Dissipative Charging of a Quantum Battery

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We show that a cyclic unitary process can extract work from the thermodynamic equilibrium state of an engineered quantum dissipative process. Systems in the equilibrium states of these processes serve as batteries, storing energy. The dissipative process that brings the battery to the active equilibrium state is driven by an agent that couples the battery to thermal systems. The second law of thermodynamics imposes a work cost for the process; however, no work is needed to keep the battery in that charged state. We consider simple examples of these batteries and discuss situations in which the charged state has full population inversion, in which case the extractable work is maximal, and circumstances in which the efficiency of the process is maximal.

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Introduction.—In a dissipative process, the state of a system converges asymptotically to an invariant or steady state [1]. Thus, by engineering a dissipative-open-quantum-system dynamic, a system can be driven to a given target state. This idea has been explored in quantum computation, entanglement generation, and quantum sensing [2–7], thanks to the control achieved on multipartite quantum systems [8–10].

Another application relevant to future quantum technologies is charging a quantum battery. The process of charging a battery, i.e., storing energy in a quantum system for later use, has been studied in various contexts. Some of these consider dissipative processes [11,12] but do not take the invariant state of the dynamics as the charged state. Most of the studies focus on the isolated unitary evolution emphasizing the role of quantum correlations between its components [13–17]. Because in this setup the target state is given, charging a battery by a unitary process requires fine-tuning between the initial state of the battery and the unitary process. This step is avoided by resetting the battery before the charging process. Alternatively, engineering a quantum dissipative process with a charged target state, an agent can run a protocol without the need for any information of the battery state and load it. The question that arises is how efficient such a process can be.

An energetically efficient engineered process should have no work cost to the agent if the battery has already achieved the charged state; otherwise, he dissipates the same currency that he wants to store. From a thermodynamic point of view, steady states of dissipative dynamics are either nonequilibrium states and dissipate energy or are *equilibrium* states, which are *dissipationless*. Motivated by these considerations and recent progress in quantum thermodynamics [18–21], we consider the following question: Can we engineer a dissipative process for a battery, involving auxiliary systems in the thermal Gibbs states all at the same temperature, such that its invariant state is an equilibrium state where energy can be stored and then extracted? We will show that, for systems with finite-dimensional Hilbert space, this is indeed the case. We will characterize the charged equilibrium state by its ergotropy [22] and the charging process by its efficiency.

The ergotropy of a state is the maximum work that can be extracted from it in a unitary cyclic process [22]. States with positive ergotropy are called "active states" and those with vanishing, "passive states" [23,24]. Equilibrium states reached by a relaxation process of a system in contact with a thermal bath are passive; otherwise, a perpetuum mobile of the second kind could be built in contradiction with the Kelvin Planck statement of the second law [25]. However, equilibrium states are not necessarily passive. It is possible to engineer processes with active equilibrium states. The second law implies that there is a work cost implementing the protocol that drives the system to the active equilibrium state (see Supplemental Material [26]).

When the optimal unitary cyclic process extracts the ergotropy, the battery is left in a corresponding passive state. We define the efficiency of the charging process by the ratio between the ergotropy of the equilibrium state and the work cost for the battery charging process from the corresponding passive state. If the battery is recycled after use, this will be the quantity of interest.

Usually, in thermodynamics, there is a tradeoff between the resource we are interested in and the efficiency of the process that produces it. We will show this tradeoff for the ergotropy of active equilibrium states. In particular, we find an engineered thermodynamic process that brings a battery to an equilibrium state with maximal ergotropy, as well as full population inversion but of low efficiency. A process of maximal efficiency with low ergotropy is illustrated with a 2-qubit battery, and in the Supplemental Material [26], we show that the efficiency generally goes to its maximal value only if the ergotropy goes to zero.

