-equilibrium, strongly correlated systems is at present incomplete and requires the further development of reliable techniques for nonequilibrium conditions (see, e.g., Refs. [7–9] and references therein). The influence of a nonthermal drive on phase boundaries and quantum critical points (QCP) is of particular interest.

An important class of nonequilibrium states are current-carrying steady-states (CCSS) that emerge in the long-time limit of systems coupled to reservoirs which are held at different thermodynamic potentials. These states are characterized by a steady flow of otherwise conserved quantities, such as energy, spin, or charge. They can be realized in solid-state devices [1–3] and have recently also became available in cold atomic setups [4].

For Markovian processes, substantial progress has been made due the discovery of exact solutions for boundary driven Lindblad dynamics [10–13], allowing for the characterization of certain nonequilibrium phases and phase transitions. In these cases, however, the Markovian condition substantially simplifies the dynamics. As a result, its validity is confined to extreme nonequilibrium conditions (e.g., large bias) that cannot be connected to thermal equilibrium [14,15]. Nonthermal steady states in Luttinger liquids have also been studied [16–18], but the results are less general than their equilibrium counterparts. Other methods to study CCSS include looking at the asymptotic dynamics in pairs of semi-infinite quantum wires following quenches of the hopping connecting the pairs [19–23], Bethe ansatz-based approaches [24,25] that exploit the properties of integrable systems, hybrid approaches involving Lindblad dynamics [26], and more phenomenological approximations based on Boltzmann kinetic equations [27,28].

Another guiding element is the occurrence of scaling and criticality, which signal the absence of intrinsic energy scales and make the system particularly susceptible to any nonequilibrium drive [29–35]. Phase transitions under nonequilibrium conditions [7,36–44] were shown to allow intrinsic nonequilibrium universal properties not seen at equilibrium. Nevertheless, a systematic approach describing CCSS is not available and exact solutions therefore must serve as a guiding principle.

In this Letter, we discuss an order-disorder symmetry breaking transition induced by nonequilibrium conditions in one of such exactly solvable models, i.e., a spin chain that admits an exact solution by a mapping to a noninteracting fermionic system. Besides presenting the phase diagram and a characterization of various nonequilibrium phases, we identify a remarkable mixed-order quantum phase transition, where a discontinuous jump of the order parameter occurs in the presence of a divergent correlation length. The coexistence of such defining features of first- and second-order phase transitions implies the emergence a universality class specific to nonequilibrium conditions, for which an effective field-theoretic description is yet to be developed.

Model.—The model we consider is depicted in Fig. 1(a) and consists of an Ising spin chain of length L, exchange coupling J, and an applied transverse field h, coupled to two zero-temperature magnetic reservoirs at $r = r_{\mathsf{L}} \equiv 1$ and $r = r_{\mathsf{R}} \equiv L$, respectively. The total Hamiltonian is given by

$$H = -J \sum_{r=1}^{L-1} \sigma_r^x \sigma_{r+1}^x - h \sum_{r=1}^{L} \sigma_r^z + \sum_{l=\mathsf{L},\mathsf{R}} (H_l + H_{\mathsf{C},l}), \qquad (1)$$

