Topological Amplification in Photonic Lattices

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We characterize topological phases in photonic lattices by unveiling a formal equivalence between the singular value decomposition of the non-Hermitian coupling matrix and the diagonalization of an effective Hamiltonian. Our theory reveals a relation between topological insulators and directional amplifiers. We exemplify our ideas with an array of photonic cavities which can be mapped into an AIII topological insulator. We investigate stability properties and prove the existence of stable topologically nontrivial steady-state phases. Finally, we show numerically that the topological amplification process is robust against disorder in the lattice parameters.

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Introduction.—Topological photonics builds on ideas from electronic band theory [1], such as the classification of topological phases based on symmetries [2–5]. After Haldane and Raghu's pioneering work [6], early realizations of topological phases were implemented in photonic spin Hall systems [7]. In the last years topological phases have been investigated in photonic lattices by breaking time-reversal symmetry with magnetic fields [8–12] or periodic drivings [13–18]. Analogous ideas have been explored in optomechanical systems [19–23] or even in purely vibronic or mechanical systems [24–26], as well as in spin-cavity setups [27,28].

Photonic lattices present distinctive features with respect to their electronic counterparts like dissipative decay or pumping (loss or gain), as well as coherent driving [29,30]. The breaking of time-reversal invariance that is a typical ingredient of topological phases leads to nonreciprocal photon transport [31] and topological quantum fluctuations [32]. Loss or gain in photonic lattices leads to non-Hermitian coupling matrices in which the direct application of topological insulator theory is highly nontrivial. Theoretical works have shown zero-energy edge states in non-Hermitian systems [33–36], and even extended topological band theory to non-Hermitian lattices [37,38]. Theoretical work has so far focused on topological properties of eigenvectors of non-Hermitian matrices. Recent experiments have detected photonic zero edge modes in the transmission properties of photonic lattices with a chiral symmetry originated by a bilattice structure [39–42], and shown nonreciprocal transmission induced by a synthetic magnetic field in an optomechanical system [43].

In this Letter we present a novel approach to the study of topological photonic phases that focuses on the singular value decomposition (SVD) of the non-Hermitian coupling matrix, H. Our approach establishes a link between the existence of nontrivial topological phases and the amplification of a coherent input signal. We show that the SVD of H is formally equivalent to the diagonalization of an effective Hamiltonian, $\mathcal{H}$, which possesses an intrinsic chiral symmetry and potentially also a set of topological protected edge states. The latter govern the response to a coherent drive and they lead to an exponential amplification effect. Our formalism allows us to exploit the existing classification of Hermitian lattices into symmetry classes [2–5] and predict parameter regimes in which a photonic lattice acts as an amplifier. We present an example consisting of an array of coupled photonic cavities and we find topologically nontrivial phases that are stable over a wide range or parameters. Numerical calculations show that our scheme is topologically protected against disorder. Finally, we discuss a physical implementation of our ideas in a photonic lattice subject to periodic drivings.

Dissipative photonic lattice.—We consider a dissipative lattice of local photonic modes with annihilation and creation operators a_j and $a_j^\dagger$, respectively, whose density matrix operator dynamics is described by ($\hbar = 1$)

$$\begin{aligned}\mathcal{L}(\rho) = &\sum_{j,l}\Gamma^{(\mathrm{d})}_{jl}(2a_j\rho a_l^\dagger - a_l^\dagger a_j\rho - \rho a_l^\dagger a_j)\\ &+\sum_{j,l}\Gamma^{(\mathrm{p})}_{jl}(2a_j^\dagger\rho a_l - a_l a_j^\dagger\rho - \rho a_l a_j^\dagger)\\ &- i\left[\sum_{j,l}G_{jl}a_j^\dagger a_l, \rho\right] - i\left[\sum_j(\epsilon_j^* a_j + \epsilon_j a_j^\dagger), \rho\right]. \quad (1)\end{aligned}$$

