Pair Creation with Strong Laser Fields, Compton Scale X Rays, and Heavy Nuclei

B. Hafizi, D. F. Gordon, and D. Kaganovich
Naval Research Laboratory, Washington, D.C. 20375, USA

(Received 30 October 2018; revised manuscript received 14 February 2019; published 13 June 2019)

Electron-positron pair creation is considered when intense laser pulses collide head-on with <1 MeV x-ray photons in the presence of stationary Coulomb charges $Z(-e)$. The analysis employs Coulomb-corrected Volkov states and is not limited to Born's approximation in Z. The cross section and the yield increase dramatically with increasing Z, potentially enabling (i) measurable yields with petawatt lasers and (ii) sensitive tests of strong-field QED.

DOI: 10.1103/PhysRevLett.122.233201

Vacuum has characteristics resembling a tangible medium due to the coupling of its *virtual* particle-antiparticle field with external fields [1,2]. It has complex-valued permittivity and permeability tensors [3–9] and manifests nonlinear properties in uncommonly intense fields [10–15]. Figure 1(a) shows pair creation *ex nihilo* by collision of two photons $\gamma + \gamma \to e^- + e^+$ in what is referred to as the Breit-Wheeler process [2,16].

With intense lasers the rates for many QED processes enter the realm of observation, e.g., [15–32]. SLAC's pioneering photoproduction $\gamma + n\omega \to e^- + e^+$ experiments ushered in a new era by colliding high-energy photons γ with n TW-class laser photons ω [16,33–35]. For this variant of the Breit-Wheeler process [2], e.g., [36–44] to be allowed the Mandelstam invariant $\sqrt{s}$ must exceed $2m$, imposing a lower bound on the integer n. The high-energy photons were generated by Compton scattering laser photons from ~50GeV electrons, enabling the reaction with $n > 4$. Analysis of QED processes in intense fields is facilitated by known *exact* solutions of Dirac's equation representing dressed electron-positron states (Volkov states) [2,16].

Positrons can be also produced in the presence of an external charge $Z(-e)$ via the Bethe-Heitler process $\gamma + Z \to Z + e^- + e^+$, Fig. 1(b)[2,16] and, e.g., [45–51]. Variants of Schwinger, of Breit-Wheeler, of Bethe-Heitler processes, and combinations thereof, have been analyzed, e.g., [27,52–71]. Lorentz boosts when *relativistic nuclei* are employed enhance the production rate, e.g., [49,53–56,58,68]; the yield has been determined for LHC-class nuclei in the Born approximation, assuming $Z\alpha \ll 1$ ($\alpha = e^2/\hbar c$) [72,73].

The pair production rate depends sensitively on the ratio of the laser electric field E_L to the Schwinger electric field $E_S = m^2c^3/\hbar|e|$. When E_L/E_S is small the process is in the tunneling regime where the rate is exponentially small, scaling as $\sim e^{-E_S/E_L}$, e.g., [59,60]. In the opposite limit the exponential factor does not appear, and the rate is far more prolific, e.g., [68]. In order to access the prolific regime of the Breit-Wheeler process using currently available lasers, GeV-class photons are required [33–35]. The process considered here is sketched in Fig. 2(a), where laser photons collide head-on with high-energy ($<2mc^2$) photons in the presence of stationary nuclei with *high Z*—wherein the Born approximation is inapplicable [74]. For this process the assist given by x-ray photons with frequency ω_X to leptons to tunnel through the barrier is conveniently measured by parameters κ, ξ defined by

$$\kappa \equiv \hbar\omega_X/2mc^2,$$
$$\xi = \sqrt{1 - (\sqrt{8+\kappa^2} - \kappa)\kappa/2}. \tag{1}$$

The pair production rate is now proportional to $\sim C_2(Z)e^{-\beta}$, where $\beta = 4\sqrt{3}\xi^3(E_S/E_L)/(1+\xi^2)$ [Eqs. (7) and (8) below]. When $\beta \gg 1$ the rate is again exponentially small. However, the factor in the exponential involving ξ rapidly vanishes as $\hbar\omega_X \to 2mc^2(\kappa \to 1, \xi \to 0)$, thus enhancing the production rate. Additionally, $C_2(Z)$ is a very strong function of Z. This shows that (i) the rate is strongly enhanced by the presence of Coulomb charges, and (ii) the prolific regime can be accessed with ~1 MeV photons. This Letter explores the regime $\beta \gg 1$ leaving the opposite limit for future research. There are numerous analyses of QED in the presence of strong laser fields. The very large increase in the pair production yield with Z to be demonstrated here can be useful for testing

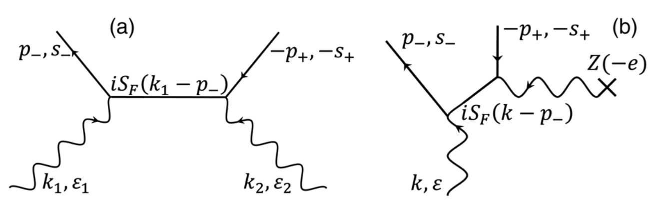

FIG. 1. Production of e^--e^+ pair, described by plane waves with momenta $p_-, -p_+$. Tree-level diagrams with weak photon fields. Bare-fermion propagator labeled by iS_F. (a) Breit-Wheeler process with photons (k_1, ε_1) and (k_2, ε_2). (b) Bethe-Heitler process with photon (k, ε) and nucleus $Z(-e)$. Time axis points upward.

