Thermal Transport in One-Dimensional Electronic Fluids

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We study thermal conductivity for one-dimensional electronic fluids. The many-body Hilbert space is partitioned into bosonic and fermionic sectors that carry the thermal current in parallel. For times shorter than the bosonic umklapp time, the momenta of Bose and Fermi components are separately conserved, giving rise to the ballistic heat propagation and imaginary heat conductivity proportional to $T/i\omega$. The real part of thermal conductivity is controlled by decay processes of fermionic and bosonic excitations, leading to several regimes in frequency dependence. At lowest frequencies or longest length scales, the thermal transport is dominated by Lévy flights of low-momentum bosons that lead to a fractional scaling, $\omega^{-1/3}$ and $L^{1/3}$, of heat conductivity with the frequency ω and system size L, respectively.

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In interacting systems, the Wiedemann-Franz law is violated and there is no universal relation between the thermal and electric conductivities. As a result, thermal transport reveals information that cannot be accessed by measuring charge transport. Because of experimental and theoretical challenges, thermal transport is far less explored compared to charge transport.

In recent years, the situation started to change, and energy transport was measured in several experiments. The universal value of thermal conductance $g_0 = \pi^2 T/3h$ was observed [1–4] in various devices with ideal point contacts. Heat Coulomb blockade was observed in Ref. [5], directly demonstrating a controllable energy-charge separation. The propagation of heat in the quantum-Hall-effect regime was intensively investigated [6–11]. Frequency dependence of the thermal conductivity was studied in various materials [12–14].

The thermal transport in low-dimensional classical fluids and 1D anharmonic chains (of which the Fermi-Pasta-Ulam-Tsingou model [15] is a prominent example) has been studied in the framework of (nonlinear) fluctuating hydrodynamics. The corresponding renormalization group (RG) [16] or self-consistent mode-coupling [17] analysis predicts that the thermal dc conductivity scales with the system size as $\sigma \sim L^{1/3}$ in 1D models with momentum conservation. This prediction was verified by means of numerical simulations in Ref. [18]. The $\sigma \sim L^{1/3}$ behavior of the heat conductivity is closely related to the anomalous broadening, $\Delta x \sim t^{1/z} \sim t^{2/3}$, (on top of ballistic propagation) of the sound peak in the density-density correlation function of the system. The dynamical exponent $z = 3/2$ entering here is a manifestation of the Kardar-Parisi-Zhang (KPZ) fixed point in the RG flow that governs the propagation of the sound mode [19]. The KPZ scaling of the density-density correlation function was also predicted (within the classical Gross-Pitaevskii formalism) to occur in 1D Bose gas at finite temperature [20]. In the context of 1D Fermi systems the KPZ scaling was advocated in Ref. [21].

The experimental progress combined with open fundamental questions prompts us to study thermal transport in a 1D quantum electronic fluid. In the low-energy limit such fluids are often described within the Tomonaga-Luttinger model that linearizes the spectrum of fermions near Fermi points and treats the interaction of fermions as pointlike. Via the bosonization procedure [22], the Tomonaga-Luttinger model maps to free bosons with a linear dispersion relation. The Luttinger liquid (LL) fixed point is thus a free theory with essentially trivial kinetics. It turns out however, that the corrections to it, a finite curvature of fermionic spectrum and a finite range of interactions, albeit irrelevant in RG sense, can have profound effect on the low-energy behavior of the system's *dynamical* correlation functions [23–28]. Therefore, those perturbations are to be taken into account in the discussion of the thermal conductivity; i.e., the conventional LL paradigm is not sufficient for this problem.

With the aforementioned corrections included, the Tomonaga-Luttinger model turns into an interacting theory both in fermionic and bosonic [29–33] languages. One then has to resort to perturbative treatment of the model. Two choices of basis for such a perturbation theory are available [34]: (i) one can choose to work with the bosonized version of the theory treating the nonlinearity of fermionic spectrum (that translates into interaction of bosons) perturbatively; (ii) the bosonic theory can be refermionized [26,34,35] giving rise to the description of the system in terms of fermionic quasiparticles that are related to original electrons via nonlinear unitary transformation. In the latter approach the curvature of the bosonic spectrum translates into the interaction of fermions [36].

It was shown in Ref. [34] that the applicability of perturbation theory for *thermal* (with energy of order T) excitations in the fermionic and bosonic approaches depends on temperature. Specifically, the effective mass of fermionic excitations m_* and a length l quantifying the curvature of bosonic spectrum [see Eq. (2)] define a temperature scale $T_{FB} = 1/m_* l^2$. At $T < T_{FB}$ the thermal fermions are long-living excitations; the perturbation theory in their interaction is well-behaved and controlled by the small parameter T/T_{FB}. At higher temperatures, $T > T_{FB}$, the proper thermal excitations are bosons and the bosonic perturbation theory possesses a small parameter T_{FB}/T.