$\Gamma^{(d)}_{jl}$ and $\Gamma^{(p)}_{jl}$ are Hermitian matrices that describe collective photon losses and incoherent pumping, respectively. Diagonal terms, $\Gamma^{(d)}_{jj}$ and $\Gamma^{(p)}_{jj}$, appear naturally because of local photon losses or gain, being the latter induced by an active medium or illumination with an incoherent source. Nondiagonal dissipative terms ($j \neq l$) can be controlled by using additional degrees of freedom (for example, auxiliary modes [19,21–23]). G_{jl} are coherent couplings describing the tunneling of photons between cavities. Finally, the last term describes a resonant coherent drive with a site-dependent complex amplitude, ϵ_j. The latter can be induced by direct illumination of the photonic lattice with a coherent field.

We define the non-Hermitian matrix $H = \Gamma - iG$, where $\Gamma_{jl} = \Gamma^{(p)}_{jl} - \Gamma^{(d)}_{lj}$, which allows us to express the evolution of the coherences, $\alpha_j = \langle a_j \rangle$,

$$\dot{\alpha}_j = \sum_{l=1}^{N} H_{jl}\alpha_l + \epsilon'_j, \tag{2}$$

with $\epsilon'_j = i\epsilon_j$. Equation (2) is a closed set of linear equations that exactly describes the evolution of field coherences in the lattice. The steady-state solution $\alpha^{\rm ss}_j$ is found by solving $\dot{\alpha}_j = 0$, and it can be expressed in terms of the SVD, $H = USV^\dagger$, where U and V are unitary matrices and S is a diagonal matrix, $S_{nm} = s_n\delta_{nm}$, with $s_n \geq 0$,

$$\alpha^{\rm ss}_j = -\sum_{n,l} V_{jn} s_n^{-1} U^*_{ln}\epsilon'_l. \tag{3}$$

Small values of s_n give a larger contribution to $\alpha^{\rm ss}_j$.

Mapping to an effective Hamiltonian.—Our work relies on the observation that the SVD of H is equivalent to the diagonalization of an effective Hamiltonian $\mathcal{H}$,

$$\mathcal{H} = H \otimes \sigma^+ + H^\dagger \otimes \sigma^-, \tag{4}$$

where we have introduced ladder spin operators acting on an auxiliary spin-1/2, $\{|\uparrow\rangle, |\downarrow\rangle\}$. We define singular vectors $u^{(n)}$, $v^{(n)}$, corresponding to the columns of U and V, $u^{(n)}_j = U_{jn}$, $v^{(n)}_j = V_{jn}$,

$$\mathcal{H}(u^{(n)} \otimes |\uparrow\rangle \pm v^{(n)} \otimes |\downarrow\rangle) = \pm s_n(u^{(n)} \otimes |\uparrow\rangle \pm v^{(n)} \otimes |\downarrow\rangle). \tag{5}$$

The eigenvalues of $\mathcal{H}$ come in pairs $\pm s_n$, due to the chiral symmetry,

$$(\mathbb{1} \otimes \sigma_z)\mathcal{H}(\mathbb{1} \otimes \sigma_z) = -\mathcal{H}, \tag{6}$$

which exists by the very definition of $\mathcal{H}$, independently of the physical symmetries of the lattice.

The mapping $H \to \mathcal{H}$ allows us to use the theoretical machinery of topological band theory (TBT) [2–5] and classify topological steady states in translational invariant lattices. We consider periodic boundary solutions and express $\mathcal{H}$ in a plane-wave basis,

