Quantifying Operations with an Application to Coherence

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To describe certain facets of nonclassicality, it is necessary to quantify properties of operations instead of states. This is the case if one wants to quantify how well an operation detects nonclassicality, which is a necessary prerequisite for its use in quantum technologies. To do so rigorously, we build resource theories on the level of operations, exploiting the concept of resource destroying maps. We discuss the two basic ingredients of these resource theories, the free operations and the free superoperations, which are sequential and parallel concatenations with free operations. This leads to defining properties of functionals that are well suited to quantify the resources of operations. We introduce these concepts at the example of coherence. In particular, we present two measures quantifying the ability of an operation to detect, i.e., to use, coherence, one of them with an operational interpretation, and provide methods to evaluate them.

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Introduction.—In recent years, there has been an increasing interest in quantum technologies. To investigate rigorously which properties of quantum mechanics are responsible for potential operational advantages, quantum resource theories were developed, see e.g., Refs. [1–10]. These resource theories originate from constraints that are imposed in addition to the laws of quantum mechanics, motivated either by physical or by practical considerations. From the constraints follow the free states and the free operations, which are the ones that can be prepared and executed without violation of the constraints. These two main ingredients allow for the formulation of a rigorous theoretical framework in which to analyze quantitatively the amount of the resource present in quantum *states* and its usefulness in operational tasks [11–13]. In addition, there exist quantum operations that can be considered resources as well, because they are not free. Therefore, a complementary question to ask is how valuable these operations are [14]. This question is often approached by the evaluation of quantities such as the resource generation capacity, i.e., the maximal increase of the resource in an input state under application of the operation, or the resource cost, i.e., the minimal amount of resources needed to simulate a nonfree operation by means of free operations [15–22]. As we will discuss later and in the Supplemental Material [23], these methods cannot be used to quantify all relevant properties of quantum operations. Hence the situation merits a broader approach and this is why we are examining a broader framework. More concretely, we will build formal resource theories on the level of operations, allowing us to quantify the value of operations directly. This is also interesting from a conceptual point of view: the goal of quantum technologies is to perform *tasks* that are impossible using classical technologies. This includes sensing at high precision [35], efficient processing of information, and securing the transmission of data [36]. Ultimately, this is all achieved by quantum operations, i.e., dynamical resources. Hence it seems natural to quantify the value of operations directly without the detour through states as the latter are static resources that have to be transformed into dynamic resources using free operations. Since quantum states can be seen as quantum operations with no input and a constant output (describing a quantum mechanical preparation apparatus), a resource theory on the level of operations can quantify the value of states, too, leading to a unified resource theoretic treatment of states and operations. Therefore we expect that resource theories on the level of operations will be a key method to the systematic exploration of quantum advantages. In this Letter, we will exemplify the concepts and advantages of resource theories of operations at the example of coherence.

A fundamental ingredient to the departure of quantum mechanics from classical physics is the omnipresence of the superposition principle [37,38]. This has led to the development of rigorous resource theories of coherence [3,9,13,39], which allow us to investigate the role of coherence in quantum technological applications [40–42]. These theories are formulated on the level of states and mainly focused on the inability to create coherence. However, this is only half of the picture: to exploit coherences or more generally quantum superpositions [38,43] in technologies, it is both necessary to have access to operations that can create coherence and operations that can detect it in the sense that its presence makes a difference in the measurement statistics [44,45]. If we cannot detect or equivalently use coherence or, more generally, nonclassicality, there cannot be an operational advantage in its presence. This is also reflected in ongoing efforts to describe detectors for nonclassicality [46–50]. As discussed in Refs. [44,51] and the Supplemental Material [23], this is particularly clear in interferometric experiments. Therefore an answer to the question "How well can a quantum operation detect coherence?" is needed to understand quantum advantages. To the best of our knowledge, and as we will discuss now, this question cannot be addressed using a resource theory on the level of states.

