Dark Energy from Quantum Gravity Discreteness

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We argue that discreteness at the Planck scale (naturally expected to arise from quantum gravity) might manifest in the form of minute violations of energy-momentum conservation of the matter degrees of freedom when described in terms of (idealized) smooth fields on a smooth spacetime. In the context of applications to cosmology, such "energy diffusion" from the low energy matter degrees of freedom to the discrete structures underlying spacetime would lead to the emergence of an effective dark energy term in Einstein's equations. We estimate this effect using a (relational) hypothesis about the materialization of discreteness in quantum gravity which is motivated by the strict observational constraints supporting the validity of Lorentz invariance at low energies. Arguments based on a simple dimensional analysis lead to an estimate of an effective cosmological constant agreeing in order of magnitude with its observed value. If correct, this would constitute remarkable empirical evidence for a Planckian granular aspect of spacetime.

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The discovery that the Universe is undergoing an accelerated expansion [1,2] is the source of one of the greatest puzzles in our present understanding of cosmology which goes under the name of the dark energy problem. While the assumption of the presence of a cosmological constant Λ remains the most successful phenomenological model, naive theoretical reasoning predicts a value for Λ that is either 120 orders of magnitude too big, or is strictly vanishing when a protective symmetry principle is at play [3]. It would be desirable to have a concrete fundamental calculation leading to $\Lambda_{\mathrm{obs}} \approx 1.19 \times 10^{-52}\ \mathrm{m}^{-2}$, the value indicated by observations [4].

A recent work [5] proposed a framework where violations of energy momentum conservation produce a dark energy contribution. The key result of that work was to characterize the effective framework where violations of energy conservation are made compatible with general relativity. As an illustration, we applied it to two models, previously considered in the literature, that propose such violations. However, neither of these two could be taken as truly realistic. On the one hand, the cosmological time at which the effects would start was not intrinsically defined by the models, and, on the other hand, the strength of the violations of energy conservation were encoded in a phenomenological adjustable parameter with no explicit link to known fundamental constants. Therefore, while these examples were illustrative of the idea that small violations can accumulate and contribute non-negligibly to Λ, they could not be used to predict its value.

In this Letter, we bridge this gap by proposing a mechanism to generate Λ leading to quantitative estimates based entirely on known fundamental features of the physics involved. The origin of the cosmological term, we suggest, is to be found in the microscopic structure of spacetime and its interaction with matter. We will work under the hypothesis that discreteness of geometry and Lorentz invariance at low energies are fundamental aspects of quantum gravity. Based on these two fundamental features, we propose a phenomenological model for quantum-gravity-induced violations of energy conservation depending only on the fundamental constants G, c, $\hbar$, and a few parameters of the standard model (SM). We show that our simple proposal resolves the two limitations of the previous examples and predicts a contribution to the cosmological constant of the correct order of magnitude.

One of the most important constraints on the form of quantum discreteness at Planck scales comes from the observed validity of Lorentz invariance at quantum field theory scales. As shown in [6,7], this rules out the simple atomistic view of a spacetime foam selecting a preferred "rest frame" at the Planck scale. This result, which severely constrains phenomenological ideas, is corroborated by a large collection of empirical evidence [8]. A more subtle theoretical characterization of spacetime discreteness at Planck's scale is necessary.

We think that the key for understanding Planckian discreteness lies in the relational nature of physics partly uncovered by Einstein's theory of gravity [9]. In general relativity, geometry can only be probed by the matter degrees of freedom (d.o.f.). The metric has a clear physical meaning only when rulers and clocks are introduced.

More precisely, the construction of observables (diffeomorphism invariant quantities) requires the use of relational notions involving a mixture of geometric and matter d.o.f. The difficulty of actually defining such quantities is, in fact, one of the most severe technical problems in formal approaches to quantum gravity [10]. In our view, such a relational perspective is essential for understanding discreteness at the Planck scale.

Thus, we are rejecting the notion of a spacetime foam acting as an empty arena where matter, if placed there, would reveal its preexisting features. Quantum discreteness should arise primarily via the interactions of gravity with those other d.o.f. which, by their nature, are able to select a preferential rest frame where the fundamental scale ℓ_p acquires an invariant meaning. In other words, within the relational approach we are advocating, it is clear that, in order to be directly sensitive to the discreteness scale ℓ_p, the probing d.o.f. must themselves carry their intrinsic scale. Thus, massless (scale-invariant) fields are ruled out as leading probes of discreteness simply because they cannot be associated with any local notion of rest frame, and thus, of a fundamental length scale. This argument identifies massive fields as the natural candidates for probes of spacetime discreteness. Thus, such discreteness must be thought of as becoming relevant, or as "awakened," by the interactions of gravity with such scale-invariance-breaking fields. The immediate possibility arising from such considerations (and framed in a phenomenological perspective) is that low energy quantum field theoretical excitations of massive fields could interact with the underlying quantum gravity microstructure and exchange "energy" with it. (Some ideas with similar conceptual underpinning have been explored in the context of laboratory searches for quantum gravity phenomenology [11–13]. For a discussion of the implications for the information problem in black hole evaporation, see [14,15].).