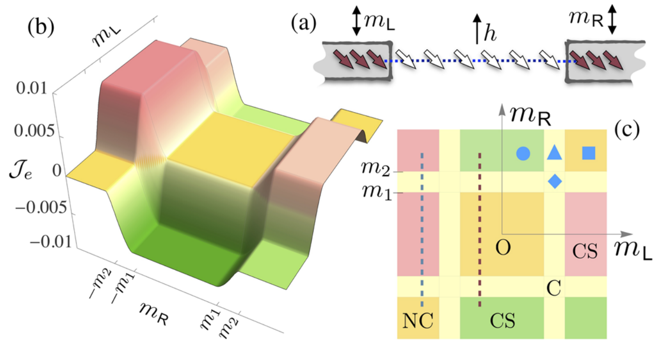

FIG. 1. (a) Sketch of the model—transverse field Ising chain coupled at its edges to magnetic reservoirs, L and R, held at magnetizations m_L and m_R, respectively. (b) Energy current, $\mathcal{J}_e$, flowing through the chain as function of m_L and m_R. (c) Schematic phase diagram—color coding matches that of (b). The phase labels are O for ordered, NC for nonconducting, C for conducting, and CS for conducting saturated. The properties of these phases are discussed in the text. Properties displayed in Figs. 2 and 3 correspond to the parameters along the dashed lines; geometric symbols mark the parameters used in Fig. 4. Here $\Gamma_{L,R} = 0.01$.

where $\sigma_r^{x,y,z}$ are the Pauli matrices acting on site r. The reservoirs are described by isotropic XY models, $H_l = -J_l \sum_{r\in\Omega_l}(\sigma_r^x\sigma_{r+1}^x + \sigma_r^y\sigma_{r+1}^y) - m_l M_l$ with $\Omega_L = \{-\infty, \ldots, 0\}$, $\Omega_R = \{L+1, \ldots, \infty\}$, and the magnetization $M_l = \sum_{r\in\Omega_l}\sigma_r^z$ (which is a good quantum number, i.e., $[H_l, M_l] = 0$). The chain-reservoirs coupling Hamiltonians are $H_{C,l} = -J'_l(\sigma_{r'_l}^x\sigma_{r_l}^x + \sigma_{r'_l}^y\sigma_{r_l}^y)$, with $r'_L = 0$ and $r'_R = L+1$. Each reservoir is characterized by a set of gapless magnetic excitations within an energy bandwidth J_l and the average value of M_l is set by the magnetic potential m_l. Below we use J as our unit of energy, i.e., $J = 1$.

Nonequilibrium order-disorder phase transition.—The ground-state of the chain Hamiltonian H_C [the first two terms of Eq. (1)] has a continuous phase transition for $h = \pm 1$ that separates a $\mathbb{Z}_2$ symmetry broken state from a paramagnetic one. The symmetry-broken state can be characterized by an order parameter $\phi = \lim_{h_x\to 0}\lim_{L\to\infty}\langle\sigma_r^x\rangle$, $\forall\, r$, with h_x a magnetic field along x that explicitly breaks the $\mathbb{Z}_2$ symmetry. ϕ vanishes as $|\phi| = (1-h^2)^{1/8}$ [45] as the transition point is approached from the ordered side, i.e., $|h| \to 1$, with the critical exponent $\beta = 1/8$. The correlation length diverges as $\xi \propto (1-h^2)^{-\nu}$ with $\nu = 1$. This phase transition is in the universality class of the 2D classical Ising model and thus the QCP is described by a ϕ^4 theory.

Our primary concern in this Letter is the steady-state phase diagram that emerges far from equilibrium when $J'_l \neq 0$. The energy drained from the left reservoir is $\mathcal{J}_e = -i\langle[H, H_L]\rangle$, which equals the steady-state energy current in any cross section along the chain (detailed calculations are provided in the next section). The current $\mathcal{J}_e$ is depicted in Fig. 1(b) as a function of the left and right magnetic potentials, while Fig. 1(c) schematically shows its corresponding nonequilibrium phase diagram. We consider the case $|h| < 1$, for which the equilibrium phase is ordered. Interestingly, the ordered state survives a non-vanishing coupling to the reservoirs for $|m_{L,R}| < m_1$, with $m_1 = 2(-h+1) > 0$. The order parameter along the dashed-red segment of Fig. 1(c) is depicted in Fig. 2(a). Within the ordered phase ϕ does not depend on m_R. At $|m_R| = m_1$, ϕ drops discontinuously to zero as $L \to \infty$, and this limit is approached as $\phi \sim L^{-1/2}$ in the disordered phase ($|m_R| > m_1$). In this region we have also computed the correlation length ξ, shown in Fig. 2(b). For $m_R \to \mp m_1$ from the disordered phase we find a divergent behavior $\xi \propto |m_R \pm m_1|^{-\lambda}$, compatible with a critical exponent $\lambda = 1/2$ [46]. Our results imply that the discontinuous vanishing of ϕ at $|m_R| = m_1$ in the $L \to \infty$ limit, a characteristic feature of a first-order phase transition, is accompanied by a divergent correlation length, a hallmark of continuous phase transitions. Therefore, such a behavior cannot be accommodated within an equilibrium effective description. Below, some immediate implications of this significant finding will be further substantiated and analyzed. In particular, we will present the order-disorder transition in the context of a detailed description of the model and its other interesting non-equilibrium properties.

