Cavity Quantum Eliashberg Enhancement of Superconductivity

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Driving a conventional superconductor with an appropriately tuned classical electromagnetic field can lead to an enhancement of superconductivity via a redistribution of the quasiparticles into a more favorable nonequilibrium distribution—a phenomenon known as the Eliashberg effect. Here, we theoretically consider coupling a two-dimensional superconducting film to the quantized electromagnetic modes of a microwave resonator cavity. As in the classical Eliashberg case, we use a kinetic equation to study the effect of the fluctuating, dynamical electromagnetic field on the Bogoliubov quasiparticles. We find that when the photon and quasiparticle systems are out of thermal equilibrium, a redistribution of quasiparticles into a more favorable nonequilibrium steady state occurs, thereby enhancing superconductivity in the sample. We predict that by tailoring the cavity environment (e.g., the photon occupation and spectral functions), enhancement can be observed in a variety of parameter regimes, offering a large degree of tunability.

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It has been known since the late 1960s that subjecting a superconductor to strong microwave radiation can lead to an enhancement of superconductivity [1,2]. The explanation of this was first provided by Eliashberg *et al.* [3–5], who showed that the irradiation yields a nonthermal distribution of the Bogoliubov excitations with an effectively colder band edge. The degree of enhancement can be obtained by using standard BCS theory with a nonthermal quasiparticle distribution function. In the subsequent decades, Eliashberg's theoretical explanation for this effect has been extended and applied to a variety of other systems [6–12].

In recent years there has been a renewed interest in nonequilibrium superconductivity motivated in part by a number of "pump-probe" experiments which have found that materials subjected to intense terahertz pulses exhibit transient superconducting properties up to very high sample temperatures [13–15]. Understanding these transient states has led to a variety of theoretical models which go beyond the quasiparticle redistribution effect [16–21].

All of these systems concern the interaction between quantum matter and a *classical* external field. Particularly interesting and novel, however, is the effect that a fluctuating *quantum* gauge field has on quantum matter. Indeed, it has been a long-standing focus in the field of cavity quantum electrodynamics to realize the dynamical quantum nature of the electromagnetic field through the use of resonant electromagnetic cavities [22–26]. Recently, there have been many advances in this area including the realization of exciton-polariton condensates [27,28], states formed from hybridizing cavity photons and semiconductor excitons.

This Letter extends some of these concepts to superconducting systems with an eye on cavity-induced Eliashberg-type enhancement of superconductivity. The central observation is that even in a nonequilibrium steady state the BCS self-consistency equation

$$\frac{1}{g} = \int \frac{dE}{E} \nu_{\mathrm{qp}}(E)[1 - 2n(E)] \tag{1}$$

can be solved for a nonthermal quasiparticle distribution function $n(E)$, where $\nu_{\mathrm{qp}}(E) = 2\nu_F|E|/\sqrt{E^2 - \Delta^2}$ is the quasiparticle density of states. The solution of this equation—the BCS superconducting gap Δ—is therefore a functional of the distribution function $n(E)$ as well as the BCS coupling constant g. Of particular interest are cases where the gap exceeds its equilibrium thermal value, $\delta\Delta = \Delta[n_F + \delta n] - \Delta[n_F] > 0$. In the classical Eliashberg effect, this is achieved via irradiation with a coherent microwave field. For frequencies smaller than 2Δ, pair breaking is suppressed and existing thermal quasiparticles are scattered up to higher energies, where their debilitating effect is lessened by the reduced relative density of states. This emptying of states near the band edge increases Δ above its equilibrium value. In this Letter we generalize this idea to include the dynamical fluctuations of the electromagnetic field in a microwave cavity, depicted in the inset of Fig. 1(b). Our main result is that, by appropriately tuning the parameters of the cavity environment (e.g., resonance, linewidth, temperature, etc.), an enhancement in the BCS gap strength may be obtained, now in the absence of coherent electromagnetic radiation. This gap enhancement is shown in Fig. 1(a), which illustrates the change in the BCS gap strength $\delta\Delta$ as a function of the cavity resonant frequency ω_0. The rest of the Letter is devoted to deriving this result.