Thermodynamics of open quantum systems.—We consider the following idealized scenario: We have a system of interest, the battery with a Hamiltonian H_S and many copies of the same auxiliary system, each with the same Hamiltonian H_A. Initially, all systems are uncorrelated, and every copy of the auxiliary system is in the same temperature Gibbs thermal state $\omega_\beta(H_A) = e^{-\beta H_A}/\mathrm{Tr}[e^{-\beta H_A}]$, where Tr denotes the total trace. They play the role of a thermal bath. The initial state of the battery is $\rho_S(0)$. Systems in the Gibbs state are easy to prepare by coupling them weakly to the environment with inverse temperature β (we consider units such that $k_B = \hbar = 1$). To implement the desired dissipative dynamics, an agent couples the battery to one auxiliary system for a lapse of time τ and then to another system for a subsequent lapse of time τ and so on, turning on and off the interaction between the battery and different copies of the auxiliary systems, in a repeated interaction process [27–29]. At the nth step, the battery interacts with the nth member of the auxiliary systems through the time-independent potential V. This interaction vanishes at the initial $(n-1)\tau$ and final $n\tau$ times where the interaction is off and the total Hamiltonian is simply $H_S + H_A$. The state of the battery at time $n\tau$ reads $\rho_S(n\tau) = \mathrm{Tr}_A[\rho_{\mathrm{tot}}(n\tau)]$, where Tr_X denotes a partial trace over the degrees of freedom of system X, in this case the nth auxiliary system, and

$$\rho_{\mathrm{tot}}(n\tau) = U\{\rho_S[(n-1)\tau] \otimes \omega_\beta(H_A)\}U^\dagger, \qquad (1)$$

where $U = e^{i\tau(H_S+H_A+V)}$ is the unitary time evolution operator for the composite system with the interaction on. Various dissipative processes are modeled in this way, such as streams of atoms moving across a quantum electrodynamic cavity [30] and a boundary driven system [31], to mention a few. Recently, the importance of the time-dependent coupling for a proper thermodynamic description of these processes was discussed in [28] (see also [32]), where the work performed by the agent due to switching on and off the interaction was taken into account.

Let us briefly analyze the thermodynamics of the elementary process of duration τ. The properties of the concatenated process are deduced from these (see [33,34] for details). The energetics of the nth step is characterized by the switching work W_n, and the heat Q_n is the negative energy change of the nth auxiliary bath system [28,29,32]. These quantities read as follows:

$$W_n = \mathrm{Tr}\big((H_S + H_A)\{\rho_{\mathrm{tot}}(n\tau) - \rho_S[(n-1)\tau] \otimes \omega_\beta(H_A)\}\big), \qquad (2)$$

$$Q_n = -\mathrm{Tr}\{H_A[\rho_A(n\tau) - \omega_\beta(H_A)]\}, \qquad (3)$$

where $\rho_A(n\tau) = \mathrm{Tr}_S[\rho_{\mathrm{tot}}(n\tau)]$. Their sum is the energy change of the battery (first law), calculated as

$$\Delta E_n = Q_n + W_n = \mathrm{Tr}\big(H_S\{\rho_S(n\tau) - \rho_S[(n-1)\tau]\}\big). \qquad (4)$$

Considering the von Neumann entropy $S(\rho_S) = -\mathrm{Tr}[\rho_S \ln \rho_S]$, the entropy change of the battery in the nth step can be expressed as [35]

$$\Delta S_n = \Sigma_n + \beta Q_n, \qquad (5)$$

where

$$\Sigma_n = D[\rho_{\mathrm{tot}}(n\tau) || \rho_S(n\tau) \otimes \omega_\beta(H_A)] \geq 0 \qquad (6)$$

is the entropy production with $D(a||b) \equiv \mathrm{Tr}[a \ln a] - \mathrm{Tr}[a \ln b]$ as the "relative entropy," which is nonnegative for any density matrices a and b [1]. The inequality in Eq. (6) corresponds to the second law.

In each time step, the battery evolves under the completely positive trace preserving map $\mathcal{E}(\rho) = \mathrm{Tr}_A[U\rho \otimes \omega_\beta(H_A)U^\dagger]$, and we have characterized the thermodynamic properties for this elementary process. The engineered dissipative dynamics is obtained by concatenating this elementary process $(\mathcal{E}\circ\mathcal{E}\circ\cdots)$.

We engineer an $\mathcal{E}$ with a unique invariant state, $\pi = \mathcal{E}(\pi)$, attractive due to the contractive character of the relative entropy under the action of the map [1]. Concatenating it a large number of times, every initial state of the battery will converge to π, i.e., $\lim_{n\to\infty}\mathcal{E}^n[\rho_S(0)] = \pi \ \forall \ \rho_S(0)$; the work, heat, and entropy produced in this process $\rho_S(0) \to \pi$ are the sum of the corresponding quantities, Eqs. (2), (3), and (6), for each step.