Methodology.—The full Hamiltonian, H, can be represented in terms of fermions through the so-called Jordan-Wigner mapping [47], $\sigma_r^+ = e^{i\pi\sum_{r'=0}^{r-1} c_{r'}^\dagger c_{r'}} c_r^\dagger$, where $c_r^\dagger/c_r$ creates(annihilates) a spinless fermion at site r. This leads to a Kitaev chain [48,49] in contact with two metallic reservoirs at chemical potentials $\mu_{L,R} = 2m_{L,R}$. The topological nontrivial phase corresponds to the ordered phase of the original spin model. The transformed Hamiltonian is quadratic and the chain contribution is given by $H_C = \frac{1}{2}\boldsymbol{\Psi}^\dagger \boldsymbol{H}_C \boldsymbol{\Psi}$, with $\boldsymbol{\Psi}^\dagger = (c_1^\dagger, \ldots, c_L^\dagger, c_1, \ldots, c_L)$, and where $\boldsymbol{H}_C$ is a $2L \times 2L$ Hermitian matrix respecting the particle-hole symmetry condition $\boldsymbol{S}^{-1}\boldsymbol{H}_C^T\boldsymbol{S} = -\boldsymbol{H}_C$ with $\boldsymbol{S} = \tau^x \otimes \mathbf{1}_{L\times L}$ and where τ^x interchanges particle and hole subspaces. In the fermionic representation, any

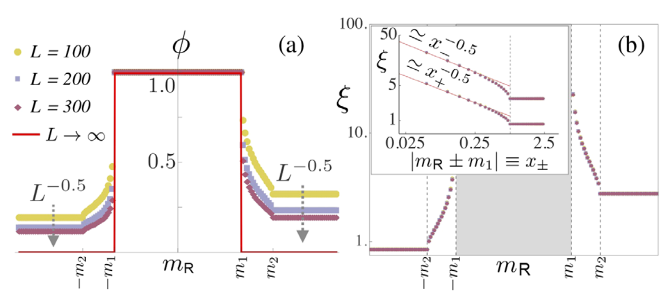

FIG. 2. (a) Order parameter, ϕ, computed for parameters along the red-dashed line in Fig. 1(c) for different system sizes. (b) Correlation length, ξ, for parameters along the red-dashed line in Fig. 1(c) and for the same system sizes in panel (a). The inset shows the log scaling of ξ near the transition points $m_R = \pm m_1$.

correlation function can be described in terms of the retarded, advanced, and Keldysh components of the single-particle Green's function [50].

In the following we make the simplifying assumption that the bandwidths of the reservoirs, $J_{l=\mathsf{L,R}}$, are much larger than all other energy scales ("wide band limit"). In this limit, the coupling to each reservoir l is completely determined by $\Gamma_l = \pi J_l'^2 D_l$, the hybridization energy scale, with D_l being the local density of states of the reservoir. Furthermore, we can define the non-Hermitian single-particle operator $\boldsymbol{K} = \boldsymbol{H}_\mathsf{C} - i\sum_{l=\mathsf{L,R}}(\boldsymbol{\gamma}_l + \hat{\boldsymbol{\gamma}}_l)$, with $\boldsymbol{\gamma}_l = \Gamma_l|r_l\rangle\langle r_l|$ and $\hat{\boldsymbol{\gamma}}_l = \Gamma_l|\hat{r}_l\rangle\langle\hat{r}_l|$, and where $|r\rangle$ and $|\hat{r}\rangle = \boldsymbol{S}|r\rangle$ are single-particle states. We assume that $\boldsymbol{K}$ is diagonalizable, having right and left eigenvectors $|\alpha\rangle$ and $\langle\tilde{\alpha}|$, with associated eigenvalues λ_α.