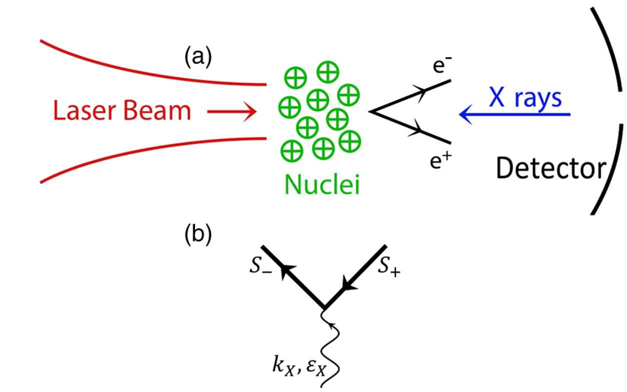

FIG. 2. (a) Experimental arrangement for pair production using laser (k), x ray (k_X), and Coulomb charges [$Z(-e)$]. Laser beam propagates along the z axis with electric field polarized along the y axis (vertical). Figure-eight orbit of leptons in the laser field lies in the plane of the paper. (b) Leptons described by actions S_-, S_+. External (thick) lines in (b) depict Coulomb-corrected Volkov states (Furry representation [2]). X-ray field is assumed weak.

predictions of QED in the presence of strong Coulomb fields without requiring high-energy photons in the same class as SLAC's or LHC-class nuclei.

The S-matrix element for pair creation mediated by x rays with vector potential A_X^μ is given by [75,76] $S_{fi} \equiv -i(e/c\hbar)\int d^4y \bar{\Psi}_f(y) \not{A}_X(y)\Psi_i(y), (f \neq i)$, with the interpretation that a positron $\Psi_i(y) \equiv \sqrt{mc^2/E_+V} v(p_+, s_+) \times \exp[iS_+(y)/\hbar]$ with 4 momentum p_+ $(E_+/c = p_+^0 > 0)$ and polarization s_+ is created at y along with an electron $\Psi_f(y) \equiv \sqrt{mc^2/E_-V} u(p_-, s_-) \times \exp[iS_-(y)/\hbar]$ with 4 momentum p_- $(E_-/c = p_-^0 > 0)$ and polarization s_-. Feynman's slash notation $\not{A}_X \equiv \gamma \cdot A_X$ is employed, γ^μ are the Dirac matrices, $e(<0)$ is the charge on an electron and $u(p,s), v(p,s)$ are bispinor plane wave amplitudes normalized in a box of volume V.

The action $S(y)$, governed by the vector potential 4 vectors of the laser $A_L^\mu = (0, \mathbf{A})$ and of the Coulomb charge $(V, \mathbf{0})$, determine the Coulomb-corrected Volkov states $\Psi(y)$. The laser field, in the radiation gauge, has wave number 4 vector k satisfying $\partial \cdot A_L = k \cdot A_L = k \cdot k = 0$. Use is made of the expansion $S(y) = S_0(y) + (\hbar/i)S_1(y) + \cdots$ reflecting the quasiclassical nature of the solution. There is a further expansion $S_j(y) = S_j^{(0)}(y) + S_j^{(1)}(y) + \cdots (j = 0, 1, ...)$ wherein the effects of the Coulomb field are incorporated perturbatively but in the exponent [77]. Thus,

$$S_0^{(0)}(y) = \mp p \cdot y - \int^{k \cdot y} d\varphi \left(\frac{e p \cdot A_L}{ck \cdot p} \mp \frac{e^2 A_L \cdot A_L}{2c^2 k \cdot p} \right),$$

$S_1^{(0)}(y) = \pm e \not{k} \not{A}_L / 2ck \cdot p$, give the usual Volkov solution [2,16], where $\varphi = k \cdot y$ and the upper (lower) sign refers to electrons (positrons), and the Coulomb action $S_0^{(1)}(y) = \mp (e/c^2) \int^{k \cdot y} d\varphi V[y(\varphi)] E[y(\varphi)]/k \cdot p$ is ordered to be smaller but its contribution in the *exponent* supersedes Born's approximation. These three contributions to the action define the thick external lines in Fig. 2(b), constituting Coulomb-corrected Volkov (dressed) states.

The laser field $A_L^\mu(y) = -A_L \varepsilon_L^\mu \cos(k \cdot y)$ is linearly polarized, where $k^\mu = (\omega/c, \mathbf{k})$, ω is the frequency, $\hat{\mathbf{k}}$ is the unit 3 vector along the propagation direction, and ε_L^μ is the polarization. The x rays are described by electric field $\mathbf{E}_X(k_X \cdot y)$, frequency $\omega_X = ck_X^0$, and vector potential $A_X^\mu(y) = (-\mathbf{y}.\mathbf{E}_X, -\hat{\mathbf{k}}_X \mathbf{y}.\mathbf{E}_X)$ in the *electric field gauge* [77–79], Supplemental Material [80], where $\hat{\mathbf{k}}_X$ is a unit 3 vector in the propagation direction and $A_X \cdot A_X = 0$. Taking $\mathbf{E}_X = \sqrt{2\pi\hbar\omega_X/V} \hat{\boldsymbol{\varepsilon}}_X (e^{ik_X \cdot y} + e^{-ik_X \cdot y})$, where $\hat{\boldsymbol{\varepsilon}}_X$ is a unit 3 vector along the polarization direction, it follows that $A_X^\mu(y) = -\mathbf{y}.\mathbf{E}_X \varepsilon_X^\mu$, with $\varepsilon_X^\mu = (1, \hat{\mathbf{k}}_X)$, with one photon of energy $\hbar\omega_X$ in volume V.