In this work we employ the combination of the fermionic and bosonic frameworks to study low-temperature thermal conductivity of the electronic fluid. It turns out that *subthermal* excitations dominate the thermal transport at low frequencies and one is faced with the problem of understanding their kinetics. We show that at lowest frequencies the behavior of thermal conductivity is anomalous and has the universal scaling $\sigma(\omega) \propto \omega^{-1/3}$. This corresponds to length dependent dc thermal conductivity $\sigma(L) \propto L^{1/3}$ consistent with the classical hydrodynamic limit [16,17]. At higher frequencies, we identify a variety of new regimes characterized by the power-law dependence of the thermal conductance on frequency, temperature, and system size.

We consider a model of spinless right- and left-moving fermions

$$H = \sum_\eta \int dx \psi_\eta^\dagger(x)\left(-i\eta v_F \partial_x - \frac{1}{2m}\partial_x^2\right)\psi_\eta(x) + \frac{1}{2}\int dx dx' g(x-x')\rho(x)\rho(x'), \quad (1)$$

where $g(x)$ is the short-range interaction potential, and the total density $\rho(x)$ is a sum of the chiral components, $\rho(x) = \rho_R(x) + \rho_L(x)$. In the low momentum limit ($ql \ll 1$), the interaction potential is $g_q - g_0 \propto q^2 l^2$.

After bosonization, the original Hamiltonian [Eq. (1)] is mapped to an interacting bosonic model [22,34,37–40], see Supplemental Material (SM) [41], Sec. I. The interaction between electrons, Eq. (1), controls the dispersion of the bosonic modes. At small momenta

$$\omega_q^B = u_q^B |q|, \qquad u_q^B = u(1 - l^2 q^2), \quad (2)$$

where $u = v_F\sqrt{1 + g_0/\pi v_F}$ denotes the sound velocity.

We now construct the kinetic equation for bosonic and fermionic quasiparticles. These equations can be derived from the fermionic [Eq. (1)] and bosonized [Eq. (1.2) of SM] versions of the Hamiltonian in a standard way, see SM [41], Secs. 1, 2.1, and 3.2,

$$\frac{\partial N_\alpha(q)}{\partial t} + u_q^\alpha \frac{\partial N_\alpha(q)}{\partial x} = I_{\alpha,q}[N_\alpha]. \quad (3)$$

Here $\alpha = F/B$ specifies the type of the quasiparticles (Fermi or Bose), N_α is a distribution function, and I_α is the collision integral. A combination of two equations [Eq. (3)] permits us to extend the Bose-Fermi duality framework [34] away from thermal equilibrium. We next linearize the kinetic equation using the ansatz $N_\alpha = n_\alpha + \delta N_\alpha$, where n_α is the quasiequilibrium distribution and δN_α is a deviation from a local equilibrium, see SM [41], Sec. II.1 (for bosons) and Sec. III.2 (for fermions). To determine the heat conductivity the linearized kinetic equation should be solved for δN_α with the temperature gradient introduced into n_α. In either the fermionic or bosonic approach the energy current can then be computed as

$$J_\alpha(\omega) = \int (dq) u_q^\alpha \omega_q^\alpha \delta N_\alpha(q) \quad (4)$$

where for fermions

$$\omega_q^F = \pm uq + q^2/2m^*, \qquad u_q^F = \pm u + q/m^*. \quad (5)$$

In Eq. (5) the $\pm$ sign refers to the right and left movers; $(dq) \equiv (dq/2\pi\hbar)$ and we set $\hbar = 1$ through the Letter. The explicit formula for the effective mass m_* is given in the SM [41], Eq. (1.5).

Before discussing the relaxation of fermionic and bosonic modes in more detail, we note that the model Eq. (1) as stated preserves, apart from the charge conservation, also the *difference* of the number of right- and left-moving fermions. In the bosonic language this corresponds to the conservation of the total momentum of the bosonic excitations [42–45]. Correspondingly, the linearized collision integral (both in the fermionic and bosonic formalisms) has a zero mode that gives rise to a ballistic transport of heat [46]. In a more accurate description of the electronic fluid the chiral branches merge at the bottom of the energy band enabling the equilibration of the number of left and right fermions. Within the bosonic description this process corresponds to the umklapp scattering. The associated time scale is exponentially long [42–45]

$$\tau_U^{-1} \sim T^{3/2} \epsilon_F^{-1/2} e^{-\epsilon_F/T}. \quad (6)$$

Here $\epsilon_F \sim mv_F^2$ is an ultraviolet energy scale and we omitted a nonuniversal numerical coefficient that is determined by interaction strength and by details of the spectrum at the bottom of the band. The contribution of the (almost) zero mode associated to the conservation of the bosonic momentum can then be extracted either in a fermionic or bosonic framework [47,48],

$$\sigma^{\text{bal}}(\omega) = \frac{\pi}{3}\frac{uT}{i\omega + \tau_U^{-1}}. \tag{7}$$