We can distinguish two kinds of maps: maps with or without equilibrium [33,34]. If the action of $\mathcal{E}$ over π gives $\Sigma_n > 0$, then π is a nonequilibrium steady state sustained by dissipated work $(W_n = -Q_n = \Sigma_n/\beta > 0)$ performed by the agent. Conversely, if the action of the map over π gives $\Sigma_n = 0$, then π is an equilibrium state. In equilibrium, the heat, work, and entropy production vanish $(W_n = Q_n = \Sigma_n = 0)$: no work is needed to sustain the state π. In this case, we say that $\rho_S(0) \to \pi$ is an "equilibration process." The unitary time evolution operator U of a map with equilibrium satisfies [33]

$$[U, H_0 + H_A] = 0, \qquad (7)$$

with H_0 as an operator on the Hilbert space of the system, and the equilibrium state is $\pi = e^{-\beta H_0}/\mathrm{Tr}[e^{-\beta H_0}] = \omega_\beta(H_0)$.

Among the properties of maps with equilibrium, an important one is that the thermodynamic quantities (2), (3), and (6) can be written in terms of system operators only. Heat, work, and entropy production take the form

$$Q_n = \mathrm{Tr}\big(H_0\{\rho_S(n\tau) - \rho_S[(n-1)\tau]\}\big), \qquad (8)$$

$$W_n = \mathrm{Tr}\big((H_S - H_0)\{\rho_S(n\tau) - \rho_S[(n-1)\tau]\}\big), \qquad (9)$$

$$\Sigma_n = D\{\rho_S[(n-1)\tau] || \pi\} - D[\rho_S(n\tau) || \pi] \geq 0. \qquad (10)$$

We see that, if $H_0 = H_S$, then $W_n = 0$, and $\rho_S(0) \to \pi$ is a "relaxation process." In this case, the equilibrium state is the passive Gibbs state. If H_0 is an operator different from H_S, the equilibrium state $\pi = \omega_\beta(H_0)$ may be an active state and the equilibration process $\rho_S(0) \to \pi$ has a total work cost $W = \sum_n W_n = \mathrm{Tr}\{(H_S - H_0)[\pi - \rho_S(0)]\}$, as follows from Eq. (9). Thus, a condition necessary to have a charged battery in an equilibrium state is to engineer a map $\mathcal{E}$ with an equilibrium state $\omega_\beta(H_0)$ with $H_0 \neq H_S$.

Ergotropy.—To quantify the energy stored in a battery, we consider the ergotropy [22]

$$\mathcal{W}(\rho_S) = \max_{\mathrm{u}} \mathrm{Tr}(H_S[\rho_S - u\rho_S u^\dagger]) \tag{11}$$

of its state ρ_S. This is the maximal amount of work that can be extracted in a unitary cyclic process, where the state evolves unitarily with $u = \mathcal{T}_+ e^{-i\int dt[H_S + V_S(t)]}$ where $\mathcal{T}_+$ denotes the time ordering operator, and $V_S(t)$ is a time-dependent potential vanishing at the beginning and end of the process, accounting for a cyclic external work source. For passive [22,24] states, one has $\mathcal{W}(\rho_S) = 0$. States are active if $\mathcal{W}(\rho_S) > 0$.

If we order the eigenvalues of $H_S = \sum_i E_i |E_i\rangle\langle E_i|$ (assumed to be nondegenerate for simplicity, but see Supplemental Material [26] for the general case) in increasing order, $E_1 < E_2 < \cdots < E_N$, and the eigenvalues of $\rho_S = \sum_i r_i |r_i\rangle\langle r_i|$ in decreasing order, $r_1 \geq r_2 \geq \cdots \geq r_N$, then the ergotropy of ρ_S is given [22] by

$$\mathcal{W}(\rho_S) = \sum_{jk} r_j E_k (|\langle r_j | E_k\rangle|^2 - \delta_{jk}). \tag{12}$$