In SLAC's experiments with GeV-class x rays a few laser photons sufficed for photoproduction. With infrared lasers and moderate energy $\hbar\omega_X < 1$ MeV x rays, on the other hand, $>10^6$ laser photons will be required. This limit corresponds to *tunneling* through a barrier [81] and the quasiclassical approach is applicable [82]. This approach is valid if (i) laser electric field $E_L \equiv \omega A_L/c \ll E_S$ and (ii) laser photon energy $\hbar\omega \ll mc^2$. The tunneling frequency is $\omega_t = |e|E_L/mc$ [15] from which $\omega_t/\omega = |e|E_L/mc\omega$, the significance of which is that for $\omega_t/\omega \gg 1$ tunneling takes place before the field changes significantly. This *adiabatic* limit corresponds to $-a = (-e)E_L/mc\omega \gg 1$.

The S-matrix element is given by

$$S_{fi}(p_-, p_+) = -i \frac{e}{c\hbar} \sqrt{\frac{mc^2}{E_-V}} \sqrt{\frac{mc^2}{E_+V}} \times \int d^4y e^{i\Phi(y)} [\bar{u}(p_-, s_-) M_{res}(y) v(p_+, s_+)], \tag{2}$$

where $M_{\text{res}}(y)$ and $\Phi(y)$ are given in the Supplemental Material [80]. The momentum distribution of the $e^- - e^+$ pair is proportional to $|S_{fi}(p_-, p_+)|^2$. The distribution is dominated by the exponential $\exp(-2\text{Im}\tilde{\Phi})$, where $\tilde{\Phi}(p_-, p_+)$ is given by Eq. (1) in the Supplemental Material [80]. For a laser beam with $k^\mu = (\omega/c)(1, 0, 0, 1)$ and $\varepsilon_L^\mu = (0, 0, 1, 0)$ the distribution peaks near the vertex of the elliptic paraboloid

$$\frac{p_{z0}}{mc} = \frac{\xi^2}{2\sqrt{1+\xi^2}} + \frac{1}{2}\sqrt{1+\xi^2}\left(\frac{p_y}{mc}\right)^2 + \frac{1}{2}\frac{\sqrt{1+\xi^2}}{3+\xi^2}\left(\frac{p_x}{mc}\right)^2, \tag{3}$$

where

$$\mathrm{Im}\tilde{\Phi}(p_x,p_y,p_z)\approx\frac{-\lambda/\pi\bar{\lambda}_C}{3a\sqrt{1-(p_y/mca)^2}}\left\{\frac{(\sqrt{3}\xi)^3}{1+\xi^2}+\frac{2\sqrt{3}(3+\xi^2)\xi}{[2+\xi^2+(p_y/mc)^2(1+\xi^2)]^2}\times\left(\frac{p_z-p_{z0}}{mc}\right)^2+\frac{3\sqrt{3}\xi}{2}\left(\frac{p_x}{mc}\right)^2\right\}, \quad (4)$$

and $\bar{\lambda}_C=\hbar/mc$ is the reduced Compton wavelength.

Incorporating the contribution of the Coulomb potential $V=Z(-e)/\sqrt{\mathbf{y}(\varphi)\cdot\mathbf{y}(\varphi)}$, the phase in Eq. (2) is

$$\Phi(y)=\tilde{\Phi}-\frac{1}{\hbar}(p_-^1+p_+^1)x-\frac{1}{\hbar}(p_-^2+p_+^2)y+qz+\nu\log\frac{[\tilde{p}_s r+zp_z(\varphi_s)+yp_y(\varphi_s)]_-}{[\tilde{p}_s r-zp_z(\varphi_s)-yp_y(\varphi_s)]_+}, \quad (5)$$

where $p_z(\varphi_s), p_y(\varphi_s), \tilde{p}_s, q$ are given by Eqs. (3)–(5) and (7) in Supplemental Material [80], $r=\sqrt{z^2+y^2}$ and

$$\nu=\mp iZ\alpha\kappa/\sqrt{1-\kappa^2} \quad (6)$$

is the analog of Sommerfeld's parameter [83,84]. [The $\pm$ signs in Eq. (6) are associated with branches of the square-root function and not with the particles.] For the high laser intensities of interest portions of the figure-eight orbit along *both* the laser electric field *and* the $\mathbf{E}\times\mathbf{B}$ directions contribute to the Coulomb phase (last) term in Eq. (5). Large x-ray momenta κ enhance the Coulomb coupling to the formed dipole pair—manifested by the appearance of κ in the numerator of Eq. (6). For x-ray momenta $\kappa\to 1$, however, the escaping pair tends to "stall" and the momentum $\tilde{p}_s/mc=\pm i\sqrt{1-\kappa^2}\to 0$, effectively enhancing the influence of the charge Z. This well-known behavior turns out to hold in the classically forbidden, under the barrier motion here. The time integration in Eq. (2) is carried out along the steepest descent paths and the spatial integration is effected by employing Eq. (5) (Supplemental Material [80]). The phase is modified by the full logarithmic form of Coulomb wave functions [85–89], extending previous analyses.