At $\omega\tau_U \gg 1$, $\sigma^{\text{bal}}(\omega)$ is purely imaginary and does not contribute to the dissipative real part of the total thermal conductance. In the opposite limit, $\omega\tau_U \ll 1$ the contribution of $\sigma^{\text{bal}}(\omega)$ becomes a (large) frequency-independent constant, $\sigma^{\text{bal}} = \pi\tau_U uT/3$.

Let us now turn to the analysis of the relaxing modes in the kinetic Eq. (3). Employing the relaxation-time approximation for its solution we find

$$\text{Re}\sigma^B(\omega) \simeq \frac{T^4 l^4}{u^2}\text{Re}\int_0^{T/u}\frac{(dq)}{\tau_B^{-1}(q) - i\omega}, \tag{8}$$

for the bosonic and

$$\text{Re}\sigma^F(\omega) \simeq \frac{T^2}{m_*^2 u^2}\text{Re}\int_0^{T/u}\frac{(dq)}{\tau_F^{-1}(q) - i\omega}, \tag{9}$$

for the fermionic representation of the theory, respectively, see SM [41], Sec. 2.2, 3.2, and 3.3 for details. In Eqs. (8) and (9), $\tau_B(q)$ and $\tau_F(q)$ denote the relaxation times for the bosons and fermions. Note, that the prefactors in Eqs. (8) and (9) match at $T = T_{FB}$, which is a manifestation of Bose-Fermi duality [34]. One thus might think that for $T > T_{FB}$ only bosonic excitations are relevant, and for $T < T_{FB}$ only fermionic ones. However, this is not true. As we discuss below, the bosonic lifetime $\tau_B(q)$ diverges in low q limit, while $\tau_F(q)$ remains constant. This makes the two channels of heat transport profoundly different: the bosonic one *always* dominates the low-frequency thermal conductivity, irrespectively of the relation between T and T_{FB}.

Equations (8) and (9) represent contributions of bosonic and fermionic quasiparticles to the thermal conductivity. Thus, taking into account also the ballistic contribution σ^{bal} discussed above, we approximate the total thermal conductivity of the electronic fluid by

$$\sigma(\omega) = \sigma^{\text{bal}}(\omega) + \sigma'(\omega), \tag{10}$$

$$\sigma'(\omega) = \sigma^F(\omega) + \sigma^B(\omega) \simeq \max[\sigma^F(\omega), \sigma^B(\omega)]. \tag{11}$$

To evaluate Eqs. (8) and (9), one needs to compute the decay rates $\tau_\alpha^{-1}(q)$ for the fermionic and bosonic sectors and $q \lesssim T/u$. Let us discuss the bosonic excitations first. The simplest process of the bosonic decay obeying energy and momentum conservation is shown in the left panel of Fig. 1. It corresponds to the decay of one boson mode into three and involves three bosons of the same chirality (e.g., right) and one boson of the opposite chirality [34,49]. The resulting decay rate of subthermal bosons is given by (see SM [41], Sec. 2.1; we omit numerical factors)

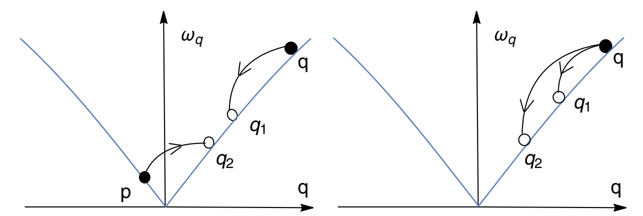

FIG. 1. Left: Two-into-two bosonic scattering process involving three bosons of the same chirality and one boson of opposite chirality. Right: The self-consistent decay process of one boson into two, yielding the decay rate $\tau^{-1}(q) \propto q^{3/2}$ for $q < q_{\text{thr}}$.

$$\frac{1}{\tau_B(q)} \sim \begin{cases} \frac{\gamma q^{5/3}T^2}{u^5 l^{\frac{4}{3}} m_*^4}, & q < \frac{l^2T^3}{u^3}, \\ \frac{\gamma q^2 T^3}{m_*^4 u^6}, & \frac{l^2T^3}{u^3} < q < \frac{T}{u}. \end{cases} \tag{12}$$

