Prescaling in a Far-from-Equilibrium Bose Gas

Christian-Marcel Schmied,[1,2] Aleksandr N. Mikheev,[1] and Thomas Gasenzer[1,*]
[1]Kirchhoff-Institut für Physik, Ruprecht-Karls-Universität Heidelberg, Im Neuenheimer Feld 227, 69120 Heidelberg, Germany
[2]Dodd-Walls Centre for Photonic and Quantum Technologies, Department of Physics, University of Otago, Dunedin 9016, New Zealand

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Nonequilibrium conditions give rise to classes of universally evolving configurations of quantum many-body systems at nonthermal fixed points. While the fixed point and thus full scaling in space and time is generically reached at very long evolution times, we propose that systems can show prescaling much earlier in time, in particular, on experimentally accessible timescales. During the prescaling evolution, some well-measurable properties of spatial correlations already scale with the universal exponents of the fixed point while others still show scaling violations. Prescaling is characterized by the evolution obeying conservation laws associated with the remaining symmetry which also defines the universality class of the asymptotically reached nonthermal fixed point. Here, we consider $N = 3$ species of spatially uniform three-dimensional Bose gases, with identical inter- and intraspecies interactions. During prescaling, the full $U(N)$ symmetry of the model is broken to $U(N-1)$ while the conserved transport, reflecting explicit and emerging symmetries, leads to the buildup of rescaling quasicondensate distributions.

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Far from equilibrium, comparatively little is known about the possibilities nature reserves for the structure and states of quantum many-body systems. Much progress has been made recently in the context of prethermalization [1,2], generalized Gibbs ensembles [3,4], many-body localization [5], critical and prethermal dynamics [6–9], decoherence and revivals [10], and (wave) turbulence [11–13].

Quantum systems quenched far from equilibrium can show relaxation behavior distinctly different from what is known in classical statistics. In particular, a system can approach a nonthermal fixed point [14] exhibiting universal scaling in time and space [15–17]. Universal behavior has been predicted to occur in various different systems ranging from the postinflationary early Universe [18,19], via the dynamics of quark-gluon matter created in heavy-ion collisions [20,21], to the evolution of dilute quantum gases starting from a far-from-equilibrium initial state [15,22–24]. The concept of nonthermal fixed points paves the way to a unifying description of universal dynamics. It remains, though, an unresolved question how in general quantum many-body systems evolve from a given initial state to such a fixed point. In this Letter, we propose *prescaling* as a generic feature of that evolution.

Universal scaling dynamics associated with a nonthermal fixed point is characterized by scaling evolution of correlation functions. For example, the occupation number $n_a(\mathbf{k},t) = \langle\Phi_a^\dagger(\mathbf{k},t)\Phi_a(\mathbf{k},t)\rangle$ of an (N component) Bose field $\Phi_a(\mathbf{k},t)$, *at* the fixed point, evolves in a self-similar manner according to

$$n_a(\mathbf{k},t) = (t/t_{\mathrm{ref}})^{\alpha} f_{\mathrm{S},a}([t/t_{\mathrm{ref}}]^{\beta}\mathbf{k}), \tag{1}$$

with universal scaling function $f_{\mathrm{S},a}(\mathbf{k}) = n_a(\mathbf{k},t_{\mathrm{ref}})$ depending on a single d-dimensional variable only, scaling exponents α, β, and some reference time t_{ref} within the temporal scaling regime [15]. In particular, the scaling exponent β defines the time evolution of a single characteristic length scale $L_\Lambda(t) \sim t^\beta$. Strictly speaking, the fixed point itself is reached only in a certain scaling limit, such as, for $\beta > 0$, at asymptotic times and infinite volume. However, the question arises how the scaling limit is reached and to what extent and when scaling is already seen at finite times.

In equilibrium, fixed points of renormalization-group flows describe correlations at a continuous, e.g., second-order phase transition. They correspond to a pure rescaling of the correlations, in momentum or position space, under the change of the flow parameter such as a scale beyond which fluctuations are averaged over. In the context of critical phenomena as well as fundamental particle physics, renormalization flows are known which are first attracted to a partial fixed point [25]. In such situations, still away from the actual fixed point, scaling violations can occur for some quantities while others already show scaling and the further flow be strongly constrained by a symmetry the system is subject to.