The differential cross section $d\sigma$ for e^--e^+ creation is proportional to $|S_{fi}(p_-,p_+)|^2$ and varies as Z^2 for small Z as in standard Bethe-Heitler process (Supplemental Material [80]). The total cross section is obtained by integrating over the density of final states. While polarized leptons are critical to the discovery of new physics [90–96], for the purposes here it suffices to sum over final spins. Noting that $\lambda\gg\lambda_X$, where $\lambda_X\equiv 2\pi/k_X^0$ is the x-ray wavelength, the yield per laser shot is $Y=\pi(c\alpha/\lambda)\tau N_X(n_C L_x\bar{\lambda}_C^4/\lambda_X^2)\times\sqrt{\bar{\lambda}_C/\lambda}C_1C_2(Z)C_{\mathrm{trace}}$. Here, τ is the laser pulse duration, $N_X=2Z_{RL}\pi w^2 n_X$ is the number of x-ray photons in the interaction volume $2Z_{RL}\pi w^2$, w and $Z_{RL}=\pi w^2/\lambda$ are the laser spot radius and Rayleigh range, respectively, n_X is the x-ray photon density, n_C is the density of charges, L_x is the x-axis width,

$$C_1=\frac{\pi^{3/2}e^{-\beta}}{6\sqrt{-2a}\sqrt[4]{3}}\sqrt{\frac{\sqrt{\kappa^2+8}+\kappa}{\sqrt{\kappa^2+8}-3\kappa}}\sqrt{\frac{1+\xi^2}{\xi^5(1+\xi^2/3)}},$$
$$\beta=4\sqrt{3}\xi^3(E_S/E_L)/(1+\xi^2)=2\lambda\sqrt{3}\xi^3/(-a)\pi\bar{\lambda}_C(1+\xi^2), \quad (7)$$

$$C_2(Z)=\frac{2^{7-4|\nu|}\pi^{3/2}|\nu|}{\rho}\left(\frac{(2|\nu|-1)\Gamma(\frac{3}{2}-2|\nu|)}{\Gamma(-2|\nu|)}\eta^{4|\nu|}+\frac{2^{8|\nu|}(1+2|\nu|)\Gamma(\frac{3}{2}+2|\nu|)}{\Gamma(2|\nu|)}\frac{1}{\eta^{4|\nu|}}\right),$$
$$C_{\mathrm{trace}}=e^{\beta}\int_{-\sqrt{\beta}}^{\sqrt{\beta}}dsM(s)(\vartheta^2+s^2/\beta)^{-1}\exp(-\beta/\sqrt{1-s^2/\beta}), \quad (8)$$

$M(s)=2(1+\xi^2)\{1+a^2/2+a^2s^2/\beta+(2a^4s^2/\beta)\times\cosh[2\mathrm{Im}\varphi_s(s)]+(a^4/8)\cosh[4\mathrm{Im}\varphi_s(s)]\}$, $\rho=(\sqrt{8+\kappa^2}-7\kappa)^5(\kappa^2-\sqrt{8+\kappa^2}\kappa+2)^{3/2}/6\sqrt{6}$, $\eta=\sqrt{(1-3\kappa/\sqrt{8+\kappa^2})/2}$, $\vartheta=\sqrt{(2+\xi^2)/(1+\xi^2)}/(-a)$, and $\varphi_s(s)=\cos^{-1}[s/\sqrt{\beta}+i\sqrt{3(\sqrt{8+\kappa^2}-3\kappa)(\sqrt{8+\kappa^2}+\kappa)}/4a]$.

The x-ray pulse duration is $\geq 4Z_{RL}/c\gg\tau$.

Two restrictions of the theory must be noted. First, the quasiclassical approximation requires $\beta\gg 1$. This parameter depends on the laser amplitude [Eq. (7)] and dominates the yield since it appears in the exponent of C_1. Reducing the assist provided by the x rays [i.e., reducing κ in Eq. (1)] decreases the yield. However, the yield can be restored by increasing the laser intensity (i.e., E_L). Second, the circle of convergence for $C_2(Z)$ in Eq. (8) imposes the restriction $|\nu|<\frac{3}{4}$ on the Sommerfeld parameter defined in Eq. (6) [Eqs. (6) and (8) in Supplemental Material [80]]. Equation (6) then imposes a maximum value of Z for a