In order to study the phenomenological implications of these ideas, one needs a "mean field" or macroscopic description of the quantity parametrizing the phenomenon. An obvious choice is the trace of the energy-momentum tensor $\mathbf{T}$—which for a fluid in thermal equilibrium is simply given by $\mathbf{T} = -\rho + 3P$—which signals the breaking of conformal invariance and, hence, the presence of massive d.o.f. Via Einstein's equations, $\mathbf{T}$ is related to the scalar curvature $\mathbf{R} = -8\pi G\mathbf{T}$. Therefore, the presence of massive fields (suitable probes of discreteness according to our rationale) is geometrically captured by a nontrivial $\mathbf{R}$.

The effect on the propagation of massive fields must be realized in a deviation from the geodesic motion of free particles due to a "frictionlike" force encoding the noisy interaction with the underlying spacetime granularity. As argued in the previous paragraphs, the force must be proportional to $\mathbf{R}$. In addition, the force should depend on the mass m, the four-velocity u^μ, the spin s^μ of the classical particle (the only spacetime related proper features characterizing a particle), and a timelike unit vector ξ^μ specifying the local frame defined by the matter that curves spacetime. For instance, in cosmology, $\xi = \partial_t$ is naturally associated with the time arrow of the comoving cosmic fluid. In addition, and according to our preceding arguments, the force should be proportional to the particle's mass, endowing it with a characteristic length scale: the Compton wavelength. Dimensional analysis gives an essentially unique expression which is compatible with the above requirements

$$u^\mu \nabla_\mu u^\nu = \alpha \frac{m}{m_p^2} \mathrm{sgn}(s\cdot\xi)\mathbf{R} s^\nu, \tag{1}$$

where $\alpha > 0$ is a dimensionless coupling (Higher curvature corrections could be added, but these are highly suppressed by the Planck scale and, thus, are negligible for the central point of this Letter. A term proportional to $\epsilon^\nu_{\mu\gamma\sigma}\xi^\mu s^\gamma u^\sigma$ is also allowed but does not affect the results).

The factor $\mathrm{sgn}(s\cdot\xi)$ makes the force genuinely frictionlike. This is apparent when one considers the change of the mechanical energy of the particle $E \equiv -mu^\nu \xi_\nu$ (defined in the frame defined by ξ^μ) along the particles world line, namely,

$$\begin{aligned}\dot{E} &\equiv -mu^\mu \nabla_\mu (u^\nu \xi_\nu)\\ &= -\alpha \frac{m^2}{m_p^2} |(s\cdot\xi)|\mathbf{R} - mu^\mu u^\nu \nabla_{(\mu}\xi_{\nu)}.\end{aligned} \tag{2}$$

The last term in (2) encodes the standard change of E associated to the non-Killing character of ξ^μ. The first term on the right encodes the friction that damps out any motion with respect to ξ^μ. Energy is lost into the fundamental granularity until $u^\mu = \xi^\mu$, and the particle is at rest with respect to the cosmological fluid, and thus, $\dot{E} = 0$.

The simplest dynamics for the spin that is consistent with (1), the conservation of $s\cdot s$, and $s\cdot p = 0$, is

$$u^\mu \nabla_\mu s^\nu = \alpha \frac{m}{m_p^2} \mathrm{sgn}(s\cdot\xi)\mathbf{R}(s\cdot s)u^\nu. \tag{3}$$

This is only a minimalistic solution, other terms can be added to (3). We will investigate these aspects elsewhere as they might be important for phenomenology; however, they do not affect the main point in this Letter.

In this respect, it is also important to point out that the violations of the equivalence principle and Lorentz invariance implied by (1) and (3) can be readily checked not to be in conflict with well known observational bounds by many orders of magnitude [16] for $\alpha \sim O(1)$. A simple indication comes from comparison of the value of $\mathbf{R}$ at the electroweak (EW) transition in cosmology (a regime where our effects will be important) to that associated with, say, the gravitational effect of a piece of lead: this gives $(\mathbf{R}_{\mathrm{lead}}/\mathbf{R}_{\mathrm{EW}}) \sim 10^{-24}$.