Motivated by the general concept of partial fixed points [26], we propose the existence of prescaling [27]. This means that certain correlation functions, already at comparatively early times and within a limited range of distances scale with the universal exponents predicted for the fixed point which itself is reached only much later in time and in a finite-size system may not be reached at all.

During the stage of prescaling, (weak) scaling violations occur in correlations at distances outside this range. Such violations only slowly vanish as time evolves. In analogy to the case of partial fixed points, we expect the underlying symmetries of the system to play a key role for the realization of prescaling. While part of the symmetries can be broken, symmetries reflecting the conservation laws associated with the nonthermal fixed point remain intact during prescaling.

To reveal the existence of prescaling we employ an isolated, $(N = 3)$-component dilute Bose gas in $d = 3$ spatial dimensions, quenched far out of equilibrium. Numerically solving the field equations of motion within a semiclassical Truncated-Wigner approach we find that, during the approach of a nonthermal fixed point, the system prescales. The phenomenon becomes visible in the short-distance properties of correlation functions that measure, e.g., the spatial coherence of the local phase-angle differences between different components. We emphasize that scaling violations affect not only the scaling exponents but in particular also the shape of the scaling functions.

The spatially uniform Bose gases consist of identical particles distinguished only by a single property such as the hyperfine magnetic quantum numbers of the atoms forming the gas. The system in three spatial dimensions is described by a $U(3)$ symmetric Gross-Pitaevskii (GP) model with quartic contact interaction in the total density,

$$H = \int d^3x \left(-\Phi_a^\dagger \frac{\nabla^2}{2m} \Phi_a + \frac{g}{2} \Phi_a^\dagger \Phi_b^\dagger \Phi_b \Phi_a \right), \qquad (2)$$

where we use units implying $\hbar = 1$, space-time field arguments are suppressed, m is the particle mass, and it is summed over the Bose fields, a, $b = 1, 2, 3$, obeying standard commutators $[\Phi_a(\mathbf{x}, t), \Phi_b^\dagger(\mathbf{y}, t)] = \delta_{ab}\delta(\mathbf{x} - \mathbf{y})$. The gases are thus assumed to occupy the same space and be subject to identical inter- and intraspecies contact interactions quantified by g.

Universal scaling of the N-component Bose gas *at* the nonthermal fixed point can be described analytically in terms of a low-energy effective theory for the phase-angle excitations of the Bose fields $\Phi_a(\mathbf{x}, t) = [\rho_a^{(0)} + \delta\rho_a(\mathbf{x}, t)]^{1/2} \exp\{i\delta\theta_a(\mathbf{x}, t)\}$, on constant mean background phases $\theta_a^{(0)} = 0$ and densities $\rho_a^{(0)}$. After integrating out the density fluctuations $\delta\rho_a$, the linear modes of this effective model are given by the total phase $\sum_{a=1}^{N} \delta\theta_a$, with Bogoliubov dispersion $\omega_B(\mathbf{k}) = \sqrt{\varepsilon_\mathbf{k}(\varepsilon_\mathbf{k} + 2g\rho^{(0)})}$, $\varepsilon_\mathbf{k} = \mathbf{k}^2/2m$, and $N - 1$ gapless Goldstone excitations of the relative phases, e.g., $\delta\theta_a - \delta\theta_1$, with free-particle dispersion $\omega_G(\mathbf{k}) = \varepsilon_\mathbf{k}$. A scaling analysis of the kinetic equation $\partial_t f_a(\mathbf{k}, t) = I[f](\mathbf{k}, t)$ governing the momentum-space redistribution of the phase-angle excitations $f_a(\mathbf{k}, t) = \langle \delta\theta_a(\mathbf{k}, t) \delta\theta_a(-\mathbf{k}, t) \rangle$ at the fixed point provides an analytical prediction for α and β [28,29]. Here, $I[f]$ is a quantum-Boltzmann-type collision integral involving scattering terms nonlinear in the distributions f_a, arising from the nonlinear couplings of the $\delta\theta_a$. One obtains, for $N \to \infty$ as well as $N = 1$, the values [28,29]

$$\beta = 1/2, \qquad \alpha = \beta d = 3/2, \qquad (3)$$

consistent with the results of [15,24] for $N \to \infty$. The relation between α and β reflects the conservation of the d-dimensional integral $\int_\mathbf{k} f_a(\mathbf{k}, t)$. This particular fixed point has Gaussian character, i.e., in the limit $t \to \infty$, correlation functions factorize and the scaling of $f_a(\mathbf{k}, t)$ implies the scaling of $n_a(\mathbf{k}, t)$ as well as of higher-order correlators of the Φ_a [28].