Equal-time observables can be obtained from the single-particle density matrix defined as $\boldsymbol{\chi} \equiv \langle \boldsymbol{\Psi}\boldsymbol{\Psi}^\dagger\rangle$, which is explicitly given by

$$\boldsymbol{\chi} = \frac{1}{2} + \sum_{l=\mathsf{L,R}}\sum_{\alpha\beta}|\alpha\rangle\langle\beta| \times \langle\tilde{\alpha}|[\boldsymbol{\gamma}_l I_l(\lambda_\alpha,\lambda_\beta^*) - \hat{\boldsymbol{\gamma}}_l I_l(-\lambda_\alpha,-\lambda_\beta^*)]|\tilde{\beta}\rangle \tag{2}$$

where $I_l(z,z') = -(1/\pi)\{[g(z-2m_l) - g(z'-2m_l)]/z - z'\}$ with $g(z) = \ln(-i\mathrm{sgn}[\mathrm{Im}(z)]z)$.

The current of energy which drains from the left reservoir is equal to the steady-state energy current in any cross section along the chain; thus it can be obtained from $\boldsymbol{\chi}$ as $\mathcal{J}_e = -\frac{1}{2}\mathrm{Tr}[\boldsymbol{J}_r\boldsymbol{\chi}]$, where r is arbitrary and

$$\boldsymbol{J}_r = -2ihJ[(1+\boldsymbol{S})|r-1\rangle\langle r|(1+\boldsymbol{S}) - \mathrm{H.c.}]. \tag{3}$$

The linear and nonlinear thermal conductivity, as well as other thermoelectric properties of the chain, are determined by $\mathcal{J}_e$.

Results.—As anticipated, $\mathcal{J}_e$ is able to discriminate between different phases. We have shown in Fig. 1(b) an example for $h = 0.2$, illustrating the typical behavior and leading to the phase diagram sketched in Fig. 1(c). Two phases with $\mathcal{J}_e = 0$, NC and O, arise around the condition $m_\mathsf{L} = m_\mathsf{R}$. Note, however, that this condition does not correspond to equilibrium for the fermionic system away from $m_\mathsf{R} = m_\mathsf{L} = 0$. This is due to the fact that the noninteracting p-wave superconductor does not conserve the number of particles which, in the spin representation, translates to the nonconservation of the total magnetization. A conducting phase, C, characterized by a nonzero conductance, $\partial_{m_{l=\mathsf{L,R}}}\mathcal{J}_e \neq 0$, arises for $|m_\mathsf{L}|$ or $|m_\mathsf{R}| \in (m_1, m_2)$, where $m_2 > 0$ is defined as $m_2 = 2(h+1)$. A set of phases to which we refer as current saturated, or CS, arise for $|m_\mathsf{L}|$ or $|m_\mathsf{R}| > m_2$ and are characterized by a finite current, $\mathcal{J}_e \neq 0$, and a vanishing conductance $\partial_{m_{l=\mathsf{L,R}}}\mathcal{J}_e = 0$.