We begin with a model of an s-wave superconductor described by the BCS Hamiltonian (setting $\hbar = k_B = 1$)

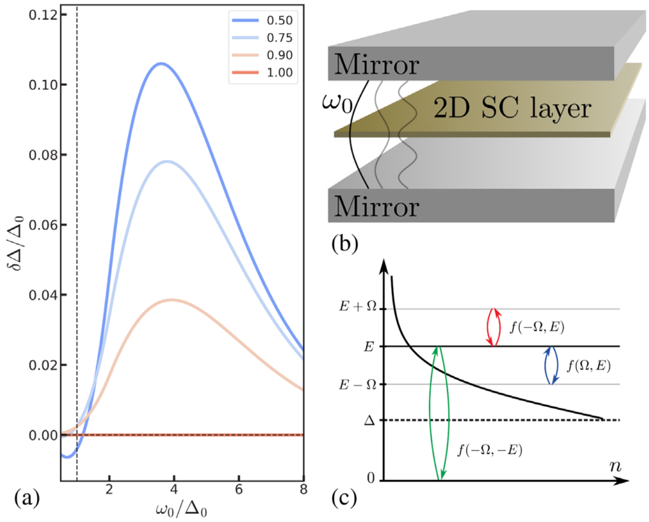

FIG. 1. (a) Relative enhancement of the gap function as a function of cavity frequency ω_0 for a particular value of the overall scaling constant $\pi\alpha XD\tau_{\rm in}/c^2$ (we take $X = 133$ and $\pi\alpha D\tau_{\rm in}T_c^2/c^2 = 9.17\times10^{-5}$ with T_c set to unity). Curves are colored and labeled according to the ratio $T_{\rm cav}/T_{\rm qp}$, comparing the photon and quasiparticle temperatures. The enhancement is seen set in after the cavity frequency surpasses the pair-breaking energy $2\Delta_0$. (b) Schematic picture of the system used for calculation. The lowest cavity resonator mode with cutoff frequency ω_0 is shown, as is the 2D superconducting (SC) layer. (c) Depiction of the various processes which contribute to the quasiparticle collision integral, plotted against the equilibrium $n(E)$. The blue arrows depict the down-scattering terms captured by $f(\Omega,E)$, the red arrows depict the up-scattering terms captured by $f(-\Omega,E)$, and the green arrows represent the pair processes captured by $f(-\Omega,-E)$.

$$H = \int d^2r\left[\psi_\sigma^\dagger\left(-\frac{\mathbf{D}^2}{2m}-\mu\right)\psi_\sigma - g\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow\right], \quad (2)$$

where ψ_σ is the electron field operator, which is minimally coupled to the electromagnetic vector potential $\mathbf{A}$ through the gauge covariant derivative $\mathbf{D} = \nabla + ie\mathbf{A}$. Throughout, we will employ the radiation gauge $\nabla\cdot\mathbf{A} = 0$. The interaction is decoupled via standard mean-field theory, and the resulting Hamiltonian is diagonalized with a Bogoliubov transformation

$$\begin{pmatrix}\psi_{\mathbf{p},\uparrow}\\ \psi^\dagger_{-\mathbf{p},\downarrow}\end{pmatrix} = \begin{pmatrix}u_{\mathbf{p}} & -v_{\mathbf{p}}\\ v_{\mathbf{p}} & u_{\mathbf{p}}\end{pmatrix}\begin{pmatrix}\gamma_{\mathbf{p},+}\\ \gamma^\dagger_{-\mathbf{p},-}\end{pmatrix},$$
$$u, v = \sqrt{\frac{1}{2}\left(1\pm\frac{\xi}{E}\right)}, \quad (3)$$

where $\gamma_{\mathbf{p}\pm}$ are the Bogoliubov quasiparticle (BQP) annihilation operators, $E_{\mathbf{p}} = \sqrt{\xi_{\mathbf{p}}^2+\Delta^2}$ is the BQP dispersion, and $\xi_{\mathbf{p}} = \mathbf{p}^2/2m - \mu$. The electromagnetic field $\mathbf{A}$ is subject to cavity quantization of the transverse momentum, leading to a dispersion relation for in-plane momentum $\mathbf{q}$ of

$$\omega_{n,\mathbf{q}} = \sqrt{\left(\frac{n\pi c}{L}\right)^2 + c^2\mathbf{q}^2} \equiv \sqrt{n^2\omega_0^2 + c^2\mathbf{q}^2}, \quad (4)$$

where $n = 1, 2, 3, \ldots$ indexes the harmonic of the confined mode. For simplicity, we will only consider the fundamental $n = 1$ harmonic and place the superconducting sample at the antinode where the coupling to the field is strongest, as depicted in Fig. 1(b).