$$\mathcal{H}(\vec{k}) = \Gamma(\vec{k})\sigma_x + G(\vec{k})\sigma_y, \tag{7}$$

where $\Gamma(\vec{k})$ and $G(\vec{k})$ are real functions due to the Hermiticity of the coupling matrices. TBT relies on symmetry operators T (time reversal) and C (charge conjugation), written like $T = U_T K$, $C = U_C K$, where U_T, U_C are unitary matrices and K is the complex conjugation operator ($K^2 = 1$, $KiK = -i$). Condition $U_T U_C \propto \sigma_z$ must be fulfilled to account for the chiral symmetry expressed in Eq. (6). Time-reversal and/or charge conjugation symmetries are fulfilled if there exist unitary matrices U_T, U_C, such that $T\mathcal{H}(\vec{k})T^{-1} = \mathcal{H}(-\vec{k})$ and/or $C\mathcal{H}(\vec{k})C^{-1} = -\mathcal{H}(-\vec{k})$, respectively. We find the following possible symmetry classes [3]: (i) $\Gamma(\vec{k})^2 + G(\vec{k})^2 \neq \Gamma(-\vec{k})^2 + G(-\vec{k})^2 \to$ AIII class (no T, C symmetry). (ii) Vectors $(\Gamma(\vec{k}), -G(\vec{k}))$ and $(\Gamma(-\vec{k}), G(-\vec{k}))$ are related by a rotation with angle θ on the x-y plane $\to$ BDI class ($T^2 = C^2 = 1$) with $U_T = \exp(i\sigma_z\theta/2)$, $U_C = \exp[i\sigma_z(\theta+\pi)/2]$. (iii) $\Gamma(\vec{k}) = \Gamma(-\vec{k})$, $G(\vec{k}) = G(-\vec{k}) \to$ CI class ($T^2 = 1$, $C^2 = -1$) with $U_T = \sigma_x$, $U_C = \sigma_y$. This is the case of real couplings matrices, $\Gamma_{jl} = \Gamma_{lj}$, $G_{jl} = G_{lj}$. (iv) $\Gamma(\vec{k}) = -\Gamma(-\vec{k})$, $G(\vec{k}) = -G(-\vec{k}) \to$ DIII class ($T^2 = -1$, $C^2 = 1$) with $U_T = \sigma_y$, $U_C = \sigma_x$. This classification allows us to predict the existence or not of edge states. Whereas the chiral symmetry in Eq. (6) is always fulfilled by construction, the physical symmetries of the coupling matrices Γ, G, determine the symmetry class above. For example, if we have real dissipative couplings Γ_{jl}, but complex photon tunneling terms, we get $G(\vec{k}) \neq G(-\vec{k})$, such that the system falls into the AIII symmetry class.

Edge singular vectors and amplification.—Nontrivial topological properties of $\mathcal{H}$ have dramatic consequences in the steady state. In particular, TBT predicts the existence of zero-energy eigenstates of $\mathcal{H}$ in nontrivial topological phases (see, for example, Ref. [44]), which in turn implies the appearance of zero-singular values, s_{n_E} that are separated by a gap from the bulk singular values, $s_{n\neq n_E}$. The characterization of topological insulators in terms of symmetry classes (see Refs. [2–5]) can be used to predict the existence of those edge states. From TBT applied to $\mathcal{H}$, we also expect the emergence of right or left *edge* singular vectors, $u^{(n_E)}_j/v^{(n_E)}_j$, whose amplitude is localized at the edges of the lattice.

We assume for simplicity that there is a single zero-singular value $n_E = N$, corresponding to a single zero-energy state of $\mathcal{H}$ (as is the case in the one-dimensional model below). In a finite size lattice, this typically implies that $s_N \propto e^{-L/\chi}$, that is, the edge singular value decays exponentially with the length of the system L divided by a typical length, χ. The sum in Eq. (3) can thus be approximated by $\alpha^{\rm ss}_j \approx -\sum_l v^{(N)}_j s_N^{-1} u^{(N)*}_l \epsilon'_l$. That expression can be simplified further in translationally invariant systems, in which the parity inversion operator, Π, fulfills that $\Pi H \Pi = H^{\rm T}$. In this case we find the condition $V = \Pi U^*$, which leads to

$$\alpha_j^{ss} = -\Pi u_j^{(N)*} s_N^{-1} \sum_l u_l^{(N)*} \epsilon'_l. \tag{8}$$

We have arrived at our main result: the existence of edge states of $\mathcal{H}$ leads to the amplification of a coherent drive. Equation (8) implies that α_j^{ss} is amplified by a factor $1/s_N \propto e^{L/\chi}$, and it is also proportional to the overlap between ϵ'_j and $u_l^{(N)}$. Furthermore, field coherences α_j^{ss} are distributed following the parity inverted singular edge-state vector, $\Pi u_j^{(N)}$. This implies that amplification is a directional process triggered by a coherent drive in one of the system's edges and leading to large values of the field in the opposite edge. By measuring the coherent component of the field in the photonic cavity, for example, by means of homodyne detection methods, the existence of a singular edge state can be experimentally proved.