In order to study the onset of order under nonequilibirum conditions, we have extended the equilibrium expression of the correlation function [47] to the general nonequilibrium case [50]. In particular, the two-point correlation function, $\mathbb{C}^{\alpha\beta}_{r,r'} = \langle\sigma^\alpha_r\sigma^\beta_{r'}\rangle - \langle\sigma^\alpha_r\rangle\langle\sigma^\beta_{r'}\rangle$, for $\alpha = \beta = x$ can be found in terms of $\boldsymbol{\chi}$ as follows:

$$\mathbb{C}^{xx}_{r,r'} = \det[i(2\boldsymbol{\chi}_{[r,r']} - 1)]^{\frac{1}{2}}, \tag{4}$$

where, for $r > r'+1$, $\boldsymbol{\chi}_{[r,r']}$ is a $2(r-r')$ matrix obtained as the restriction of $\boldsymbol{\chi}$ to the subspace in which $\mathbb{P}^T_{rr'} = \sum_{u=r'+1}^{r-1}(|u\rangle\langle u| + |\hat{u}\rangle\langle\hat{u}|) + |r_+\rangle\langle r_+| + |r'_-\rangle\langle r'_-|$ acts as the identity, with $|r_\pm\rangle = (|r\rangle \pm |\hat{r}\rangle)/\sqrt{2}$. The full derivation of Eq. (4) is given in the Supplemental Material [50].

Except for $\mathbb{C}^{xx}_{r,r'}$ in the ordered phase, O, all the other components of $\mathbb{C}^{\alpha\beta}_{r,r'}$, for $\alpha,\beta = x, y$, decay exponentially. ξ in Fig. 2(b) was obtained by fitting an exponentially decaying $\mathbb{C}^{xx}_{r,r'} \propto e^{-|r-r'|/\xi}$ to the numerical data generated by Eq. (4). For a finite system with $h_x = 0$, since the $\mathbb{Z}_2$ symmetry is never broken, ϕ can be computed by the relation $\phi^2 = \lim_{L\to\infty}\mathbb{C}^{xx}_{uL,u'L}$, with $u, u' \in (0,1)$. ϕ in Fig. 2(a) was computed in this way. Whenever m_R or m_L approaches the boundary m_1 of the ordered phase, we find that $\lambda(h) = 1/2$ for $0 < h < 1$, except for $h = 1/2$ where $\lambda(h = 1/2) = 2.5$ (we discuss this point in the Supplemental Material [50]).

Under nonequilibrium conditions we have also investigated the critical exponent ν, defined by $\xi \propto (h - h_c)^{-\nu}$ at fixed $m_{\mathsf{L,R}}$ [50]. Our numerical data indicate $\nu = \lambda = 1/2$, which differs from the equilibrium value, $\nu = 1$.

Entanglement entropy.—We now turn to the entropy content of the nonequilibrium state. The entropy of a subsystem S_ℓ, here taken to be a segment of the chain of length ℓ, is given by $E_{\mathsf{S}_\ell} = -\mathrm{Tr}[\hat{\rho}_{S_\ell}\ln(\hat{\rho}_{S_\ell})]$, with $\hat{\rho}_{S_\ell}$ the reduced density matrix. As the spin system can be mapped to noninteracting fermions, the entropy can be calculated from the fermionic model [53] and is given by $E_{\mathsf{S}_\ell} = -\mathrm{Tr}[\chi_{\mathsf{S}_\ell}\ln\chi_{\mathsf{S}_\ell}]$, where χ_{S_ℓ} is the single-particle density matrix restricted to S_ℓ. In the limit $\ell \to \infty$, the entropy behaves as [54]

$$E_{\mathsf{S}_\ell} = l_0\ell + c_0\log(\ell) + c_1. \tag{5}$$

Ground states of gapped systems in equilibrium obey the area law, i.e., $l_0 = c_0 = 0$, while gapless fermions and spin chains show a universal logarithmic violation of the area law with $c_0 = 1/3$ [53,55]. This result is a consequence of the violation of the area law in $1+1$ conformal theories in which case $c_0 = c/3$, where c is the central charge. For a nonequilibrium Fermi gas, it was shown that both l_0 and c_0 can be nonzero [56,57] and that c_0 depends on the system-reservoir coupling and is a nonanalytic function of the bias [57].