Although there exist mathematical frameworks for coherence theories on the level of states in which the free operations cannot make use of coherence [39,44,52–56], this is problematic from a conceptual point of view: ideally, the presence of resources in states should be detectable by free operations, because this is a necessary prerequisite that such states can allow for operational advantages over free operations alone. If this is not possible, then it is misleading to consider a state to be resourceful (see also Ref. [57]), as is the situation in the theories cited above. This also implies that it is not possible to address the coherence detection capabilities of operations in these frameworks via the resource cost of states. In frameworks where coherence is useful, its detection is, as mentioned above, necessarily free, leading to a zero resource cost, which therefore cannot be used to address the coherence detection capabilities of operations either. On the other hand, as we are interested in the question of how well an operation can detect coherence, its coherence generation capacity cannot be the figure of merit, and therefore we cannot address the coherence detection capabilities of an operation based on a resource theory on the level of states. We refer to the Supplemental Material [23] for more details and proofs of these observations, including a discussion why we cannot use the coherence of the corresponding Choi state [58] to quantify the coherence of operations and why this should not be expected.

In contrast, in this Letter, we will show that the coherence detection capability of operations can be quantified rigorously and that the conceptual problem discussed above vanishes if we use a resource theory on the level of operations. We will first introduce the two basic ingredients to such a theory: the free operations and the free superoperations, which map operations to operations and consist naturally of sequential and parallel concatenations with free operations [59–61]. From these ingredients we deduce defining properties of functionals which are well suited to quantify the value of operations. Then we present two such functionals quantifying how well an operation can detect coherence: one based on the diamond norm that can be calculated efficiently, and another one based on the induced trace norm, which has a clear operational interpretation. We give examples for the value of operations according to these measures and conclude with an outlook on open questions.

The framework we introduce can be extended easily to operations that cannot create coherence and operations that can neither detect nor create it. We comment on results in this direction in the Supplemental Material. In a forthcoming work, our theoretical results will be used in the analysis of an experiment based on a photodetector with a tunable degree of coherence detection capability [62]. All proofs can be found in the Supplemental Material [23].

Basic framework.—Since coherence is a basis dependent concept, we fix for all systems A an orthonormal basis $|i^A\rangle$ which we call incoherent. This basis is singled out by the physics of an actual system or the computational basis in a quantum algorithm. From now on, coherences and populations will be seen with respect to the incoherent basis. The incoherent basis of a system composed of two subsystems A and B is given by the product basis of their incoherent bases. If it is clear from the context, we will omit the labels indicating the systems from here on. All states ρ that are a statistical mixture of the incoherent basis states, i.e.,

$$\rho = \sum_i p_i |i\rangle\langle i|, \tag{1}$$

are called incoherent. In the following, we make frequent use of the total dephasing operation Δ

$$\Delta(\rho) = \sum_i |i\rangle\langle i|\rho|i\rangle\langle i|, \tag{2}$$

which is a resource destroying map [63] in coherence theory; i.e., its output is always incoherent. The total dephasing operation on a composed system is the tensor product of the total dephasing operations on the subsystems. If we concatenate operations, we will always implicitly assume that they match; i.e., the output dimension of the first operation equals the input dimension of the second operation. In addition, we will not write the concatenation operator $\circ$ if not necessary.

To construct a resource theory that allows us to answer the question how well a quantum operation can detect coherences, we need to define the free operations and superoperations. Let us begin with the free operations. First we notice that a positive operator-valued measure (POVM) cannot detect coherences if the measurement statistics are independent of them. This leads us to the following definition:

Definition 1.—A POVM given by $\{P_n\}: P_n \geq 0$, $\sum_n P_n = \mathbb{1}$ is free if and only if

$$\mathbf{tr} P_n \Delta\rho = \mathbf{tr} P_n \rho \quad \forall\, \rho, n. \tag{3}$$

As one expects, all free POVMs are of the following form:

Proposition 2.—A POVM is free if and only if

$$P_n = \sum_i P_i^n |i\rangle\langle i| \quad \forall\, n. \tag{4}$$

Next we define general free operations, where we need to address subselection (by measurement results) in a consistent manner. Since the ability to do subselection depends on the actual experimental implementation, we adopt the point of view that this is a resource in itself. In general, we can have a quantum instrument $\mathcal{I}$, which allows us to do subselection according to a variable x; i.e., we obtain with probability $p_x = \mathbf{tr}(\mathcal{E}_x(\rho))$ an output $\rho_x = \mathcal{E}_x(\rho)/p_x$. From the definition of the free POVMs, it follows that we can store the outcome x in the incoherent basis of an ancillary system, which we write as

$$\tilde{\mathcal{I}}(\rho) = \sum_x \mathcal{E}_x(\rho) \otimes |x\rangle\langle x|, \tag{5}$$

and implement the subselection later using a free POVM. In the special case of a POVM $\mathcal{P}$, we can represent it by

$$\tilde{\mathcal{P}}(\rho) = \sum_n \mathbf{tr}(P_n\rho)|n\rangle\langle n|. \tag{6}$$

Treating subselection in this way, we can reduce our analysis to trace preserving operations.