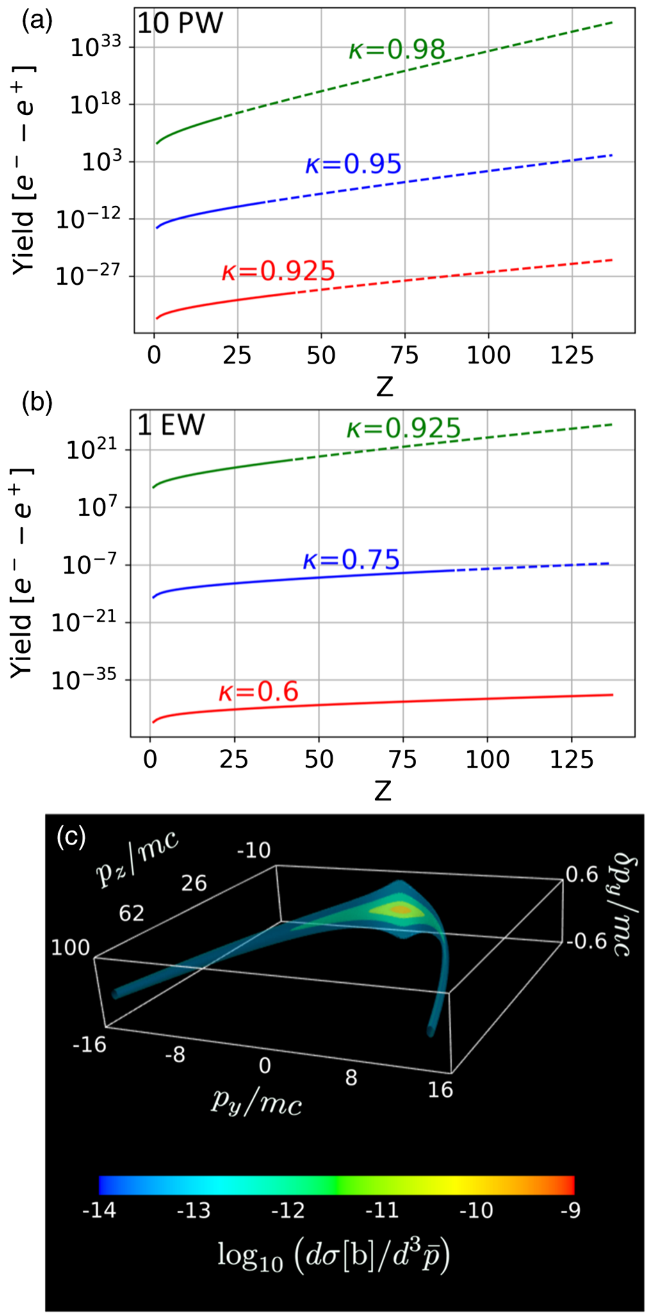

FIG. 3. (a) Yield (pairs per laser shot) versus nuclear charge Z for 10 PW laser and x-ray photons with normalized energy $\kappa = 0.925$, 0.95, 0.98, with other parameters listed in Table I. (b) Yield versus nuclear charge Z for 1 EW laser and x-ray photons with normalized energy $\kappa = 0.6$, 0.75, 0.925. In (a) and (b) strict convergence is maintained on the solid line segments only. (c) Momentum distribution of e^--e^+ for 10 PW case, $\kappa = 0.98$, and $Z = 18$. The laser beam propagates along the z axis with electric field polarized along the y axis. Leptons are ejected in opposite directions along the y axis and $\delta p_y \equiv p_-^2 + p_+^2$ measures the *difference* between their y momenta [Eq. (2) in Supplemental Material [80]]. Distribution in p_y-p_z plane, given by Eq. (4), is symmetric about $p_y = 0$ with vertex on p_z axis given by Eq. (3).

given x-ray momentum $\hbar k_X^0$. This limitation of the analysis, however, is not fundamental to the physics.

With these provisos, Fig. 3(a) shows the yield for a 10 PW laser and for x-ray photons with $\kappa = 0.925, 0.95, 0.98$. Figure 3(b) shows the results for a higher-power laser—1 EW—and for x-ray photons with $\kappa = 0.6, 0.75, 0.925$. The plots highlight the fact that the yield depends on laser power and x-ray momentum through the combination captured by the expression for β in Eq. (7) [73]. What is new is the very rapid increase in the yield with both x-ray momentum and Coulomb charge Z. This expands the window for probing strong-field QED since comparison between the theory here and experiment extends to Coulomb charges exceeding the validity of Born's approximation. In this connection it should be pointed out that—as noted in the previous paragraph—the circle of convergence for $C_2(Z)$ Eq. (8) is limited by $|\nu| < \frac{3}{4}$. The solid portions of the curves in Figs. 3(a) and 3(b) lie within the circle of convergence, while the dashed portions go beyond it. For the 1EW, $\kappa = 0.6$ example the theory holds along the entire curve—i.e., for $1 \leq Z \leq 137$. In order to obtain accurate results in the space occupied by the dashed curves, approximations must be removed in both the expression for the Coulomb-corrected Volkov state and in the subsequent integrations (see Supplemental Material [80]). Numerical treatment is a possible solution.

The momentum distribution for $Z = 18$ (Ar), $\kappa = 0.98$, and a 10 PW laser is shown in Fig. 3(c). Specifically, it is a plot of the differential cross section *per nucleus* per normalized momentum cubed $d\sigma[b]/d^3\bar{p}$ [with $d^3\bar{p} \equiv dp_y dp_z d\delta p_y/(mc)^3$], integrated over $dp_x d\delta p_z$. After integrating over the remaining momenta the cross section per nucleus is ≈70 pb. The y momentum of the pair is nearly balanced—save for a small difference δp_y that develops since the electron and the positron have opposite coupling to the Coulomb charge. The central findings of this Letter exemplified by Fig. 3 are (i) the rapid rise of the yield with κ and Z implying high sensitivity—a feature that can be helpful in testing QED—and (ii) the pair occupies an organized and compact region of momentum space as shown in Fig. 3(c). The context of Fig. 3(c) can be further appreciated by comparing with momentum distributions arising in ordinary tunneling ionization of atoms (e.g., [77]).

TABLE I. Parameters for determination of e^--e^+ yield for 10 PW and for 1 EW lasers.

Laser wavelength	800 nm
Laser spot radius	2.5 μm
Laser pulse duration	35 fs
X-ray photon density	5×10^{17} cm^{-3}
X-ray pulse duration	$4Z_{RL}/c = 327$ fs
Coulomb charge density	10^{20} cm^{-3}

Several mechanisms are available for the generation of high-energy photons, e.g., [33,97,98]. Laser-driven wakefield electrons can serve to generate the <1 MeV x rays considered here. One possibility is to derive pulses for x-ray generation and pair creation from the same chirped pulse amplification system. The two pulses are routed through different compressors in order to allow for differing pulse durations.