There is a remarkable formal similarity to Eq. (1) with others arising in well understood situations. We have the Mathisson-Papapetrou-Dixon equations [17] describing the dynamics of idealized extended objects in general relativity, $u^\nu \nabla_\nu P_\mu = -\frac{1}{2}\mathbf{R}_{\mu\nu\rho\sigma} u^\nu S^{\rho\sigma}$, where u^μ represents the four velocity of the object, P^μ its four momentum, $S^{\rho\sigma}$ its spin, and $R_{\mu\nu\rho\sigma}$ is the Riemann tensor. Moreover, we note that the characterization of WKB trajectories of the Dirac theory on a pseudo-Riemannian geometry [18], to lowest order in $\hbar$, is given by $u^\nu \nabla_\nu (m u_\mu) = -\frac{1}{2}\tilde{\mathbf{R}}_{\mu\nu\rho\sigma} u^\nu \langle S^{\rho\sigma}\rangle + \mathcal{O}(\hbar^2)$. The previous is equivalent to (1) if one considers an effective $\tilde{\mathbf{R}}_{\mu\nu\rho\sigma} \propto m^2/m_p^2 \mathrm{sgn}(s\cdot\xi)\mathbf{R}\epsilon_{\mu\nu\rho\sigma}$ taken to encode a pure torsion-related structure as $\tilde{\mathbf{R}}_{[\mu\nu\rho]\sigma} \neq 0$ (from the first Bianchi identities).

Coming back to the main argument, the diffusion of energy for a single particle, induced by (1), implies the lack of energy-momentum conservation for a fluid made of a collection of such particles (we will compute this below). However, violations of energy-momentum conservation are incompatible with general covariance and, hence, with the standard general relativity description of gravity. Fortunately, there is a simple relaxation of general covariance (originally studied by Einstein) from full coordinate invariance down to spacetime volume preserving coordinate transformations. Such a modification—which we only take as an effective low energy description of a (in a suitable sense) general covariant fundamental physics—is called unimodular gravity (UG), and its field equations are just the trace-free part of the standard Einstein's equations

$$\mathbf{R}_{\mu\nu} - \frac{1}{4}\mathbf{R} g_{\mu\nu} = 8\pi G\left(\mathbf{T}_{\mu\nu} - \frac{1}{4}\mathbf{T} g_{\mu\nu}\right). \tag{4}$$

Defining $J_\mu \equiv (8\pi G)\nabla^\nu T_{\nu\mu}$, assuming UG integrability $dJ = 0$, and using Bianchi identities, one obtains [5]

$$\mathbf{R}_{\mu\nu} - \frac{1}{2}\mathbf{R} g_{\mu\nu} + \underbrace{\left[\Lambda_0 + \int_\ell J\right]}_{\Lambda} g_{\mu\nu} = 8\pi G \mathbf{T}_{\mu\nu}, \tag{5}$$

where Λ_0 is a constant of integration, and ℓ is a one-dimensional path from some reference event. Thus, the energy-violation current J is the source of a term in Einstein's equations satisfying the dark energy equation of state. An additional, often alluded to feature of UG is that the vacuum energy does not gravitate [3,19,20].

Now, we compute $J_\nu \equiv 8\pi G \nabla^\mu T_{\mu\nu}$ as implied by (1). For a particle species i one has the following contribution to the energy momentum tensor (the interactions between different species are neglected here as their effect leads to very small corrections):

$$\mathbf{T}^i_{\mu\nu}(x) \equiv \int p_\mu p_\nu f^i(x,p,s_r) Dp Ds_r, \tag{6}$$

where $f^i(x,p,s_r)$ encodes the particle distribution in phase space with s_r denoting the value of the spin of the particle in its rest frame, $Dp = \delta(p^2+m^2)dp^4$, and Ds_r is the standard measure on the sphere of the spin directions. Simple kinetic theory allows us to express $\nabla^\mu \mathbf{T}^i{}_{\mu\nu}$ as (see equation 2.113 in [21])

$$\begin{aligned}\frac{\nabla^\mu \mathbf{T}^i_{\mu\nu}}{\mathbf{T}^i} &= -\frac{\int m_i F_\nu f^i(x,p,s_r) Dp Ds_r}{m_i^2 \int f^i(x,p,s_r) Dp Ds_r} \\ &= -\alpha \frac{m_i}{m_p^2}\mathbf{R}\frac{\int [\frac{s_\nu s_0}{|s_0|}] f^i(x,p,s_r) Dp Ds_r}{\int f^i(x,p,s_r) Dp Ds_r}\end{aligned} \tag{7}$$

the components labelled by zero refer to those along ξ. Assuming thermal equilibrium at temperature T, and ignoring the negligible additional effects of the force on the distribution, we have $f^i(x,p,s_r) = f^i_T(p)$ where the latter is the standard Boltzmann distribution.