After the optimal work extraction process, the system is left in the corresponding passive state

$$\sigma_{\rho_S} = \sum_j r_j |E_j\rangle\langle E_j|. \tag{13}$$

The ergotropy of ρ_S can then be written as

$$\mathcal{W}(\rho_S) = \mathrm{Tr}[H_S(\rho_S - \sigma_{\rho_S})]. \tag{14}$$

Condition for active equilibrium.—Let us obtain the conditions for an active equilibrium state $\omega_\beta(H_0)$. First, note that the equilibrium condition $[U, H_0 + H_A] = 0$ with $U = e^{-i\tau(H_S + H_A + V)}$ is satisfied if $[H_0, H_S] = 0$ and $[H_0 + H_A, V] = 0$. On the basis of common eigenvectors of the nondegenerate H_S and H_0, the equilibrium state is

$$\omega_\beta(H_0) = \sum_{i=1}^N \frac{e^{-\beta E_i^0}}{Z_0} |E_i\rangle\langle E_i|,$$

with $Z_0 = \mathrm{Tr}[e^{-\beta H_0}]$, and if a pair (j, k) exists such that $(E_j - E_k)(E_j^0 - E_k^0) \leq 0$, the state is active. Then, its ergotropy is extracted by a process described by a permutation unitary matrix u associated with the permutation p of $\{1, \ldots, N\}$ such that $E^0_{p_1} \leq \cdots \leq E^0_{p_N}$ leaving the battery in the passive state [22]

$$\sigma_{\omega_\beta(H_0)} = u\omega_\beta(H_0)u^\dagger = \sum_{i=1}^N \frac{e^{-\beta E^0_{p_i}}}{Z_0} |E_i\rangle\langle E_i|. \tag{15}$$

Note that the total heat Q_R and work W_R obtained by Eqs. (8) and (9) characterizing a recharging process $\sigma_{\omega_\beta(H_0)} \to \omega_\beta(H_0)$ are

$$Q_R = \mathrm{Tr}\{H_0[\omega_\beta(H_0) - \sigma_{\omega_\beta(H_0)}]\}, \tag{16}$$

$$W_R = \mathrm{Tr}\{(H_S - H_0)[\omega_\beta(H_0) - \sigma_{\omega_\beta(H_0)}]\}, \tag{17}$$

and we see that the ergotropy of the state $\omega_\beta(H_0)$, obtained from Eqs. (14) and (15), is

$$\mathcal{W}[\omega_\beta(H_0)] = \sum_{i=1}^N (E_{p_i} - E_i) \frac{e^{-\beta E^0_{p_i}}}{Z_0}$$

and it is related to W_R and Q_R by

$$W_R = \mathcal{W}[\omega_\beta(H_0)] - Q_R.$$

Note that $Q_R \leq 0$ [see Eq. (16)] because $\omega_\beta(H_0)$ is the state with minimum average H_0 among states with the same entropy (Supplemental Material [26]). It follows that $W_R \geq \mathcal{W}[\omega_\beta(H_0)] \geq 0$, and thus, no perpetuum mobile of the second kind can be built. We quantify the efficiency of the charging process by the ratio

$$\eta \equiv \frac{\mathcal{W}[\omega_\beta(H_0)]}{W_R} = 1 - \frac{|Q_R|}{W_R}, \qquad 0 \leq \eta \leq 1.$$

A protocol for active equilibrium.—A particularly interesting equilibrating processes with an active equilibrium state is obtained with an interaction $V = \sum_\alpha S_\alpha \otimes A_\alpha$, where the system operators S_α and auxiliary bath operators A_α satisfy $[H_S, S_\alpha] = \lambda_\alpha S_\alpha$ and $[H_A, A_\alpha] = \lambda_\alpha A_\alpha$. In this case, $[V, -H_S + H_A] = 0$; i.e., we have $H_0 = -H_S$, and the corresponding process $\mathcal{E}$ has the equilibrium state

$$\omega_\beta(-H_S) = \sum_{i=1}^N \frac{e^{\beta E_i}}{Z_+} |E_i\rangle\langle E_i|,$$

with $Z_+ = \mathrm{Tr}[e^{+\beta H_S}]$.

Since different Hamiltonians with the same Bohr frequency spectrum $\{\lambda_\alpha\}$ are unlikely, the process should be engineered with auxiliary baths that are copies of the system, i.e., $H_S = H_A$. With this specific interaction V, we have a process $\mathcal{E}$ with a remarkable thermodynamic equilibrium between a system in the state $\omega_\beta(-H_S)$ with copies of itself in the state $\omega_\beta(H_S)$. Replacing $H_0 = -H_S$ in Eqs. (16) and (17), we see that, in the recharging process, $Q_R = W_R/2 = \mathcal{W}$ [see Eq. (14)], and the efficiency of this process is $\eta = 1/2$.