Positrons find use in positron emission tomography, testing fundamental symmetries of nature and are *sine qua nons* in lepton colliders [99,100]. For the latter they are conventionally generated by pair production in high-Z moderators [101,102]. Laser-driven configurations—with positron generation induced via ponderomotive-wakefield acceleration of electrons in plasma—can be prolific sources [97,98,103–107]. While the laser drive enhances flexibility, the laser repetition rate, positron bunch energy spread, and emittance will ultimately determine the utility of this approach [105,108].

In summary, the tunneling regime of pair creation can be accessed using PW-scale lasers alone. The PW laser pulse collides with x rays in a medium of heavy, stationary nuclei. The x-ray energy is engineered to be below the threshold for ordinary pair creation, such that many laser photons are required to make up the deficit. The positron yield is increased by orders of magnitude relative to the laser alone, and without the need for a large accelerator. In order to attempt such an experiment, a number of details remain to be explored, such as identifying sources of noise, and determining the parameters that optimize the signal to noise ratio. The process described arises automatically in any environment consisting of ultraintense fields, x rays, and nuclei.

The authors have benefited from discussions with Dr. T-G. Lee. This work was supported by the Naval Research Laboratory 6.1 Base Program, under Work Unit EL011-09-41-4987, and the Department of Energy under Interagency Agreement No. 89243018SSC000006.

[1] J. Schwinger, Phys. Rev. **82**, 664 (1951).
[2] V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, *Quantum Electrodynamics* (Butterworth-Heinemann, Oxford, 2008).
[3] J. J. Klein and B. P. Nigam, Phys. Rev. **135**, B1279 (1964).
[4] J. J. Klein and B. P. Nigam, Phys. Rev. **136**, B1540 (1964).
[5] Z. Bialynicka-Birula and I. Bialynicki-Birula, Phys. Rev. D **2**, 2341 (1970).
[6] S. L. Adler, Ann. Phys. (N.Y.) **67**, 599 (1971).
[7] E. Brézin and C. Itzykson, Phys. Rev. D **3**, 618 (1971).
[8] B. King and T. Heinzl, High Power Laser Sci. Eng. **4**, e5 (2016).
[9] S. Bragin, S. Meuren, C. H. Keitel, and A. Di Piazza, Phys. Rev. Lett. **119**, 250403 (2017).
[10] H. R. Reiss, J. Math. Phys. (N.Y.) **3**, 59 (1962).
[11] A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. **46**, 776 (1964) [Sov. Phys. JETP **19**, 529 (1964)].
[12] A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. **52**, 1707 (1967) [Sov. Phys. JETP **25**, 1135 (1967)].
[13] V. S. Popov, Pis'ma Zh. Eksp. Teor. Fiz. **13**, 261 (1971) [JETP Lett. **13**, 185 (1971)].
[14] V. S. Popov, Zh. Eksp. Teor. Fiz. **61**, 1334 (1971) [Sov. Phys. JETP **34**, 709 (1972)].
[15] V. I. Ritus, Tr. Ordena Lenina Fiz. Inst. P. N. Lebedeva Akad. Nauk SSSR **111**, 5 (1979) [J. Sov. Laser Res. **6**, 497 (1985)].
[16] W. Greiner and J. Reinhardt, *Quantum Electrodynamics.* (Springer, Berlin, 2009).
[17] L. E. Brown and T. W. B. Kibble, Phys. Rev. **133**, A705 (1964).
[18] E. Brézin and C. Itzykson, Phys. Rev. D **2**, 1191 (1970).
[19] V. N. Baĭer, V. M. Katkov, A. I. Mil'steĭn, and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. **67**, 453 (1974) [Sov. Phys. JETP **40**, 225 (1975)].
[20] V. N. Baĭer, V. M. Katkov, A. I. Mil'steĭn, and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. **68**, 405 (1975) [Sov. Phys. JETP **41**, 196 (1975)].
[21] W. Becker and H. Mitter, J. Phys. A **8**, 1638 (1975).
[22] V. N. Baĭer, V. M. Katkov, A. I. Mil'steĭn, and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. **69**, 783 (1975) [Sov. Phys. JETP **42**, 400 (1976)].
[23] V. N. Baĭer, A. I. Mil'steĭn, and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. **69**, 1893 (1975) [Sov. Phys. JETP **42**, 961 (1976)].
[24] N. B. Narozhnyĭ, S. S. Bulanov, V. D. Mur, and V. S. Popov, Pis'ma Zh. Eksp. Teor. Fiz. **80**, 434 (2004) [JETP Lett. **80**, 382 (2004)].
[25] A. I. Mil'steĭn, I. S. Terekhov, U. D. Jentschura, and C. H. Keitel, Phys. Rev. A **72**, 052104 (2005).
[26] S. S. Bulanov, N. B. Narozhnyĭ, V. D. Mur, and V. S. Popov, Zh. Eksp. Teor. Fiz. **129**, 14 (2006) [Sov. Phys. JETP **102**, 9 (2006)].
[27] S. S. Bulanov, V. D. Mur, N. B. Narozhnyĭ, J. Nees, and V. S. Popov, Phys. Rev. Lett. **104**, 220404 (2010).
[28] S. V. Bulanov, T. Zh. Esirkepov, M. Kando, J. Koga, K. Kondo, and G. Korn, Plasma Phys. Rep. **41**, 1 (2015).
[29] F. Mackenroth and A. Di Piazza, Phys. Rev. Lett. **110**, 070402 (2013).
[30] D. G. Green and C. N. Harvey, Phys. Rev. Lett. **112**, 164801 (2014).
[31] J. M. Cole *et al.*, Phys. Rev. X **8**, 011020 (2018).
[32] K. Poder *et al.*, Phys. Rev. X **8**, 031004 (2018).
[33] C. Bula, K. T. McDonald, E. J. Prebys, C. Bamber, S. Boege, T. Kotseroglou, A. C. Melissinos, D. D. Meyerhofer, W. Ragg, D. L. Burke, R. C. Field, G. Horton-Smith, A. C. Odian, J. E. Spencer, D. Walz, S. C. Berridge, W. M. Bugg, K. Shmakov, and A. W. Weidemann, Phys. Rev. Lett. **76**, 3116 (1996).
[34] D. L. Burke *et al.*, Phys. Rev. Lett. **79**, 1626 (1997).
[35] C. Bamber *et al.*, Phys. Rev. D **60**, 092004 (1999).
[36] A. Ilderton, Phys. Rev. Lett. **106**, 020404 (2011).
[37] K. Krajewska and J. Z. Kamiński, Phys. Rev. A **86**, 052104 (2012).
[38] S. Meuren, K. Z. Hatsagortsyan, C. H. Keitel, and A. Di Piazza, Phys. Rev. D **91**, 013009 (2015).