Here, we numerically study the evolution of the system towards this fixed point, starting from a far-from-equilibrium initial condition at time t_0 given by large occupations of all fields, $n_0 \gg 1$, constant up to some cutoff scale, i.e., $n_a(\mathbf{k}, t_0) = n_0 \Theta(k_q - |\mathbf{k}|)$ [30]. The initial phase angles $\theta_a(\mathbf{k}, t_0)$ of the Bose fields $\Phi_a(\mathbf{k}, t_0) = \sqrt{n_0} \exp[i\theta_a(\mathbf{k}, t_0)]$ are chosen randomly on the circle and thus uncorrelated. In practice, such an initial condition can be achieved by, e.g., a strong cooling quench or a transient instability [14,24]. Note that already this initial state does not obey the full $U(3)$ symmetry but breaks it to $U(2) \simeq [SU(2) \times U(1)]/Z_2$ as does the evolving state. The $U(3)$ symmetry of (2) gives rise to conservation laws, consistent with the reduced $U(2)$ symmetry, which will be obeyed during prescaling [28,30]. The evolution induced by such an extreme initial condition is characterized by transport of particles from $k \lesssim k_q$ towards the infrared, while their energy is deposited by a few particles at higher momenta, $k > k_q$. In this way the system, after a few collision times, shows universal scaling indicating the approach of a nonthermal fixed point [15,23,24].

While the scaling behavior at a nonthermal fixed point is commonly extracted from momentum-space correlators, we find, however, that prescaling is more clearly seen in position-space correlations. Based on momentum-space treatments of nonthermal fixed-point scaling it is intuitive to study the first-order spatial coherence function $g_a^{(1)}(\mathbf{r}, t) = \langle \Phi_a^\dagger(\mathbf{x} + \mathbf{r}, t) \Phi_a(\mathbf{x}, t) \rangle$, which is obtained as the Fourier transform of the occupation number $n_a(\mathbf{k})$. At large evolution times, close to the nonthermal fixed point, the coherence function is expected to be spherically symmetric and characterized by a universal function $f_s(x)$ as $g_a^{(1)}(\mathbf{r}, t) = f_s(k_\Lambda(t) r)$, $r = |\mathbf{r}|$. The inverse coherence length scales as $k_\Lambda(t) \sim t^{-\beta}$.

The time evolution of the first-order coherence function is shown in Fig. 1(a). We observe that the numerically extracted form clearly differs from a pure exponential, $g_a^{(1)}(\mathbf{r}, t) \sim \exp\{-k_\Lambda(t) r\}$, which is predicted analytically within the leading approximation of a low-energy effective theory of nonthermal fixed points [28], with $k_\Lambda(t)$ being the