Since $E_j^0 = -E_j$, the permutation p that orders the spectrum $\{E_j^0\}$ in an increasing order is $i \to p_i = N + 1 - i$. Therefore, the ergotropy of $\omega_\beta(-H_S)$ is $\mathcal{W} = \sum_i (E_{N+1-i} - E_i) e^{\beta E_{N+1-i}}/Z_+$, which is positive, and at low temperature, $\beta \to \infty$, it is the maximal value $\mathcal{W} = E_N - E_1$.

In general, $\mathcal{W}(\rho_S)$ is upper bounded [13] by $\mathrm{Tr}\{H_S[\rho_S - \omega_{\beta^*}(H_S)]\}$ with β^* such that $S(\rho_S) = S[\omega_{\beta^*}(H_S)]$; it is natural to ask under what conditions $\mathcal{W}[\omega_\beta(-H_S)]$ can saturate the bound. We found that batteries with symmetric spectrum with respect to some energy value $\bar{E}$, i.e., $E_{N+1-i} = 2\bar{E} - E_i$, saturate the bound. Indeed, in this case, $E^0_{p_i} = E^0_{N+1-i} = -E_{N+1-i} = E_i - 2\bar{E}$ and $Z_0 = \sum_j e^{-\beta E_j^0} = e^{2\beta\bar{E}} Z$, with $Z = \sum_j e^{-\beta E_j}$ as the canonical partition function. Thus, the passive state of $\omega_\beta(-H_S)$ as given by Eq. (15) is

$$\sigma_{\omega_\beta(-H_S)} = \sum_j \frac{e^{-\beta E^0_{N+1-j}}}{Z_0} |E_j\rangle\langle E_j| = \omega_\beta(H_S),$$

i.e., the Gibbs state with the same temperature as the bath. Note also, that the active state $\omega_\beta(-H_S)$ is, in this case, the fully inverted population state $(1/Z)\sum_i e^{-\beta E_{N+1-i}} |E_i\rangle\langle E_i|$.

Single-qubit battery.—As an example, we consider the battery and auxiliary systems all identical qubits (Hereafter $\sigma^x, \sigma^y, \sigma^z$ are Pauli matrices and $\sigma^\pm = (\sigma^x \pm i\sigma^y)/2)$; i.e., the battery Hamiltonian is $H_S = (h/2)\sigma_S^z$ (with a symmetric spectrum), and the auxiliary systems Hamiltonians are $H_A = (h/2)\sigma_A^z$, with $h > 0$. The coupling between the system and the auxiliary qubit is

$$V = a(\sigma_S^+ \sigma_A^+ + \sigma_S^- \sigma_A^-)$$

and is such that $[\sigma_A^z - \sigma_S^z, V] = 0$, i.e., $H_0 = -H_S$. The ergotropy of the battery in the equilibrium state $\omega_\beta(-H_S)$ is $\mathcal{W} = h \tanh \beta h/2$, which achieves the maximal value in the low-temperature regime $\beta h \gg 1$. In the upper panel of Fig. 1, we plot the populations of the ground (p_g) and excited (p_e) states of the battery at each elementary step n starting from the passive thermal state, and in the lower panel, the thermodynamic quantities W_n, Q_n, and Σ_n. The population inversion is achieved, and the work cost goes to zero when the system reaches its equilibrium state.

2-qubit battery.—With the previous protocol, we can achieve maximal ergotropy, especially in the low-temperature regime. We will now illustrate with another example that the maximal efficiency can also be achieved but with low ergotropy. In the Supplemental Material [26], we show that this result is general.