[39] A. Wöllert, H. Bauke, and C. H. Keitel, Phys. Rev. D **91**, 125026 (2015).
[40] S. Meuren, C. H. Keitel, and A. Di Piazza, Phys. Rev. D **93**, 085028 (2016).
[41] M. J. A. Jansen, J. Z. Kamiński, K. Krajewska, and C. Müller, Phys. Rev. D **94**, 013010 (2016).
[42] A. I. Titov, B. Kämpfer, A. Hosaka, and H. Takabe, Phys. Part. Nucl. **47**, 456 (2016).
[43] A. Di Piazza, Phys. Rev. Lett. **117**, 213201 (2016).
[44] I. Drebot, D. Micieli, E. Milotti, V. Petrillo, E. Tassi, and L. Serafini, Phys. Rev. Accel. Beams **20**, 043402 (2017).
[45] V. P. Yakovlev, Zh. Eksp. Teor. Fiz. **49**, 318 (1965) [Sov. Phys. JETP **22**, 223 (1966)].
[46] N. B. Narozhnyĭ and A. I. Nikishov, Zh. Eksp. Teor. Fiz. **63**, 1135 (1972) [Sov. Phys. JETP **36**, 598 (1973)].
[47] M. H. Mittleman, Phys. Rev. A **35**, 4624 (1987).
[48] K. Dietz and M. Pröbsting, J. Phys. B **31**, L409 (1998).
[49] K. Krajewska, J. Z. Kamiński, and F. Ehlotzky, Laser Phys. **16**, 272 (2006).
[50] A. A. Lebed' and S. P. Roshchupkin, Laser Phys. **21**, 1613 (2011).
[51] K. Krajewska, C. Müller, and J. Z. Kamiński, Phys. Rev. A **87**, 062107 (2013).
[52] R. Alkofer, M. B. Hecht, C. D. Roberts, S. M. Schmidt, and D. V. Vinnik, Phys. Rev. Lett. **87**, 193902 (2001).
[53] I. M. Dremin, Pis'ma Zh. Eksp. Teor. Fiz. **76**, 185 (2002) [JETP Lett. **76**, 151 (2002)].
[54] H. K. Avetissian, A. K. Avetissian, G. F. Mkrtchian, and Kh. V. Sedrakian, Nucl. Instrum. Methods Phys. Res., Sect. A **507**, 582 (2003).
[55] C. Müller, A. B. Voitkiv, and N. Grün, Phys. Rev. A **67**, 063407 (2003).
[56] C. Müller, A. B. Voitkiv, and N. Grün, Phys. Rev. A **70**, 023412 (2004).
[57] V. N. Baĭer and V. M. Katkov, Phys. Rep. **409**, 261 (2005).
[58] A. I. Mil'steĭn, C. Müller, K. Z. Hatsagortsyan, U. D. Jentschura, and C. H. Keitel, Phys. Rev. A **73**, 062106 (2006).
[59] M. Yu. Kuchiev and D. J. Robinson, Phys. Rev. A **76**, 012107 (2007).
[60] M. Yu. Kuchiev, Phys. Rev. Lett. **99**, 130404 (2007).
[61] A. R. Bell and J. G. Kirk, Phys. Rev. Lett. **101**, 200403 (2008).
[62] J. G. Kirk, A. R. Bell, and I. Arka, Plasma Phys. Controlled Fusion **51**, 085008 (2009).
[63] M. Ruf, G. R. Mocken, C. Müller, K. Z. Hatsagortsyan, and C. H. Keitel, Phys. Rev. Lett. **102**, 080402 (2009).
[64] F. Hebenstreit, R. Alkofer, G. V. Dunne, and H. Gies, Phys. Rev. Lett. **102**, 150404 (2009).
[65] S. J. Müller and C. Müller, Phys. Rev. D **80**, 053014 (2009).
[66] T. Heinzl, A. Ilderton, and M. Marklund, Phys. Lett. B **692**, 250 (2010).
[67] F. Hebenstreit, R. Alkofer, and H. Gies, Phys. Rev. Lett. **107**, 180403 (2011).
[68] A. Di Piazza, C. Müller, K. Z. Hatsagortsyan, and C. H. Keitel, Rev. Mod. Phys. **84**, 1177 (2012).
[69] S. Meuren, C. H. Keitel, and A. Di Piazza, Phys. Rev. D **88**, 013007 (2013).
[70] S. Augustin and C. Müller, Phys. Lett. B **737**, 114 (2014).
[71] A. A. Lebed', E. A. Padusenko, S. P. Roshchupkin, and V. V. Dubov, Phys. Rev. A **95**, 043406 (2017).
[72] C. Müller, Phys. Lett. B **672**, 56 (2009).
[73] A. Di Piazza, E. Lötstedt, A. I. Mil'steĭn, and C. H. Keitel, Phys. Rev. Lett. **103**, 170403 (2009).
[74] D. C. Ionescu, Phys. Rev. A **49**, 3188 (1994).
[75] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, NY, 1964).
[76] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, NY, 1965).
[77] M. Klaiber, E. Yakaboylu, and K. Z. Hatsagortsyan, Phys. Rev. A **87**, 023418 (2013).
[78] H. R. Reiss, Prog. Quantum Electron. **16**, 1 (1992).
[79] G. F. Gribakin and M. Yu. Kuchiev, Phys. Rev. A **55**, 3760 (1997).
[80] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.122.233201 for details of the derivation of the *S*-matrix.
[81] A. Wöllert, M. Klaiber, H. Bauke, and C. H. Keitel, Phys. Rev. D **91**, 065022 (2015).
[82] A. Di Piazza and A. I. Mil'steĭn, Phys. Lett. B **717**, 224 (2012).
[83] G. Breit, Proc. Natl. Acad. Sci. U.S.A. **57**, 849 (1967).
[84] P. Descouvemont and D. Baye, Rep. Prog. Phys. **73**, 036301 (2010).
[85] R. H. Dalitz, Proc. R. Soc. A **206**, 509 (1951).
[86] L. Rosenberg, Phys. Rev. D **8**, 1833 (1973).
[87] O. Smirnova, M. Spanner, and M. Yu. Ivanov, J. Phys. B **39**, S307 (2006).
[88] O. Smirnova, M. Spanner, and M. Yu. Ivanov, J. Phys. B **39**, S323 (2006).
[89] O. Smirnova, M. Spanner, and M. Ivanov, Phys. Rev. A **77**, 033407 (2008).
[90] W. H. McMaster, Rev. Mod. Phys. **33**, 8 (1961).
[91] Y. S. Tsai, Phys. Rev. D **48**, 96 (1993).
[92] T. Okugi, Y. Kurihara, M. Chiba, A. Endo, R. Hamatsu, T. Hirose, T. Kumita, T. Omori, Y. Takeuchi, and M. Yoshioka, Jpn. J. Appl. Phys. **35**, 3677 (1996).
[93] D. Yu. Ivanov, G. L. Kotkin, and V. G. Serbo, Eur. Phys. J. C **40**, 27 (2005).
[94] T. Omori, M. Fukuda, T. Hirose, Y. Kurihara, R. Kuroda, M. Nomura, A. Ohashi, T. Okugi, K. Sakaue, T. Saito, J. Urakawa, M. Washio, and I. Yamazaki, Phys. Rev. Lett. **96**, 114801 (2006).
[95] G. Moortgat-Pick *et al.*, Phys. Rep. **460**, 131 (2008).
[96] A. Di Piazza, A. I. Mil'steĭn, and C. Müller, Phys. Rev. A **82**, 062110 (2010).
[97] T. E. Cowan *et al.*, Laser Part. Beams **17**, 773 (1999).
[98] H. Chen, S. C. Wilks, J. D. Bonlie, E. P. Liang, J. Myatt, D. F. Price, D. D. Meyerhofer, and P. Beiersdorfer, Phys. Rev. Lett. **102**, 105001 (2009).
[99] R. Ruffini, G. Vereshchagin, and S. S. Xue, Phys. Rep. **487**, 1 (2010).
[100] I. C. E. Turcu *et al.*, Romanian reports in Physics **68**, S145 (2016).
[101] J. E. Clendenin, Proc. 1989 IEEE Part. Accel. Conf., Accel. Sci. Technol. **2**, 1107 (1989).
[102] T. Omori, T. Takahashi, S. Riemann, W. Gai, J. Gao, S. I. Kawada, W. Liu, N. Okuda, G. Pei, J. Urakawa, and