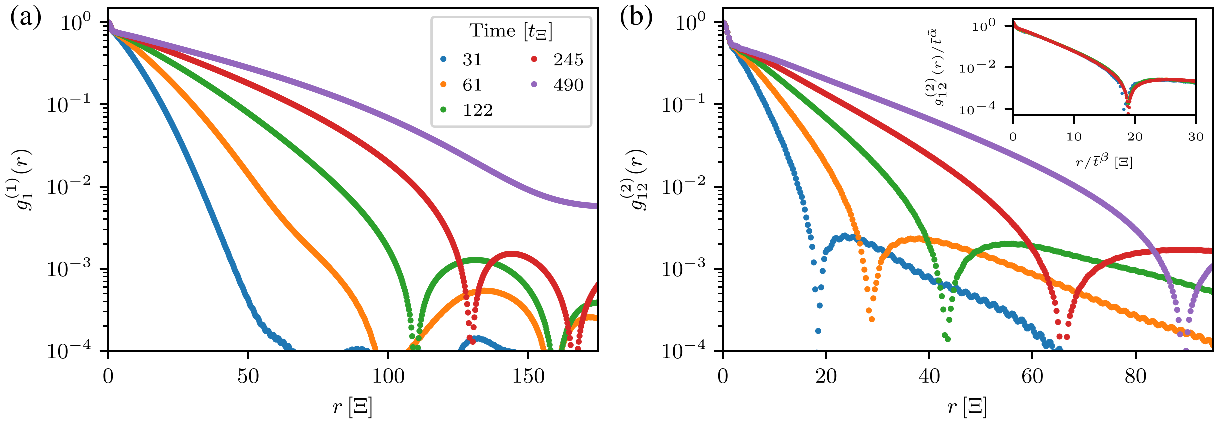

FIG. 1. (a) Time evolution of the first-order coherence function $g_1^{(1)}(r) = g_1^{(1)}(\mathbf{r}, t) = \langle\Phi_1^\dagger(\mathbf{x}+\mathbf{r}, t)\Phi_1(\mathbf{x}, t)\rangle$ at five different times (colored dots). The shape of the correlation function is reminiscent of an exponential with a multiplicative oscillatory contribution. It clearly exhibits violations of universal scaling at larger distances, which become weaker in time but still prevail even at long evolution times. At the latest time shown, finite-size effects appear. (b) Corresponding second-order coherence function measuring the spatial fluctuations of the relative phases between components 1 and 2, $g_{12}^{(2)}(r, t) = \langle\Phi_1^\dagger(\mathbf{x}+\mathbf{r}, t)\Phi_2(\mathbf{x}+\mathbf{r}, t)\Phi_2^\dagger(\mathbf{x}, t)\Phi_1(\mathbf{x}, t)\rangle$ for the same evolution times as in (a) (colored dots). The inset shows the rescaled coherence function $\bar{t}^{-\tilde{\alpha}}g_{12}^{(2)}(\bar{t}^{-\beta}r, t_{\mathrm{ref}})$, with $\beta = 0.6$, $\tilde{\alpha} = -0.2$, and $\bar{t} = t/t_{\mathrm{ref}}$, with reference time $t_{\mathrm{ref}} = 31t_\Xi$. The collapse of the data onto a single function, especially at short distances where $g_{12}^{(2)}(r, t) \gtrsim 10^{-2}$, indicates that violations of scaling are considerably weaker than for $g_1^{(1)}$. Time t is measured in units of $t_\Xi = 2\pi[g\rho^{(0)}]^{-1}$, distances r in units of the healing length scale $\Xi = [2mg\rho^{(0)}]^{-1/2}$.

inverse coherence length of the system at time t. Instead, on top of an approximately exponential falloff, the coherence function also shows oscillatory behavior in r. The oscillations indicate a structure developing in the system that causes excitations of the field to switch its sign over a distance on the order the inverse coherence length k_Λ; i.e., the phase strongly varies on that characteristic scale.

We stress that, as the nonlinear term in (2) couples the total densities, it suppresses total-density fluctuations but not fluctuations of the local density differences between the components. Hence, the spatial Goldstone excitations of the intercomponent phase differences are predicted to become relevant. As the first-order coherence function is insensitive to the relative phases $\theta_a - \theta_b$, we additionally study the second-order coherence function $g_{ab}^{(2)}(\mathbf{r}, t) = \langle\Phi_a^\dagger(\mathbf{x}+\mathbf{r}, t)\Phi_b(\mathbf{x}+\mathbf{r}, t)\Phi_b^\dagger(\mathbf{x}, t)\Phi_a(\mathbf{x}, t)\rangle$, see Fig. 1(b), for $(a, b) = (1, 2)$.