To leading order, the interaction between photons and BQPs obtained from Eq. (2) occurs through the coupling of the vector potential to the electronic current via

$$H^{\rm int} = -e\int d^dr\,\mathbf{j}\cdot\mathbf{A}.$$

Applying the Bogoliubov transformation and Fourier transforming to momentum space, this becomes

$$\mathbf{j}_{\mathbf{q}} = \int_p \frac{\mathbf{p}-\frac{1}{2}\mathbf{q}}{m}[(u_{\mathbf{p-q}}u_{\mathbf{p}} + v_{\mathbf{p-q}}v_{\mathbf{p}})\gamma^\dagger_{\mathbf{p-q},\sigma}\gamma_{\mathbf{p},\sigma}$$
$$+ (u_{\mathbf{p-q}}v_{\mathbf{p}} - v_{\mathbf{p-q}}u_{\mathbf{p}})(\gamma^\dagger_{\mathbf{p-q},+}\gamma^\dagger_{-\mathbf{p},-} - \gamma_{\mathbf{p},+}\gamma_{-(\mathbf{p-q}),-})], \quad (5)$$

where we use the shorthand $\int_p \ldots = \int d^2p\ldots/(2\pi)^2$. We see there are three types of matrix elements appearing in Eq. (5), corresponding to scattering (by both emission and absorption of photons), pair breaking, and pair recombination, respectively. Through these processes, the fluctuating cavity photon field will induce transitions among the BQP eigenstates, resulting in a redistribution of the quasiparticle occupations. This is described by a kinetic equation

$$\frac{\partial n_{\mathbf{p}}}{\partial t} = \mathcal{I}_{\rm cav}[n] - \frac{n_{\mathbf{p}} - n_F(\frac{E_{\mathbf{p}}}{T_{\rm qp}})}{\tau_{\rm in}}. \quad (6)$$

The first term on the rhs describes the photon-induced pairing or depairing and scattering of quasiparticles while the second term describes a generic inelastic relaxation mechanism which describes the coupling to a phonon bath at temperature $T_{\rm qp}$. The approximation here is that the inelastic relaxation rate $\tau_{\rm in}^{-1}$ is small compared to other energy scales, as was assumed in the original work of Eliashberg [5–7].

In this limit, we can perturbatively solve for the steady state of the kinetic equation (6) by expanding in small deviations $\delta n = n - n_F$ from equilibrium. To lowest order, the correction is $\delta n = \tau_{\rm in}\mathcal{I}_{\rm cav}[n_F]$. Utilizing the detailed balance properties of thermal equilibrium, this will end up depending on the photon occupation function $N(\omega)$ through its deviation from equilibrium:

$$\delta N_{\rm cav}(\omega) \equiv N(\omega) - n_B\left(\frac{\omega}{T_{\rm qp}}\right), \quad (7)$$

where $n_B(z)$ is the Bose occupation function.

To compute the cavity-induced collision integral, we rely on Fermi's golden rule, applied to both the pairing or depairing and the scattering processes. The result is

$$\mathcal{I}_{\text{cav}}[n] = \int_{p'} [\Gamma^{\text{pair}}_{\mathbf{p},-\mathbf{p}'}((1-n_{\mathbf{p}})(1-n_{-\mathbf{p}'})N(E_{\mathbf{p}}+E_{-\mathbf{p}'}) - \{n_{\mathbf{p}}n_{-\mathbf{p}'}[N(E_{\mathbf{p}}+E_{-\mathbf{p}'})+1]\}) + (\Gamma^{\text{scat}}_{\mathbf{p}'\to\mathbf{p}}\{n_{\mathbf{p}'}(1-n_{\mathbf{p}})[N(E_{\mathbf{p}'}-E_{\mathbf{p}})+1] - (1-n_{\mathbf{p}'})n_{\mathbf{p}}N(E_{\mathbf{p}'}-E_{\mathbf{p}})\} - (\mathbf{p}\leftrightarrow\mathbf{p}'))], \quad (8)$$