For the present case, the linear coefficient l_0 is shown in Fig. 3(a) for all phases, the details of the calculation are

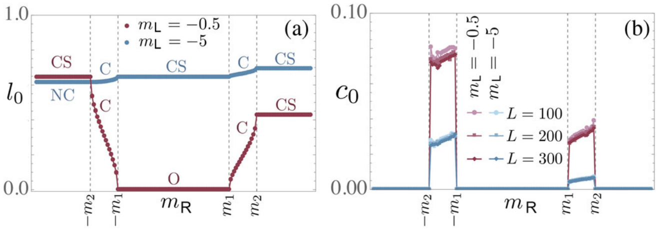

FIG. 3. Scaling analysis of the entanglement entropy, $E_{\mathsf{S}_\ell} \simeq l_0 \ell + c_0 \log \ell + c_1$, of a subsystem S_ℓ. Red (blue) data points correspond to parameters along the red-dashed (blue-dashed) line in Fig. 1(c). The color coding in panel (b) shows data points for different values of L.

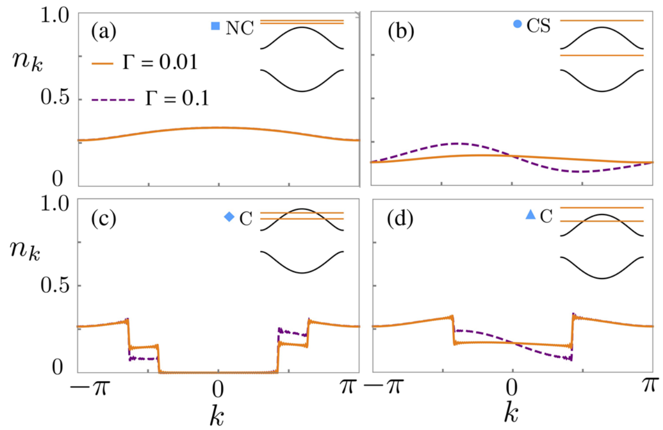

FIG. 4. Distribution of excitations with momentum k, n_k, computed for different values of the reservoir-chain couplings ($\Gamma_{\mathsf{L}} = \Gamma_{\mathsf{R}} \equiv \Gamma$). For each panel the blue geometric symbols specifies the values of m_{L} and m_{R} trough Fig. 1(c). The insets depict the energy band structure of the isolated chain (black lines), compared with the reservoirs magnetization potential (orange lines).

given in the Supplemental Material. We find that l_0 does not vary with $m_l (l = \mathsf{L}, \mathsf{R})$ away from the conducting phase, depending only on the values of h and Γ_l (not shown in the figure). Moreover, l_0 vanishes within the ordered phase. The coefficient c_0 is depicted in Fig. 3(b). It was extracted from the mutual information, $\mathcal{I}(A,B) \equiv E(\hat{\rho}_A) + E(\hat{\rho}_B) - E(\hat{\rho}_{A+B})$, of two adjacent segments A and B of total size ℓ, and using that $\mathcal{I}(A,B) \simeq c_0[2\log(\ell) - \log(2\ell)]$. We find that c_0 is nonzero in the C phase and vanishes otherwise. As in the case of a Fermi gas, c_0 depends on the strength of the reservoir-system couplings. In the present case, we find that it also depends on the bias potentials away from $m_{\mathsf{L}} = m_{\mathsf{R}} = 0$.