With subselection included into our framework, we call a quantum operation free if it cannot turn a free POVM into a nonfree one by applying the operation prior to the measurement. This is exactly the case if it cannot transform coherences into populations [52].

Definition 3.—A quantum operation $\Phi_{\text{d-inc}}$ is called detection incoherent if and only if

$$\Delta\Phi_{\text{d-inc}} = \Delta\Phi_{\text{d-inc}}\Delta. \tag{7}$$

The set of detection-incoherent operations is denoted by $\mathcal{DI}$.

Note that this condition has been called nonactivating in Ref. [63]. With our convention for treating subselection, this includes Definition 1 for POVMs. As we mentioned in the introduction, it is both important to create and to detect coherence; therefore one can define creation-incoherent operations, i.e., operations which cannot create coherence. In coherence theory, these operations are called MIO (for maximally incoherent operations) [3,64] or nongenerating in a general context in Ref. [63]. Operations that can neither create nor detect coherence are called DIO (dephasing-covariant incoherent operations) [53–56], classical operations [52], or commuting [63].

Definition 4.—A quantum operation $\Phi_{\text{c-inc}}$ from system A to B is called creation incoherent if it cannot create coherence in system B when none were present in system A,

$$\Phi_{\text{c-inc}}\Delta = \Delta\Phi_{\text{c-inc}}\Delta. \tag{8}$$

A quantum operation $\Phi_{\text{dc-inc}}$ is called detection creation incoherent if it can neither detect nor create coherence,

$$\Delta\Phi_{\text{dc-inc}} = \Phi_{\text{dc-inc}}\Delta. \tag{9}$$

Our contribution in this Letter is that we show how to quantify the abilities to create and detect coherence in a rigorous manner. Note that, formally, the three definitions of free operations lead to different resource theories. In the following, we will use "free operation" if it is unimportant which specific choice we are considering. This allows us to introduce the second ingredient to our resource theories, the free superoperations, in a unified manner. A superoperation is free if it is a sequential and/or parallel concatenation with free operations.

Definition 5.—For free operations Φ, elemental free superoperations are given by

$$\begin{aligned} \mathcal{E}_{1,\Phi}[\Theta] &= \Phi \circ \Theta, & \mathcal{E}_{2,\Phi}[\Theta] &= \Theta \circ \Phi, \\ \mathcal{E}_{3,\Phi}[\Theta] &= \Theta \otimes \Phi, & \mathcal{E}_{4,\Phi}[\Theta] &= \Phi \otimes \Theta. \end{aligned} \tag{10}$$

A superoperation $\mathcal{F}$ is free if and only if it can be written as a sequence of free elemental superoperations,

$$\mathcal{F} = \mathcal{E}_{i_n,\Phi_n}, \ldots, \mathcal{E}_{i_3,\Phi_3}\mathcal{E}_{i_2,\Phi_2}\mathcal{E}_{i_1,\Phi_1}. \tag{11}$$

This definition comes from a quantum computational setting: a free superoperation is a network of free operations into which we can plug a quantum operation. A minimal requirement on the free superoperations is that they transform free operations into free operations, otherwise it would be possible to create resources for free. This requirement can be checked directly, see the Supplemental Material [23]. It is also straightforward to show that every free superoperation can be composed using only three elemental operations (see the Supplemental Material [23] and Refs. [59,60]). Whilst we focus on the ability to detect coherence in the main text, we present a few results for the other two classes of free operations in the Supplemental Material (see also Refs. [52,64]). As mentioned in the introduction, the case of coherence treated here is an example of our general setup: if one exchanges the resource destroying map in Eq. (2), one can move on to Definitions 3, 4, and 5. It is also possible to define free operations without the usage of resource destroying maps and to use Definition 5 for free superoperations [60].