with the Γ's given by

$$\Gamma^{\text{pair}}_{\mathbf{p},-\mathbf{p}'} = \frac{e^2}{2\epsilon_0\omega_{\mathbf{p}-\mathbf{p}'}}\sum_\alpha \left|\boldsymbol{\epsilon}_{\alpha,\mathbf{p}-\mathbf{p}'}\cdot\left(\frac{\mathbf{p}+\mathbf{p}'}{2m}\right)\right|^2 \times (u_{\mathbf{p}}v_{-\mathbf{p}'} - u_{-\mathbf{p}'}v_{\mathbf{p}})^2\mathcal{A}_{\mathbf{p}-\mathbf{p}'}(E_{\mathbf{p}}+E_{-\mathbf{p}'}) \quad (9)$$

$$\Gamma^{\text{scat}}_{\mathbf{p}\to\mathbf{p}'} = \frac{e^2}{2\epsilon_0\omega_{\mathbf{p}-\mathbf{p}'}}\sum_\alpha \left|\boldsymbol{\epsilon}_{\alpha,\mathbf{p}-\mathbf{p}'}\cdot\left(\frac{\mathbf{p}+\mathbf{p}'}{2m}\right)\right|^2 \times (u_{\mathbf{p}}u_{\mathbf{p}'} + v_{\mathbf{p}'}v_{\mathbf{p}})^2\mathcal{A}_{\mathbf{p}-\mathbf{p}'}(E_{\mathbf{p}}-E_{\mathbf{p}'}). \quad (10)$$

These contain the dependence on the cavity-mode polarization vectors $\boldsymbol{\epsilon}_{\alpha\mathbf{q}}(z=L/2)$, the (momentum resolved) photon spectral function

$$\mathcal{A}_{\mathbf{q}}(\omega) = \frac{1/\tau_{\text{cav}}}{(\omega-\omega_{\mathbf{q}})^2 + (1/2\tau_{\text{cav}})^2}, \quad (11)$$

with photon lifetime τ_{cav}, and the squares of BCS coherence factors

$$(u_{\mathbf{p}}v_{-\mathbf{p}'} - v_{-p'}u_{\mathbf{p}})^2 = \frac{1}{2}\left(1 - \frac{\xi_{\mathbf{p}}\xi_{-\mathbf{p}'}+\Delta^2}{E_{\mathbf{p}}E_{-\mathbf{p}'}}\right), \quad (12)$$

$$(u_{\mathbf{p}}u_{\mathbf{p}'} + v_{\mathbf{p}}v_{\mathbf{p}'})^2 = \frac{1}{2}\left(1 + \frac{\xi_{\mathbf{p}}\xi_{\mathbf{p}'}+\Delta^2}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right). \quad (13)$$

These collision integrals are derived based on the assumption of a perfectly clean sample, and so momentum is conserved. In reality, however, impurities are always present in a quasi-two-dimensional sample and should not be ignored. Given that the photons of relevance are of long wavelengths, it is appropriate to invoke the quasiclassical approximation whereby we restrict our attention to states near the Fermi surface. In the limit of strong disorder (as compared to the gap) we then can incorporate elastic impurity scattering by replacing the photonic momentum-conserving delta function $(2\pi)^2\delta(\mathbf{q}-(\mathbf{p}-\mathbf{p}'))$ with a constant $(\nu_F/\tau_{\text{el}})^{-1}$, where $\mathbf{q}$ is the momentum transferred to the photon, ν_F is the density of states per spin at the Fermi level, and τ_{el} is the elastic scattering time [29]. We are then free to independently perform the integrations over the direction of the momentum. The validity of this heuristic may be confirmed by appealing to, e.g., the solution of the Usadel equation [30] or the Keldysh nonlinear sigma model [12,31,32], which describe the quasiclassical collective modes of the strongly disordered superconductor (as described in the Supplemental Material [33]).

The result of this procedure is a collision integral which is a function of the quasiparticle energy only. Evaluating the correction to the quasiparticle distribution function, we find

$$\delta n(E) = \tau_{\text{in}}\int_{-\infty}^{\infty} d\Omega J_{\text{cav}}(\Omega)\delta N_{\text{cav}}(\Omega)K(\Omega,E), \quad (14)$$

where $K(\Omega,E) = f(\Omega,E) + f(-\Omega,E) - f(-\Omega,-E)$, with

$$f(\Omega,E) = \theta(E-\Omega-\Delta)\frac{\nu_{\text{qp}}(E-\Omega)}{\nu_F} \times\frac{1}{2}\left(1+\frac{\Delta^2}{E(E-\Omega)}\right)\left[n_F\left(\frac{E-\Omega}{T_{\text{qp}}}\right) - n_F\left(\frac{E}{T_{\text{qp}}}\right)\right]. \quad (15)$$