Excitation numbers.—In order to conceptualize these results we turn to the fermionic representation. In the infinite-volume limit, $L \to \infty$, boundary effects vanish and the state becomes translationally invariant. The Hamiltonian of the translationally invariant chain in its diagonal representation is given by $H = \sum_k \varepsilon_k (\gamma_k^\dagger \gamma_k - 1/2)$, where the operators $(\gamma_k, \gamma_{-k}^\dagger)^T = e^{i\theta_k \sigma_x} (c_k, c_{-k}^\dagger)^T$ describe the Bogoliubov excitations, $\sin(2\theta_k) = -2J \sin(k)/\varepsilon_k$ and $\varepsilon_k = 2\sqrt{(h + J\cos k)^2 + (J \sin k)^2}$. The excitation number $n_k \equiv \langle \gamma_k^\dagger \gamma_k \rangle_{S_\ell}$ within S_ℓ can be obtained from the single-particle density matrix, χ, numerically computed at sufficiently large ℓ. The results are shown in Fig. 4, where the parameters used are labeled by the symbols marked in Fig. 1(c). Additional distributions of n_k are given in the Supplemental Material.

For the isolated chain, the ground state is characterized by $\gamma_k^\dagger \gamma_k |\mathrm{GS}\rangle = 0$, i.e., $n_k = 0$ for all k. In the open setup, $n_k = 0$ also within the ordered phase, O. All other phases are characterized by nonzero distributions of excitations, i.e., $n_k \neq 0$. For the CS phases n_k is a continuous function of k while in the C phase it may have two or four discontinuities depending on whether one or both of the magnetic potentials $m_{\mathsf{L}}/m_{\mathsf{R}}$ are located within the bands $\pm\varepsilon_k$, see Figs. 4(c) and 4(d), and their insets.

Note that n_k is asymmetric upon $k \to -k$ in all conducting phases as required to maintain a net energy flow through the chain, since $\varepsilon(k) = \varepsilon(-k)$. In Fig. 4 we illustrate this feature by using a larger value of the hybridization energy that allows for a larger energy current; thus it leads to a more asymmetric n_k (see the dashed curves).

For a translational invariant system, the entanglement entropy can be obtained using the large-ℓ asymptotics for the determinant of Toplitz matrices, see Ref. [54]. If n_k is discontinuous, the Fisher-Hartwing conjecture has to be employed. Following the steps of Ref. [54], one concludes that $n_k \neq 0$, 1 results in an extensive contribution to the entanglement entropy while every discontinuity of n_k results in a logarithmic contribution to area law violation. This explains why $c_0 \neq 0$ only within the C phase.

Discussion.—We study a spin chain that can order magnetically, driven out of equilibrium by keeping the magnetization at the two ends of the chain fixed at different values. A set of nonequilibrium phases is observed and characterized according to the conductance and the scaling of the entanglement entropy. This model offers a remarkable example of an extended, strongly interacting system that can be continuously tuned from equilibrium to nonequilibrium conditions and admits an exact solution through the generalization of the Jordan-Wigner mapping. Moreover, we demonstrated that upon increasing the reservoir magnetization, a discontinuous jump of the magnetic order parameter occurs that coincides with a divergence of the correlation length. At equilibrium, the first observation is a signature of a first-order transition, while the second is a hallmark of continuous transitions. While this seems reminiscent of the situation that can occur at the lower critical dimension and which has been discussed in long-ranged spin chains in the context of mixed-order transitions [58–60], there are notable differences. In the present case, the interaction is short-ranged and, more importantly, a second-order phase transition is recovered at equilibrium. Thus, our findings exemplify that out-of-equilibrium conditions allow for novel critical phenomena which are not possible in equilibrium. This kind of phase transition also differs from those obtained for systems where dissipation is present in the bulk which induces a change of the dynamical critical exponent [30,31,61]. Therefore, to our best knowledge, this transition belongs to a novel universality class for which an effective field theoretic description out of equilibrium is yet to be developed. The exactly solvable model presented here should prove useful in developing such a description which will elucidate the role of interactions, e.g., the presence of magnetization gradients across the chain.

From the point of view of 1D fermionic systems, the peculiar critical properties discussed here might provide alternative signatures of the topological transition. To address this question, it would be interesting to extend our study of criticality under nonequilibrium condition to concrete setups of semiconductor nanowires [62–64].

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