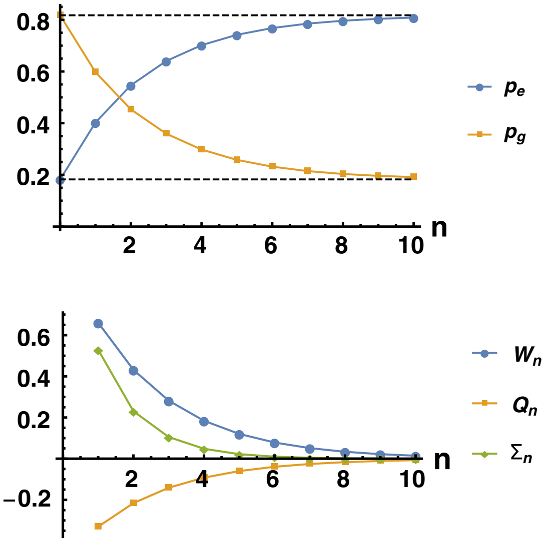

FIG. 1. Population inversion in the equilibration process for the single-qubit battery. (Upper) Populations of the ground (p_g) and excited (p_e) states for each iteration step n. (Lower) Work (W_n), heat flow (Q_n), and entropy production (Σ_n) for each step n. The initial state of the battery is a thermal state, that is, an identical state to that of the auxiliary baths. We consider for these plots $\tau = 0.1$, $a = \sqrt{10}$, $h = 1.5$, and $\beta = 1$. The continuous lines are a guide for the eyes.

We consider a 2-qubit battery with Hamiltonian

$$H_S = \frac{h}{2}(\sigma_1^z + \sigma_2^z) + J(\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y).$$

We take $2J > h > 0$. If we consider the process $\mathcal{E}$ obtained by coupling auxiliary systems of Hamiltonian $H_A = (h/2)\sigma_A^z$ to the battery of Hamiltonian H_S with

$$V = \sigma_A^x \sigma_1^x + \sigma_A^y \sigma_1^y,$$

the equilibrium state is found to be $\omega_\beta(H_0)$ with $H_0 = (h/2)(\sigma_1^z + \sigma_2^z)$, whose ergotropy is

$$\mathcal{W} = (2J - h)\frac{\sinh \beta h}{1 + \cosh \beta h}.$$

The work done in the dissipative process $\sigma_{\omega_\beta(H_0)} \to \omega_\beta(H_0)$ that recharges the battery is

$$W_R = 2J \frac{\sinh \beta h}{1 + \cosh \beta h}.$$

We see that the efficiency $\eta = \mathcal{W}/W_R \to 1$ if $h \to 0$ for all β, yet, to have a finite ergotropy, one would need $\beta \sim \mathcal{O}(h^{-1})$. Note that if $\beta h \gg 1$, but h is small, $\mathcal{W} \to 2J$, and for this system the state of maximum ergotropy is the pure state of maximal eigenenergy for which $\mathcal{W} = 4J$. Details can be found in the Supplemental Material [26].

Conclusions.—In this Letter, we have explored the role of active equilibrium states in the context of charging a battery. Active equilibrium states have been used in thermodynamic processes involving squeezing [36]. Indeed, it has been shown [37] that, for systems with a quadratic Hamiltonian, bosonic reservoirs in squeezed thermal states can act as equilibrium reservoirs, exchanging energy and squeezing (asymmetry between position and momentum uncertainties) such that these two noncommuting quantities are globally conserved and making any initial state of the system relax to an active equilibrium state in a generalized Gibbs state, the squeezed thermal state. We have shown here that, by engineering the coupling between a system, the battery, and auxiliary systems prepared in Gibbs thermal state, a bath, an agent can drive the system to an active equilibrium state. This engineering requires one to find a unitary evolution with a single globally conserved quantity, see Eq. (7). We were able to evaluate the work cost for the agent due to the coupling and decoupling between the battery and auxiliary thermal systems. As a consequence, work can be extracted from the equilibrium state, but no perpetuum mobile of the second kind could be built. The notable aspect of our result is that, because the charged state is an equilibrium state, the agent does not waste energy (work) once the battery is in the equilibrium state. One can thus consider that continuing the process once the battery is charged is a convenient way of protecting the charged battery. If a perturbation changes its state, the process will charge the battery again, spending energy only when this happens.

We have characterized the activity of these equilibrium states by their ergotropy and the efficiency of the charging process; furthermore, we showed that, in the low-temperature limit, either maximal ergotropy or efficiency could be obtained. We observe a tradeoff between ergotropy and the efficiency η of the process that produces it. Interestingly, we have found a dissipative process in which the equilibrium state of the system is $e^{\beta H}/Z_+$, while the environment is in the state $e^{-\beta H}/Z$. For this process, it is important that the Hamiltonian of the battery has a finite spectrum, but other processes for other charged states will not require that.

Finally, since all spin-spin $1/2$ interactions are possible to implement with trapped ions [38], the predictions for the qubit battery is testable with current experimental techniques.

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