A. Ushakov, Nucl. Instrum. Methods Phys. Res., Sect. A **672**, 52 (2012).
[103] E. P. Liang, S. C. Wilks, and M. Tabak, Phys. Rev. Lett. **81**, 4887 (1998).
[104] C. Gahn, G. D. Tsakiris, G. Pretzler, K. J. Witte, C. Delfin, C.-G. Wahlström, and D. Habs, Appl. Phys. Lett. **77**, 2662 (2000).
[105] H. Chen *et al.*, Phys. Rev. Lett. **105**, 015003 (2010).
[106] G. Sarri, W. Schumaker, A. Di Piazza, M. Vargas, B. Dromey, M. E. Dieckmann, V. Chvykov, A. Maksimchuk, V. Yanovsky, Z. H. He, B. X. Hou, J. A. Nees, A. G. R. Thomas, C. H. Keitel, M. Zepf, and K. Krushelnick, Phys. Rev. Lett. **110**, 255002 (2013).
[107] G. Sarri *et al.*, Nat. Commun. **6**, 6747 (2015).
[108] H. Chen, J. C. Sheppard, D. D. Meyerhofer, A. Hazi, A. Link, S. Anderson, H. A. Baldis, R. Fedosejev, J. Gronberg, N. Izumi, S. Kerr, E. Marley, J. Park, R. Tommasini, S. Wilks, and G. J. Williams, Phys. Plasmas **20**, 013111 (2013).