Here $\gamma = \alpha^2(1+\alpha)^2$ is a dimensionless interaction parameter related to the LL parameter K_0 by the relation $\alpha = (1 - K_0^2/3 + K_0^2)$. The second line of Eq. (12) agrees with Ref. [49]; the $q^{5/3}$ scaling as in the first line was obtained in the context of classical anharmonic chains [50,51]. Note that the process shown in the left panel of Fig. 1 can be interpreted as either a contribution to the relaxation of one of the "majority" bosons (q) or of the "minority" boson (p). Kinematic constraints imply $p \ll q$. As a result, the second contribution is exponentially suppressed at large momenta of the relaxing boson. It however dominates at small momenta and gives rise to the first line in Eq. (12).

The process on the left panel of Fig. 1 can be viewed as a decay of a right boson q into two other right bosons assisted by a left mover p. The participation of the later is required by kinematic constraints on the Fermi golden-rule level. However, once the bosonic spectrum is broadened by some relaxation processes, direct decay of a bosonic excitation into two bosons of the same chirality becomes possible. In particular, at $q < q_{\text{thr}}$, where q_{thr} is a threshold momentum,

$$q_{\text{thr}} = \frac{T^{1/3}}{um_*^{2/3}l^{4/3}} \equiv \frac{T}{u}\left(\frac{T_{FB}}{T}\right)^{2/3}, \tag{13}$$

the *self-consistent* treatment of the process shown in the right panel of Fig. 1 provides the dominant contribution to the relaxation of bosons [21,52,53] (see SM [41]),

$$\tau_B^{-1}(q) \sim q^{3/2} m_*^{-1} \sqrt{T/u}, \qquad q < q_{\mathrm{thr}}. \tag{14}$$

Summarizing the above analysis, we get

$$\frac{1}{\tau_B(q)} \sim \begin{cases} q^{3/2} \frac{\sqrt{T/u}}{m_*}, & q < q_{\mathrm{thr}}, \\ \frac{\gamma q^{\frac{5}{3}} T^2}{u^5 l^{\frac{4}{3}} m_*^4}, & q_{\mathrm{thr}} < q < \frac{l^2 T^3}{u^3}, \\ \frac{\gamma q^2 T^3}{m_*^4 u^6}, & \max\{\frac{l^2 T^3}{u^3}, q_{\mathrm{thr}}\} < q < \frac{T}{u}. \end{cases} \tag{15}$$

The last two regimes in Eq. (15) can be absent if the corresponding momentum interval vanishes. Specifically, for $T < T_{FB}$ the bosonic relaxation rate is always given by the first line in Eq. (15). The intermediate regime, $q_{\mathrm{thr}} < q < (l^2 T^3/u^3)$, disappears at $T < T_H = u^{3/4} l^{-(5/4)} m_*^{-(1/4)}$.

As for the fermionic excitations, their lifetime was discussed in Refs. [26,27,34,54]. It is given by

$$\frac{1}{\tau_F(q)} \sim \frac{\gamma l^4 T^7}{m_*^2 u^8}, \qquad q < \frac{T}{u}. \tag{16}$$

Note that while the bosonic decay rate vanishes at $q \to 0$ limit, the fermionic rate remains finite. This implies that the low-frequency behavior of the thermal conductivity is dominated by bosons.

We now calculate the real part of thermal conductivity as a function of ω and T, using decay rates Eqs. (15) and (16). For $T < T_{FB}$, we find

$$\sigma'(\omega) \sim \begin{cases} \frac{T^{11/3} l^4 u^{-(5/3)} m_*^{2/3}}{\omega^{\frac{1}{3}}}, & \omega < \frac{\gamma^3 T^{23} m_*^2 l^{24}}{u^{20}}, \\ \frac{u^5}{\gamma T^4 l^4}, & \frac{\gamma^3 T^{23} m_*^2 l^{24}}{u^{20}} < \omega < \frac{\gamma l^4 T^7}{m_*^2 u^8}, \\ \frac{1}{\omega^2} \frac{\gamma T^{10} l^4}{u^{11} m_*^4}, & \frac{\gamma l^4 T^7}{m_*^2 u^8} < \omega. \end{cases} \tag{17}$$

For details of the calculations and results for $T > T_{FB}$ see the SM [41], Sec. IV. To obtain the overall picture, the "ballistic" contribution [Eq. (7)] should be taken into account. At sufficiently high frequencies, $\omega \gg 1/\tau_U$, the ballistic mode associated with the conservation of the momentum of bosonic excitations does not contribute to the real part of $\sigma(\omega)$ and $\sigma(\omega) \approx \sigma'(\omega)$. At $\tau_U \omega \lesssim 1$ the ballistic channel of the energy propagation becomes gapped and contributes an exponentially large but frequency-independent constant $\sigma^{\mathrm{bal}} \simeq \pi u T \tau_U/3$ to the thermal conductance.