There is a subtle point that ought to be noted here: this part of the calculation is carried out by considering a spacetime region small enough to be covered by Riemann normal coordinates (i.e., a local inertial frame) in such a way that the standard effects of curvature can be neglected. However, the region is large in comparison with the Planck scale so that the energy diffusion effects, the nonstandard influence of $\mathbf{R}$ in our model, are encoded in the friction force underlying (1).

Isotropy of the equilibrium configuration implies that only the zeroth component of (7) is nontrivial. The relevant integration over spin is

$$\int |s_0| Ds_r = \frac{2\pi \mathbf{p}|s|}{m}\int |\cos(\theta)|\sin(\theta)d\theta = \frac{2\pi\mathbf{p}|s|}{m},$$

where $\mathbf{p}^2 \equiv \vec{p}\cdot\vec{p}$, and the factor $\mathbf{p}/m$ comes from the boost relating the comoving frame to the rest frame of the particle. The next step is

$$\frac{\int [\frac{2\pi\mathbf{p}|s|}{m}] f_T(p) Dp}{\int f_T(p) Dp} = 4\pi|s|\frac{T}{m}\left[1 + \mathcal{O}\left(\log\left(\frac{m}{T}\right)\frac{m^2}{T^2}\right)\right].$$

Therefore, in the relativistic regime $T \gg m$, one has

$$\begin{aligned} J_\nu &\equiv (8\pi G)\nabla^\mu \mathbf{T}_{\mu\nu} = 4\pi\alpha\frac{T}{m_p^2}\mathbf{R}\left[8\pi G\sum_i |s_i|\mathbf{T}^i\right]\xi_\nu, \\ &\approx 2\pi\alpha\hbar\frac{T}{m_p^2}\mathbf{R}^2 dt_\nu, \end{aligned} \tag{8}$$

where, in the first line the sum is over particle species, and in the last line, we specialize on an approximation valid for the case where a single $|s| = \hbar/2$ fermion species dominates. This approximation will used in order to simplify the formulas that follow; however, it is not necessary [the full expression is used in computing the results of Fig. (1)].

Now, we focus on the effects of (8) in the dynamics of the early Universe when its macroscopic geometry is well approximated by the flat Friedmann-Lemaître-Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2, \tag{9}$$

and where the local frame $\xi = \partial_t$ is identified with comoving observers. As only massive particles with spin are subjected to the frictional force (1), the diffusion mechanism in cosmology starts when such particles first appeared. According to the SM—whose validity is assumed from the end of inflation—this corresponds to the EW transition time. We further assume that a protective symmetry enforces $\Lambda_0 = 0$ (see, for instance, [22,23]).

Now, we are ready to estimate the effective cosmological constant predicted by our model. Using (5) and (8), one gets

$$\Lambda = \frac{2\pi\alpha\hbar}{m_p^2}\int_{t_{\rm EW}}^{t_0}[8\pi G(\rho - 3P)]^2 T dt, \tag{10}$$

with t_0 the present time. It is convenient to change the integration variable in (10) from comoving time t to temperature T given the essentially direct relation between the two quantities. During the relevant period, of radiation domination, the matter fields are assumed to be in thermal equilibrium. The density of the Universe is then given by $\rho = \pi^2 g_* T^4/(30\hbar^3)$, where $g_* \approx 100$ is the effective degeneracy factor for the temperatures of interest [24]. Taking into account that temperature scales like a^{-1}, using the Friedman equation, and $H(a) = \dot{a}/a$, one gets

$$\frac{dT}{T} = -\frac{da}{a} = -\underbrace{\sqrt{\frac{8\pi G}{3}\frac{\pi^2 g_* T^4}{30\hbar^3}}}_{H(a)}dt. \tag{11}$$