Here, $\theta(x)$ is the Heaviside step function. The three f terms appearing in $K(\Omega,E)$ are depicted schematically in Fig. 1(c), alongside the various processes they describe. After the Fermi-surface average, the coupling to the cavity is effectively characterized by the coupling function

$$J_{\text{cav}}(\Omega) = 4\pi\alpha c D\int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{\mathcal{A}_{\mathbf{q}}(\Omega)}{2\omega_q}\sum_\alpha|\hat{\boldsymbol{\epsilon}}_{\alpha\mathbf{q},\|}|^2, \quad (16)$$

where $D = v_F^2\tau_{\text{el}}/2$ is the electronic diffusion constant and $\hat{\boldsymbol{\epsilon}}_{\alpha\mathbf{q},\|}$ indicates that only the in-plane components of the polarization vector contribute. For a BCS gap of order $\Delta = 10$ K we find a corresponding resonance frequency $\omega_0 \sim 1.3$ THz. Recently, a number of advances have lead to large enhancements in the strength and tunability of the light-matter coupling strength in this frequency regime, such that $J_{\text{cav}}(\Omega)$ may potentially exceed what is expected from our simple planar cavity model by many orders of magnitude [35–38]. We incorporate this fact by rescaling the spectral function J by a phenomenological factor X, so that $J(\Omega)\to\tilde{J}(\Omega) = XJ_{\text{cav}}(\Omega)$.

In order to simplify the calculation, we will study the system in the Ginzburg-Landau regime ($T_{\text{qp}} \lesssim T_c$), which allows us to expand the gap equation in powers of Δ. Including the nonequilibrium distribution function contribution, this results in

$$\left(\frac{T_c - T_{\text{qp}}}{T_c} - \frac{7\zeta(3)}{8\pi^2}\frac{\Delta^2}{T_c^2} - 2\int_\Delta^\infty\frac{dE}{E}\frac{\nu_{\text{qp}}(E)}{\nu_F}\delta n(E)\right)\Delta = 0. \quad (17)$$

To leading order in the gap change, we obtain the correction to the BCS gap

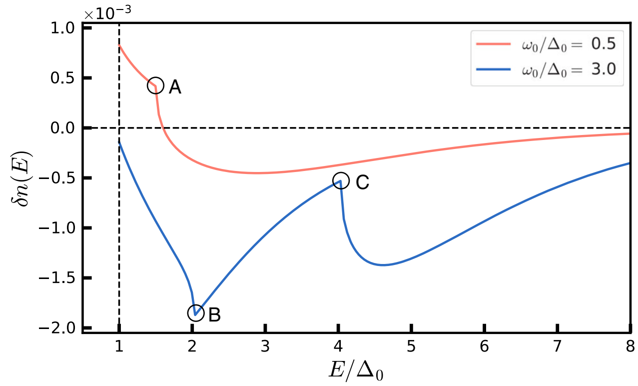

FIG. 2. Change in quasiparticle distribution function due to cavity photons. The two curves are at the same temperature ($T_{\rm cav}/T_{\rm qp} = 0.5$) but different cavity frequencies ω_0/Δ_0. For low cavity frequency (orange), the gap Δ is diminished due to an accumulation of cooler quasiparticles near the gap edge, due to a down-scattering of particles. For higher cavity frequency (blue), the recombination processes are more dominant and lead to a net reduction in quasiparticles, enhancing the gap Δ. The kink features labeled A and C reflect the onset of the term $f(\Omega, E)$ in Eq. (14), which is nonzero only for $E > \omega_0 + \Delta_0$. At higher cavity frequencies ($\omega_0 > 2\Delta_0$) an additional kink feature (located at B) emerges at $E = \omega_0 - \Delta_0$. For $E < \omega_0 - \Delta_0$, the term $f(-\Omega, E)$ (which represents the pair processes) contributes over the entire integration region of $\Omega > \omega_0$, while for $E > \omega_0 - \Delta_0$ the integral only captures some of the frequencies where this term contributes.

$$\frac{\delta\Delta}{\Delta_0} = -\frac{T_c}{T_c - T_{\rm qp}} \int_{\Delta_0}^{\infty} \frac{dE}{E} \frac{\nu_{\rm qp}(E)}{\nu_F} \delta n(E). \qquad (18)$$

This is plotted in Fig. 1(a) as a function of the cavity frequency ω_0 for different photon temperatures relative to the quasiparticle temperature $T_{\rm qp}$. The enhancement is ultimately driven by the enhanced BQP recombination rate which, for a cold photon reservoir serves to remove detrimental quasiparticles.