Detecting coherence.—To quantify the amount of a resource present in an operation, we follow the usual axiomatic approach of quantum resource theories [1–10]. From physical considerations, we collect a set of defining properties that every measure of the resource should obey. The first property is that the measure should be faithful, which means that it needs to be zero on the set of free operations and larger than zero on nonfree operations. The second property is monotonicity under the free superoperations; i.e., the amount of resource can only decrease under the application of a free superoperation. With our convention concerning subselection, this ensures monotonicity under subselection as well [65]. The third property is convexity and can be seen as a matter of convenience. It ensures that mixing does not create resources. These properties lead to the following definition.

Definition 6.—A functional M from quantum operations to the positive real numbers is called a resource measure if and only if

$$\begin{aligned} &M(\Theta) = 0 \Leftrightarrow \Theta \text{ free}, \\ &M(\Theta) \geq M(\mathcal{F}[\Theta]) \quad \forall\, \Theta, \quad \forall \text{ free superoperations } \mathcal{F}, \\ &M(\Theta) \text{ is convex}. \end{aligned} \tag{12}$$

A functional that is a measure according to the above definition is of special interest if it has a clear operational interpretation; i.e., if the number it puts on a resource is directly connected to its value in a specific application. Often resource measures are hard to evaluate; thus measures that have a closed form expression or can be calculated efficiently using numerical methods are important as well. In the following, we will give one resource measure with respect to the ability to detect coherence that can be calculated efficiently and another one with an operational interpretation. Both involve norms on quantum operations. Therefore we review some related terminology first. A norm $\|\cdot\|$ on quantum operations is called submultiplicative if and only if

$$\|\Theta_1 \circ \Theta_2\| \leq \|\Theta_1\|\|\Theta_2\| \quad \forall\, \Theta_1, \Theta_2 \tag{13}$$

and submultiplicative with respect to tensor products if and only if

$$\|\Theta_1 \otimes \Theta_2\| \leq \|\Theta_1\|\|\Theta_2\| \quad \forall\, \Theta_1, \Theta_2. \tag{14}$$

Norms with the above properties can be used to define measures.

Proposition 7.—Let $\|\cdot\|$ denote a norm on quantum operations which is both submultiplicative and submultiplicative with respect to tensor products. If $\|\Phi\| \leq 1$ for all Φ detection-incoherent operations, the functional

$$M(\Theta) = \min_{\Phi \in \mathcal{DI}} \|\Delta\Theta - \Delta\Phi\| \tag{15}$$

is a measure in the detection-incoherent setting.

Choosing a particular norm in the above proposition, the so-called completely bounded trace norm or diamond norm [66], we find a measure that can be calculated efficiently. The diamond norm is based on the trace norm, which is defined for a linear operator A by [67]

$$\|A\|_1 = \mathbf{tr}\left(\sqrt{A^\dagger A}\right). \tag{16}$$

The induced trace norm on a quantum operation (or more general a superoperator) Θ is, as the name suggests, defined by

$$\|\Theta\|_1 = \max\{\|\Theta(X)\|_1 : \|X\|_1 \leq 1\}. \tag{17}$$

Finally, the completely bounded trace norm or diamond norm of a quantum channel is given by

$$\|\Theta^{B\leftarrow A}\|_\diamond = \sup_Z \|\Theta^{B\leftarrow A} \otimes \mathbb{1}^Z\|_1 = \|\Theta^{B\leftarrow A} \otimes \mathbb{1}^C\|_1$$

with $\dim A = \dim C$ and has multiple applications in quantum information [66–68]. With these definitions at hand, we are ready to present our first measure.

Theorem 8.—The functional

$$M_\diamond(\Theta) = \min_{\Phi \in \mathcal{DI}} \|\Delta\Theta - \Delta\Phi\|_\diamond \tag{18}$$

is a measure in the detection-incoherent setting. We call this measure the diamond measure.

Rather surprisingly, we show in the Supplemental Material [23] that this measure can be calculated efficiently using a semidefinite program [69] that is based on Refs. [58,70,71]. A related measure is given in the following theorem.