Now, we will focus just on the leading contributions. In the ultrarelativistic regime, standard thermodynamics leads to the expression

$$\rho - 3P \approx \frac{m_t^2 T^2}{2\hbar^3}, \tag{12}$$

where m_t is the top mass. Replacing the leading term in (12) and (11) into (10), one gets

$$\Lambda \approx 16\alpha\sqrt{\frac{5\pi^3}{g_*}}\frac{m_t^4 {T_{\rm EW}}^3}{m_p^5\hbar^2}\epsilon(T_{\rm EW}), \tag{13}$$

where

$$\epsilon(T_{\rm EW}) = -\frac{3}{T_{\rm EW}^3}\int_{T_{\rm EW}}^{T_{\rm end}}\left(1 - \frac{T^2}{T_{\rm EW}^2}\right)^2 T^2 dT \tag{14}$$

is a dimensionless correction factor that takes into account the temperature dependence of the quark mass during the EW transition, namely, $m_t^2(T) = m_t^2(1 - T^2/T_{\rm EW}^2)$. The temperature where the contribution from the top quark ends, $T_{\rm end}$ is the one satisfying $2m_t(T_{\rm end}) = T_{\rm end}$ as it defines the moment when its abundance decreases dramatically (Massive gauge bosons do not change the order of magnitude estimate, as $m_Z/m_t \approx 1/2$ and $g_{ZW^\pm}/g_{t\bar{t}} = 3/4$. In (13), this leads to a factor $(3/4)^2(1/2)^4 \times 2$ as the spin of the bosons is twice that of the fermions; i.e., their contributions is about 7% of that coming from the top quark. From (8), one can work out the precise corrections which are included in Fig. 1). Simple dark matter models such as WIMPS will not affect the order of magnitude of the estimate as long as they acquire their masses via the Higgs mechanism and their number of species is not too large. We note that, aside from the correction factor, $\epsilon(T) \approx 10^{-3}$–$10^{-4}$ in the range of interest, Eq. (13) could have been guessed from dimensional analysis. After substitution of the different quantities involved and taking, for example, $T_{\rm EW} \approx 100$ GeV [25,26], and adding the gauge boson contributions [not included in (13)] we find

$$\Lambda \approx 4\alpha\Lambda_{\rm obs}, \tag{15}$$

where $\Lambda_{\rm obs}$ is the observed value of the cosmological constant. For other values of $T_{\rm EW}$, see Fig. 1, where we plot the value of the dimensionless coupling α needed to fit the observed values as a function of $T_{\rm EW}$. These results are an order of magnitude estimate; a refined calculation would require detailed considerations of the dynamics of the electroweak transition. However, such details are not expected to modify our result in essential ways.

We believe that our proposal has important implications of various types. At the theoretical level, it provides a novel view that could reconcile Planckian discreteness and Lorentz invariance and gives possibly valuable insights

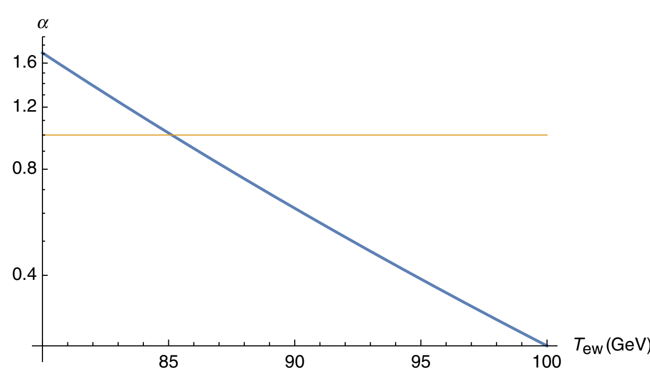

FIG. 1. The value of the phenomenological parameter α, see Eq. (1), that fits the observed value of $\Lambda_{\rm obs}$ as a function of the EW transition scale $T_{\rm EW}$ in GeV. The contributions from the massive gauge bosons of the SM have been included.

guiding the search for a theory of quantum gravity. At the empirical level, our analysis opens a path for searches of novel physical manifestations of the gravity-quantum interface.

Concerning the latter, we note that one might use (8) to estimate the amount of energy loss in local experiments. Presently (neglecting the cosmic expansion), we find $\dot{\rho} \approx -\alpha(\rho/\rho_{\rm water})^2 10^{-70}$ g/(cm^3 s) where $\rho_{\rm water}$ is the density of sea water. The amount of energy produced is maximal at the EW transition when the density of the Universe $\rho(T_{\rm EW}) \approx 10^{25}$ g/cm^3, and corresponds to a relative change of energy density in a Hubble time of $\Delta\rho/\rho \approx \alpha 10^{-51}$. Such a minuscule level of energy loss cannot have significant effects on the matter dynamics and, thus, would be very hard to detect. Nevertheless, we have seen that such small energy losses can explain the observed late time acceleration of the expansion rate of our Universe.

Finally, as the model links ρ and its evolution with the present value of the cosmological constant, and ρ directly enters in the computation of the structure formation leading to galaxies, stars, and eventually humans, this framework opens, in principle, a path that might help in addressing the long debated "coincidence problem" [24].

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