The resulting behavior of $\sigma(\omega)$ is shown in Fig. 2 for $T < T_{FB}$ and in Figs. 1 and 2 of the SM [41] for $T > T_{FB}$. At $\omega < 1/\tau_U$, we observe a universal $\omega^{-1/3}$ scaling of $\sigma(\omega)$. This behavior can be traced back to the contribution of bosons with momentum $q \ll T/u$ that have the relaxation rate specified in the first line of Eq. (15). It is consistent with the predictions of the fluctuating

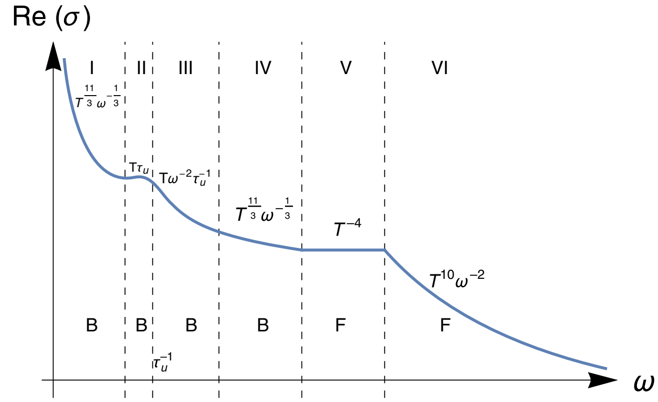

FIG. 2. Re$\sigma(\omega)$ at $T < T_{FB}$. For each regime, the T and ω scaling is shown. Labels F and B indicate whether the dominant contribution comes from the fermionic or bosonic sector. In regions II and III the contribution of the momentum zero mode [Eq. (7)] is dominant, while other regions are dominated by finite energy modes, Eq. (17). For lowest ω, region I, the $\omega^{-1/3}$ dependence translates into the $L^{1/3}$ scaling of $\sigma(L)$, analogous to that obtained for classical fluids [16,17].

hydrodynamics. This is to be expected as strongly subthermal bosonic modes correspond to classical density waves of the hydrodynamic theory.

We now discuss the scaling of the dc conductivity with the system size L. In contrast to the frequency scaling, the contribution of the zero mode associated to the conservation of the bosonic momentum is always real. In fact at scales shorter than L_U the entire dc thermal conductance is dominated by this *ballistic* contribution, $\sigma(L) = \pi T L/6$, with the other modes providing only subleading corrections. The situation changes in the limit $L \gg L_U$ where the contribution of the zero mode becomes a size-independent constant, and nonzero modes start to be dominant.

As was found above, the thermal conductivity $\sigma(\omega \to 0)$ is governed by bosons. In this limit, the bosonic lifetime diverges as power law $\tau^{-1}(q) \propto q^z$. In our case $z = 3/2$, that is consistent with KPZ scaling [21]. The divergence of the life time implies that bosons with momentum below $q < L^{-1/z}$ propagate through the system almost ballistically. The contribution of these bosons to the thermal conductance can be estimated as

$$G(L) \sim L^{-1/z}. \tag{18}$$

This implies that for $z = 3/2$ the conductivity scales as $\sigma(L) = L^{1/3}$. Note that this is a manifestation of the Lévy-flight character of the energy transport in the system, cf. Ref. [17]. To find the corresponding Lévy-flight distribution function, one needs to relate z to the Lévy-flight parameter α. This can be done by comparing the diffusion coefficient D_E of the Lévy-flight process with the thermal conductivity computed above. The thermal conductance G is related to the energy diffusion coefficient as $G = \sigma/L \sim D_E/L$. To estimate D_E we compute

$$D_E = \frac{\langle x^2 \rangle}{t} \sim \int_0^{L/u} dx x^2 x^{-\alpha-1} \sim L^{2-\alpha}, \tag{19}$$

where we used that the tails of the Lévy distribution function at time t decay with distance as $x^{-\alpha-1}t$. Comparing Eqs. (18) and (19), we obtain the following relation between the exponent z controlling decay rate and α of Lévy flight, $1 - \alpha = -z^{-1}$, so that the conductivity scales as $\sigma \sim L^{2-\alpha} = L^{1-z^{-1}}$. For $z = \frac{3}{2}$, the Lévy parameter $\alpha = 5/3$ and $\sigma(L)$ scales as $L^{1/3}$. Note that the scaling in L for the Lévy-flight regime can be obtained from the scaling in ω by the replacement $\omega \to u/L$. The $L^{1/3}$ scaling agrees with the one found in classical fluids [16,17].