Theorem 9.—The functional

$$\tilde{M}_\diamond(\Theta) = \min_{\Phi \in \mathcal{DI}} \|\Delta\Theta - \Delta\Phi\|_1 \tag{19}$$

is a measure in the detection-incoherent setting. We call it the NSID measure (nonstochasticity in detection).

As we prove in the Supplemental Material [23], this measure has an operational interpretation in our framework: assume you obtain a *single* copy of a quantum channel which is equal to Θ_0 or Θ_1 with probability $\frac{1}{2}$ each. The optimal probability $P_c(1/2, \Theta_0, \Theta_1)$ to correctly guess $i = 0,\ 1$ if one can perform only detection-incoherent measurements is given by (see also Ref. [72])

$$P_c(1/2, \Theta_0, \Theta_1) = \frac{1}{2} + \frac{1}{4}\max_{|\psi\rangle} \|\Delta(\Theta_0 - \Theta_1)|\psi\rangle\langle\psi|\|_1.$$

Therefore, in a single shot regime, $1/2 + 1/4\tilde{M}_\diamond(\Theta)$ is the optimal probability to guess correctly if one obtained Θ or the least distinguishable free operation, provided we can use only free measurements. Accordingly, the measure $\tilde{M}_\diamond$ quantifies how well the visible part of an operation can be approximated by a free one. This operational interpretation is the reason for the choice of the name NSID measure. Note that a similar interpretation holds for the diamond measure with the only difference that, in the auxiliary system, nonfree measurements are allowed as well. Therefore the diamond measure is an upper bound on the NSID measure. The operational interpretation of this measure (which satisfies faithfulness) proves that we can distinguish at no cost all operations that can detect coherence from those that cannot. As we argue in the introduction and the Supplemental Material [23], this is an important property that cannot be achieved using any coherence theory on the level of states. In the Supplemental Material, we give details of how this measure can be evaluated and some examples.

Now that we have described a measure with an operational interpretation, a natural question is which quantum operations maximize this measure. The answer is given by the following proposition.

Proposition 10.—The maximum value of $\tilde{M}_\diamond(\Theta)$ for Θ a quantum channel with input of dimension n and output of dimension m is given by

$$\frac{2(N_0 - 1)}{N_0}, \tag{20}$$

where $N_0 = \min\{n, m\}$. It is both saturated by a Fourier transform in a subspace of dimension N_0 and by a measurement in the Fourier basis, encoding the outcomes in the incoherent basis.

For transformations on qubits, this means that the Hadamard gate is best suited to detect coherence in the sense of the NSID measure. This can be seen as a reason why e.g., the Deutsch-Jozsa algorithm [73,74] not only starts but also finishes with Hadamard gates. It is not enough to create coherence, it also has to be detected, i.e., used, in order to exploit it.

Conclusions.—We argued why the formulation of resource theories on the level of operations are a valuable unifying concept and demonstrated at the example of coherence theory how to construct them rigorously using resource destroying maps [63]. These theories are based on two main ingredients, the free operations and the free superoperations. The free superoperations are sequential and parallel concatenations with free operations, i.e., the embedding into a network of free operations. Based on physical considerations, we defined properties that a measure of resources in an operation should obey, e.g., monotonicity under the free superoperations. We focused particularly on the question how well a quantum operation can detect coherence. This is important, since both the ability to create and to detect coherence are necessary prerequisites for operational advantages of quantum computation over classical computation, and the latter cannot, as we have shown, be addressed using resource theories on the level of states. We presented two measures quantifying the ability of an operation to detect coherence. The first can be calculated efficiently using a semidefinite program. The second, named the NSID measure, can be evaluated in an iterative manner and has a clear operational interpretation. Its value determines how well we can distinguish the given quantum operation from the free operations in a single try. Finally, we proved that Fourier transforms and measurements in a Fourier basis maximize the NSID measure and can therefore be considered optimal in the task of measuring coherence.

Completion of the resource theories provided here is a sizable task. It includes the question of manipulation, quantification, and exploitation of the resourceful operations using free superoperations. A thorough answer to these questions may lead to a better understanding of operational advantages provided by quantum devices, which in turn may lead to improved designs. Working out our approach in scenarios different from coherence theory will shed new light on other quantum properties.

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