A temporal scaling analysis of the numerically determined functions $g_1^{(1)}(\mathbf{r}, t)$, $g_{12}^{(2)}(\mathbf{r}, t)$ provides a direct way to extract the scaling exponent β via the single scale $k_\Lambda(t)$. As long as, however, the fixed-point scaling is not yet fully developed, the time evolution of the correlations is not given by such a single scale. To account for that we provide a general scheme for determining how the scaling behavior is being approached. In order to approximate the correlation functions, within a certain regime of r, without any restriction to a particular scaling form we expand them into a general Taylor series such that they take the form $g^{(l)}(r,t) = c_0^{(l)} + \sum_{n=1}^{\infty} c_n^{(l)}(t)(r - r_0)^n$. Here, $r_0 \geq 0$ marks the expansion point, and $l = 1, 2$ denotes the two different types of correlators [30]. The time-dependent coefficients of the series, dropping the l index, are written as $c_n(t) = c_n[k_{\Lambda,n}(t)]^n$, rescaling in time according to $k_{\Lambda,n}(t) \sim t^{-\beta_n}$. In consequence, the coefficients of the expansion rescale as $c_n(t) \sim t^{-n\beta_n}$. Each order of the expansion can be seen as a probe for the scaling of the correlations at a different distance r. The corresponding scaling exponents can be written as $\beta_n(t) = \beta + \delta\beta_n(t)$. A particular order of the expansion shows scaling with the fixed-point exponent β when $\delta\beta_n(t)$ becomes small and approximately constant in time. The system prescales when $\beta_n \approx \beta$ for at least one order n of the expansion. The fixed point itself is, in a strict sense, only reached if the statement holds for all orders of the expansion.

For our system we expect prescaling to emerge on short distances and to subsequently spread towards longer distances. Therefore, we truncate the expansion at the fourth order and extract the coefficients $c_n(t)$, with $n = 1, 2, 3, 4$, from a fit of the expansion to the data at various instances of time t. To focus on short-distance scaling properties of the system the fit is applied at distances $5\Xi \lesssim r \ll \mathcal{L}$, with linear system size $\mathcal{L}$. The lower bound of the fit range is used in order to not be affected by the nonuniversal short-distance thermal peak around zero distance. Taking the negative of the logarithmic derivative of $c_n(t)$ with respect to t and dividing by n gives the scaling exponent β_n at a particular instance in time. To reduce fluctuations of the locally in time extracted exponents we average the β_n over a fixed time window. Taking into account possible fluctuations of the scaling exponents arising from the choice of the fit range we

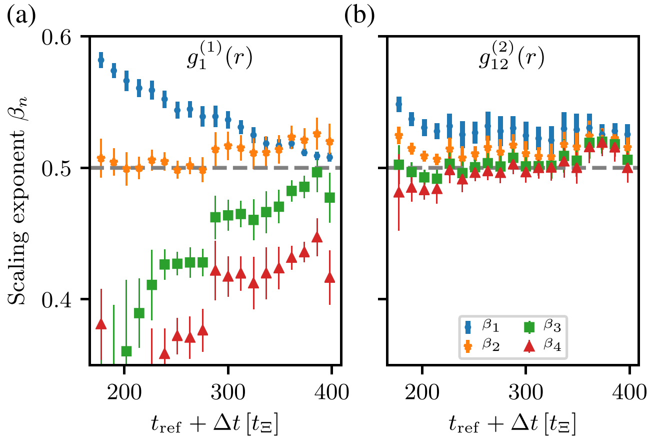

FIG. 2. Prescaling of position-space correlations. (a) Scaling exponents β_n describing the time evolution of $k_{\Lambda,n}(t) \sim t^{-\beta_n}$ with $n = 1, 2, 3, 4$. The different exponents are deduced from Taylor series coefficients $c_n(t) = c_n[k_{\Lambda,n}(t)]^n$ which are obtained by means of a fit of the first-order coherence function $g_1^{(1)}(r, t)$, shown in Fig. 1(a), at small distances r. The index n marks the corresponding order of the Taylor series. The jump of the exponents at $t_{\text{ref}} + \Delta t \approx 285 t_\Xi$ results from a sign change of the fitted third- and fourth-order coefficients. This indicates that the shape of the scaling form is altered more significantly on large distances as compared to short distances as can already be expected from Fig. 1(a). (b) Scaling exponents β_n deduced from an analogous Taylor series fit of $g_{12}^{(2)}(r, t)$ [Fig. 1(b)]. Prescaling is quantitatively seen by the scaling exponents β_n settling into, within errors, equal stationary values for the lower orders of the fit. While $g_1^{(1)}(r)$, up to order r^4 shows scaling violations, $g_{12}^{(2)}(r)$ already scales, to a good approximation, with the predicted exponent $\beta = 1/2$ for $t_{\text{ref}} + \Delta t \gtrsim 250 t_\Xi$. For an individual fit, the β_n result from averaging over times $[t_{\text{ref}}, t_{\text{ref}} + \Delta t]$ with $\Delta t = 146 t_\Xi$. The final data points shown are obtained by additionally averaging over a set of fits with different fit ranges. Errors are given by the corresponding standard deviation of the exponents of the set [30].

furthermore average over different such ranges [30]. Performing the whole analysis procedure gives the scaling exponents β_n shown in Fig. 2, for $n = 1, \ldots, 4$, for both, $g_1^{(1)}$ and $g_{12}^{(2)}$.