To summarize, we computed the thermal conductivity of 1D electronic fluids as a function of frequency ω, temperature T, and system length L. For energy scales below bosonic umklapp time, the momentum of bosonic and fermionic fluids are separately conserved. The momentum zero modes give rise to the ballistic heat conductivity Eq. (7). This corresponds to purely imaginary $\sigma(\omega)$ and results in the LL thermal conductance $\pi^2 T/3h$ [47] for a finite sample. The massive modes of the fermionic and bosonic collision integrals contribute to the real part of the heat conductivity, yielding subleading (in $1/L$) corrections to the thermal conductance. However, they may be detected via measuring the real part of $\sigma(\omega)$ at $\omega > u/L$. The real part of $\sigma(\omega)$ exhibits several regimes. For temperatures $T < T_{FB}$ it is determined by fermionic modes for not too low frequencies, see regions V and VI in Fig. 2. At the lowest frequencies, the conductivity is determined by low-momentum bosonic modes, yielding $\sigma(\omega) \propto \omega^{-1/3}$. The length dependence of the thermal conductance depends on the relation between L and the bosonic umklapp length L_U. For $L \ll L_U$, the transport is ballistic, $\sigma(L) \propto L$, as expected for LL. On the other hand, for $L \gg L_U$, we find $\sigma(L) \propto L^{1/3}$, as expected for a classical fluid [16,17].

We close by briefly discussing prospective research directions. First, while our computations were done within kinetic theory, these results can be found also within the hydrodynamic approach. While for $L > L_U$ the hydrodynamic theory has three modes (particle-number, momentum, and energy conservation), for $L < L_U$ the number of modes is four, due to the additional zero mode discussed above. In both regimes, kinetic coefficients are expected to be anomalous. The hydrodynamic framework is particularly convenient for computing scaling functions describing heat conductivity and pulse evolution [55]. Second, whereas our computations were done for a strictly 1D system, we expect that an anomalous scaling of heat conductance should hold also for other low-dimensional quantum electronic fluids (quasi-1D and 2D geometries).

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Note added.—Recently, a related preprint [56] appeared, which addresses the same problem solely within the fermionic approach. Results of Ref. [56] for $\sigma(\omega)$ match ours in what concerns the "plateaus" II and V in Fig. 2 but do not capture other regions, where $\sigma(\omega)$ is controlled by a slow relaxation of bosonic modes.

[1] K. Schwab, E. Henriksen, J. Worlock, and M. Roukes, Measurement of the quantum of thermal conductance, Nature (London) **404**, 974 (2000).
[2] M. Meschke, W. Guichard, and J. P. Pekola, Singlemode heat conduction by photons, Nature (London) **444**, 187 (2006).
[3] S. Jezouin, F. D. Parmentier, A. Anthore, U. Gennser, A. Cavanna, Y. Jin, and F. Pierre, Quantum limit of heat flow across a single electronic channel, Science **342**, 601 (2013).
[4] L. Cui, W. Jeong, S. Hur, M. Matt, J. C. Klöckner, F. Pauly, P. Nielaba, J. C. Cuevas, E. Meyhofer, and P. Reddy, Quantized thermal transport in single-atom junctions, Science **355**, 1192 (2017).
[5] E. Sivre, A. Anthore, F. D. Parmentier, A. Cavanna, U. Gennser, A. Ouerghi, Y. Jin, and F. Pierre, Heat Coulomb blockade of one ballistic channel, Nature (London) **14**, 145 (2018).
[6] C. Altimiras, H. le Sueur, U. Gennser, A. Anthore, A. Cavanna, D. Mailly, and F. Pierre, Energy Relaxation in the Integer Quantum Hall Regime, Phys. Rev. Lett. **109**, 026803 (2012).
[7] V. Venkatachalam, S. Hart, L. Pfeiffer, K. West, and A. Yacoby, Local thermometry of neutral modes on the quantum Hall edge, Nat. Phys. **8**, 676 (2012).
[8] H. Inoue, A. Grivnin, Y. Ronen, M. Heiblum, V. Umansky, and D. Mahalu, Proliferation of neutral modes in fractional quantum Hall states, Nat. Commun. **5**, 4067 (2014).
[9] M. Banerjee, M. Heiblum, A. Rosenblatt, Y. Oreg, D. Feldman, A. Stern, and V.Umansky, Observed quantization of anyonic heat flow, Nature (London) **545**, 75 (2017).
[10] A. Rosenblatt, F. Lafont, I. Levkivskyi, R. Sabo, I. Gurman, D. Banitt, M. Heiblum, and V. Umansky, Transmission of heat modes across a potential barrier, Nature (London) **8**, 2251 (2017).
[11] M. Banerjee, M. Heiblum, V. Umansky, D. Feldman, Y. Oreg, and A. Stern, Observation of half-integer thermal Hall conductance, Nature (London) **559**, 205 (2018).
[12] Y. K. Koh and D. G. Cahill, Frequency dependence of the thermal conductivity of semiconductor alloys, Phys. Rev. B **76**, 075207 (2007).
[13] A. J. Minnich, J. A. Johnson, A. J. Schmidt, K. Esfarjani, M. S. Dresselhaus, K. A. Nelson, and G. Chen, Thermal Conductivity Spectroscopy Technique to Measure Phonon Mean Free Paths, Phys. Rev. Lett. **107**, 095901 (2011).