This can be seen explicitly in Fig. 2, which shows the change in the distribution function δn for two different cavity frequencies. When the cavity frequency is too low, scattering processes dominate and the photons cool the existing BQPs, leading to a buildup of particles near the gap edge. At higher cavity frequencies the pair processes dominate, leading to an enhancement as photons now cool the system by reducing the total number of harmful BQPs.

While the effect we predict here essentially relies on the cooling ability of the cold photon reservoir, we also remark that our formula for $\delta n(E)$, presented in Eq. (14), is valid for a wide variety of photon spectral functions. In particular, switching from a multimode planar cavity, where $J_{\rm cav}(\Omega) \sim \omega_0(1 + \omega_0^2/\Omega^2)\theta(\Omega - \omega_0)$ is roughly constant for $\Omega > \omega_0$, to a simpler single-mode cavity, where $J_{\rm cav} \sim \omega_0^2\{2\kappa/[(\Omega - \omega_0)^2 + \kappa^2]\}$ is peaked at the resonant frequency, will allow for an enhancement in $\delta\Delta$ even when the photon reservoir is hotter than the sample. This is explicitly

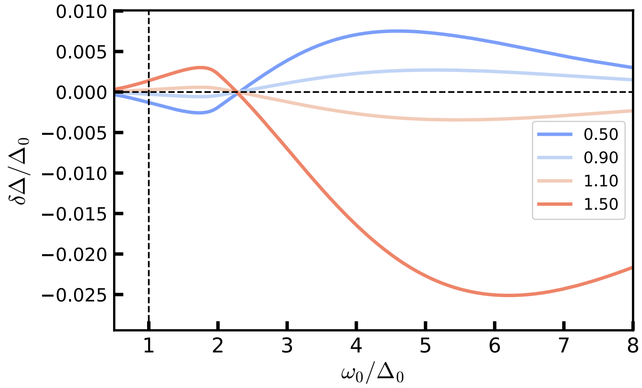

FIG. 3. Gap enhancement $\delta\Delta_0$ for a single-mode cavity, for both cold and hot photons. The y axis is determined by the overall scale $4\pi\alpha D\tau_{\rm in}T_c^2/((\pi\sqrt{3})^3c^2)X$; with the same values chosen for X and $\tau_{\rm in}$, $\tau_{\rm el}$, v_F/c as in Fig. 1. Curves are colored and labeled according to the ratio $T_{\rm cav}/T_{\rm qp}$, comparing the photon and quasiparticle temperatures. Here, the cavity width is held fixed at $1/2\tau_{\rm cav} = 10\omega_0$.

demonstrated in Fig. 3, where we plot $\delta\Delta$ against ω_0 for the case of a single-mode $J_{\rm cav}(\Omega)$. The enhancement in $\delta\Delta$ due to hot photons is now qualitatively similar to the classical Eliashberg effect, albeit with a narrow spectral broadening applied to the driving. For cold photons, the enhancement is similar to that seen in the multimode system and results from the photons cooling the sample via enhanced BQP recombination.

In conclusion, we have generalized the classical Eliashberg effect to include both quantum and thermal fluctuations, as realized by a thermal microwave resonator cavity. In the appropriate parameter regime, we show that the photonic reservoir can be used to drive the quasiparticles into a nonequilibrium state which enhances the superconducting gap Δ. In our calculation, we assumed that the cavity relaxation rate $\tau_{\rm cav}^{-1}$ was fast, allowing us to essentially ignore the dynamics and kinetics of the photons themselves. We should not expect this to remain the case when we go to the limit of a high-quality cavity, in which the relaxation rate $\tau_{\rm cav}^{-1}$ is no longer small compared to all the other energy scales in the problem. In the high-quality limit, a more elaborate treatment which treats the joint evolution of fermion-photon system is required. Though potentially much more complicated, the inclusion of photons as a participating dynamical degree of freedom may unveil many new and interesting phenomena. These range from the formation of new collective modes (including polaritons) [39,40], superradiant phases [24,41], and potentially photon-mediated superconductivity [42]. The prospect of exploring the full breadth of these joint matter-gauge systems is an exciting development in the fields of quantum optics and condensed matter physics.

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