The particular value $\beta_n \simeq 0.5$ found, at late times, for the scaling of $k_{\Lambda,n}(t)$, for $n = 1, 2$, parametrizing $g_1^{(1)}$, and for $n = 1, \ldots, 4$ in the case of $g_{12}^{(2)}$, is in good agreement with the analytically predicted value of $\beta = 1/2$, cf. (3) [15,24]. Note that the finite size of the system does not lead to scaling beyond $t \simeq 400 t_\Xi$.

For $g_1^{(1)}$ we find that scaling in the higher orders of the expansion is not yet fully developed within our time window. This causes the scaling violations on larger distances observed in Fig. 1. The converging flow of the scaling exponents indicates the slow approach of a full scaling form. In consequence, the system appears close to the nonthermal fixed point but is still away from it.

Comparing Figs. 2(a) and 2(b) we conclude that different correlators can enter the stage of prescaling on different timescales. Therefore, establishing the full scaling function and the associated scaling exponents is observable-dependent. This can also be intuitively concluded from comparing Figs. 1(a) and 1(b). In general, we expect the fixed-point scaling to first show up in correlators of observables that are most sensitive to the relevant degrees of freedom of the underlying universal behavior. Hence, our results indicate that the fixed-point scaling of the model considered is dominated by relative-phase fluctuations, forming the Goldstone modes of the broken $U(3)$ symmetry [28,30]. Note that these excitations are much less energetically constrained than the soundlike excitations of the *total* density, which are suppressed by the interaction term in (2) and associated with the overall $U(1)$ symmetry. If N is large, the relative-phase fluctuations, corresponding to spatial reshuffling of the local density differences between the different components, will in general dominate the nonequilibrium evolution of the system, also of the single-component correlators $g_1^{(1)}$. As $N = 3$, however, is comparatively small, a clear difference in the scaling violations for $g_1^{(1)}$ and $g_{12}^{(2)}$ is seen.

We emphasize that the evolution during the stage of prescaling already obeys the conservation laws associated with the nonthermal fixed point. Both, $n_a(\mathbf{k}, t)$ and the Fourier transform of $g_{ab}^{(2)}(\mathbf{r}, t)$ allow a scaling collapse according to (1) with exponents $\alpha \simeq d\beta$. This is consistent with number conservation reflecting the $U(3)$ symmetry of the Hamiltonian, as well as an emerging symmetry which ensures the invariance of $g_a^{(1)}(0, t)$ and $g_{ab}^{(2)}(0, t)$, respectively [30].

It is remarkable that the $N = 3$ prescaling exponents β_i found for $g_a^{(1)}$ and $g_{ab}^{(2)}$ as shown in Fig. 2 agree with the (for $N \to \infty$) analytically predicted value $\beta = 1/2$ to a very good accuracy. This suggests that the universality class of the model is independent of the number of components N, reflecting that the $U(N)$ symmetry is broken during prescaling to $U(N-1)$ and the dispersion of the dominating Goldstone relative-phase modes is independent of N.

A similar value for the scaling exponent β has been found in recent experiments on a quasi one-dimensional three-component spinor Bose gas [16] which have motivated us to consider the $U(3)$ GP model. In this experiment, additional spin-changing interactions and Zeeman shifts break the $U(3)$ symmetry, freezing out one of the relative-phase degrees of freedom at low k. Nonetheless, given the experimental parameters, the measured momentum range is within a regime well described by the $U(3)$ model and prescaling is expected to be detectable.

Prescaling, observable in the relatively early evolution after a quench far from equilibrium, is expected to play an important role in universal scaling evolution and its accessibility in experiments with ultracold atomic gases. Furthermore, from a renormalization-group perspective and with respect to the given underlying symmetries we expect prescaling during the time evolution of various types of quantum many-body systems.

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*t.gasenzer@uni-heidelberg.de

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