[14] K. T. Regner, D. P. Sellan, Z. Su, C. H. Amon, A. J. H. McGaughey, and J. A. Malen, Broadband phonon mean free path contributions to thermal conductivity measured using frequency domain thermoreflectance, Nat. Commun. **4**, 1640 (2013).

[15] E. Fermi, J. Pasta, S. Ulam, and M. Tsingou, Studies of nonlinear problems, FPU, Document No. LA-1940, Los Alamos National Laboratory, 1955.

[16] O. Narayan and S. Ramaswamy, Anomalous Heat Conduction in One-Dimensional Momentum-Conserving Systems, Phys. Rev. Lett. **89**, 200601 (2002).

[17] H. Spohn, Nonlinear fluctuating hydrodynamics for anharmonic chains, J. Stat. Phys. **154**, 1191 (2014).

[18] T. Mai, A. Dhar, and O. Narayan, Equilibration and Universal Heat Conduction in Fermi-Pasta-Ulam Chains, Phys. Rev. Lett. **98**, 184301 (2007).

[19] H. van Beijeren, Exact Results for Anomalous Transport in One-Dimensional Hamiltonian Systems, Phys. Rev. Lett. **108**, 180601 (2012).

[20] M. Kulkarni and A. Lamacraft, From GPE to KPZ: Finite temperature dynamical structure factor of the 1D Bose gas, Phys. Rev. A **88**, 021603(R) (2013).

[21] M. Arzamasovs, F. Bovo, and D. M. Gangardt, Kinetics of Mobile Impurities and Correlation Functions in One-Dimensional Superfluids at Finite Temperature, Phys. Rev. Lett. **112**, 170602 (2014).

[22] T. Giamarchi, *Quantum Physics in One Dimension* (Clarendon Press, Oxford, 2004).

[23] I. V. Protopopov, D. B. Gutman, M. Oldenburg, and A. D. Mirlin, Dissipationless kinetics of one-dimensional interacting fermions, Phys. Rev. B **89**, 161104(R) (2014).

[24] I. V. Protopopov, D. B. Gutman, P. Schmitteckert, and A. D. Mirlin, Dynamics of waves in one-dimensional electron systems: Density oscillations driven by population inversion, Phys. Rev. B **87**, 045112 (2013).

[25] A. Imambekov and L. I. Glazman, Universal theory of nonlinear luttinger liquids, Science **323**, 228 (2009); Phenomenology of One-Dimensional Quantum Liquids Beyond the Low-Energy Limit, Phys. Rev. Lett. **102**, 126405 (2009).

[26] A. Imambekov, T. L. Schmidt, and L. I. Glazman, One-dimensional quantum liquids: Beyond the Luttinger liquid paradigm, Rev. Mod. Phys. **84**, 1253 (2012).

[27] M. Khodas, M. Pustilnik, A. Kamenev, and L. I. Glazman, Fermi-Luttinger liquid: Spectral function of interacting one-dimensional fermions, Phys. Rev. B **76**, 155402 (2007).

[28] M. Filippone, F. Hekking, and A. Minguzzi, Violation of the Wiedemann-Franz law for one-dimensional ultracold atomic gases, Phys. Rev. A **93**, 011602(R) (2016).

[29] L. D. Landau, J. Phys. USSR **5**, 71 (1941); Theory of superfluidity of Helium-II, Sov. Phys. ZhETF **11**, 592 (1941).

[30] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics, Part 2* (Elsevier, Oxford, 1980).

[31] M. Schick, Flux Quantization in a One-Dimensional Model, Phys. Rev. **166**, 404 (1968).

[32] B. Sakita, *Quantum Theory of Many-variable Systems and Fields* (World Scientific, Singapore, 1985).

[33] A. Jevicki and B. Sakita, The quantum collective field method and its application to the planar limit, Nucl. Phys. **B165**, 511 (1980).

[34] I. V. Protopopov, D. B. Gutman, and A. D. Mirlin, Relaxation in Luttinger liquids: Bose-Fermi duality, Phys. Rev. B **90**, 125113 (2014).

[35] A. V. Rozhkov, Density-density propagator for one-dimensional interacting spinless fermions with nonlinear dispersion and calculation of the Coulomb drag resistivity, Phys. Rev. B **77**, 125109 (2008); Class of exactly soluble models of one-dimensional spinless fermions and its application to the Tomonaga-Luttinger Hamiltonian with nonlinear dispersion, Phys. Rev. B **74**, 245123 (2006); Fermionic quasiparticle representation of Tomonaga-Luttinger Hamiltonian, Eur. Phys. J. Spec. Top. **47**, 193 (2005).

[36] The fermionic quasiparticles introduced in this way are advantageous over the original electrons of the model because their interaction vanishes in the low-energy limit. In particular, they have flat density of states at the Fermi surface.

[37] F. D. M. Haldane, Luttinger liquid theory of one-dimensional quantum fluids, J. Phys. C **14**, 2585 (1981).

[38] M. Stone, *Bosonization* (World Scientific, Singapore, 1994).

[39] J. von Delft and H. Schoeller, Bosonization for beginners-refermionization for experts, Ann. Phys. (Berlin) **7**, 225 (1998).

[40] A. O. Gogolin, A. A. Nersesyan, and A. M. Tsvelik, *Bosonization in Strongly Correlated Systems* (Cambridge University Press, Cambridge, England, 1998).

[41] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.122.206801 for technical details of bosonization, kinetic theory, and calculation of thermal conductivity.

[42] K. A. Matveev and A. V. Andreev, Equilibration of a spinless Luttinger liquid, Phys. Rev. B **85**, 041102(R) (2012).

[43] T. Micklitz, J. Rech, and K. A. Matveev, Transport properties of partially equilibrated quantum wires, Phys. Rev. B **81**, 115313 (2010).

[44] K. A. Matveev and A. V. Andreev, Scattering of hole excitations in a one-dimensional spinless quantum liquid, Phys. Rev. B **86**, 045136 (2012).

[45] K. A. Matveev, A. V. Andreev, and A. D. Klironomos, Scattering of charge and spin excitations and equilibration of a one-dimensional Wigner crystal, Phys. Rev. B **90**, 035148 (2014).

[46] K. A. Matveev and A. V. Andreev, Hybrid Sound Modes in One-Dimensional Quantum Liquids, Phys. Rev. Lett. **121**, 026803 (2018).

[47] C. L. Kane and M. P. A. Fisher, Thermal Transport in a Luttinger Liquid, Phys. Rev. Lett. **76**, 3192 (1996).

[48] A. Levchenko, T. Micklitz, J. Rech, and K. A. Matveev, Transport in partially equilibrated inhomogeneous quantum wires, Phys. Rev. B **82**, 115413 (2010).

[49] J. Lin, K. A. Matveev, and M. Pustilnik, Thermalization of Acoustic Excitations in a Strongly Interacting One-Dimensional Quantum Liquid, Phys. Rev. Lett. **110**, 016401 (2013).

[50] A. Pereverzev, Fermi-Pasta-Ulam β lattice: Peierls equation and anomalous heat conductivity, Phys. Rev. E **68**, 056124 (2003).

[51] H. Spohn and J. Lukkarinen, Anomalous energy transport in the FPU-β chain, Commun. Pure Appl. Math. **61**, 1753 (2008).
[52] A. F. Andreev, The hydrodynamics of two and one dimensional liquids, Sov. Phys. JETP **51**, 1038 (1980).
[53] K. Samokhin, Lifetime of excitations in a clean Luttinger liquid, J. Phys. Condens. Matter **10**, L533 (1998).
[54] A. M. Lunde, K. Flensberg, and L. I. Glazman, Three-particle collisions in quantum wires: Corrections to thermopower and conductance, Phys. Rev. B **75**, 245418 (2007).
[55] S. Rickmoy, I. V. Protopopov, D. B. Gutman, and A. D. Mirlin, Pulse propagation in electronic fluid (to be published).
[56] K. A. Matveev and Z. Ristivojevic, Thermal conductivity of the degenerate one-dimensional Fermi gas, arXiv: 1901.08136.