One-dimensional example: Nonreciprocal photonic chain.—We consider an array of cavities with nearest-neighbor dissipative couplings leading to

$$\begin{aligned} \Gamma_{jl} &= (\gamma_p - 2t_d)\delta_{jl} + t_d\delta_{l,j+1} + t_d\delta_{l,j-1}, \\ G_{jl} &= t_c e^{i\phi}\delta_{l,j+1} + t_c e^{-i\phi}\delta_{l,j-1}. \end{aligned} \tag{9}$$

This photonic lattice is related to the Hatano-Nelson model [45,46]. Dissipative terms with rate t_d couple nearest neighbors. The diagonal element $\Gamma = \gamma_p - 2t_d$ is parametrized in terms of γ_p, which determines the net incoherent pumping of photons into the lattice. Finally, coherent couplings couple nearest-neighbor sites with complex tunneling phase ϕ. The effective Hamiltonian in the plane-wave basis is

$$\mathcal{H}(k) = [\gamma_p - 2t_d + 2t_d\cos(k)]\sigma_x + 2t_c\cos(k+\phi)\sigma_y. \tag{10}$$

The cases $\phi = 0, \pi$ belong to the CI class. The generic case with $\phi \neq 0$ belongs to the AIII class and we can expect nontrivial topological phases to appear [5]. We characterize the properties of Hamiltonian (10) by using the winding number ν as a topological invariant [47]. For this we write $\mathcal{H}(k) = \Gamma(k)\sigma_x + G(k)\sigma_y$, such that $\nu = 1$ (nontrivial topological phase) if the ellipse formed by the two-dimensional vector $(\Gamma(k), G(k))$ encompasses the origin as k varies from 0 to 2π, and $\nu = 0$ otherwise. Without loss of generality we can assume $0 \leq \phi < \pi$. The condition for $\nu = 1$ reads

$$2t_d(1 - \sin\phi) < \gamma_p < 2t_d(1 + \sin\phi), \tag{11}$$

and $t_c \neq 0$. Since the case $\phi = 0$ and/or $\gamma_p = 0$ do not admit nontrivial solutions, our model requires complex photon tunneling couplings together with incoherent pumping in the original photonic lattice. A numerical calculation confirms the appearance of zero singular value modes, see Fig. 1.

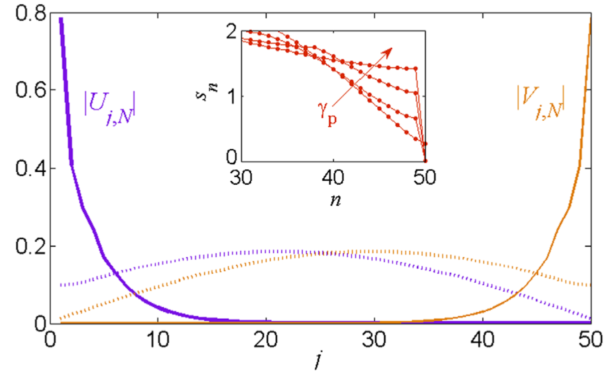

FIG. 1. Purple (orange) line is the absolute value of the $n = N$ right (left) singular vector of the 1D model (9) with $t_c = t_d = 1$, $\phi = \pi/3$, $N = 50$. Dotted lines: $\gamma_p = 0$ (topologically trivial case). Continuous lines: $\gamma_p = 1$ (topologically nontrivial case). Inset: Singular values for $\gamma_p = 0,\ 1,\ 1.5,\ 2.0$, showing the emergence of a zero singular value state at $n_E = N$.

We focus now in the case $\phi = \pi/2$, $t_d = t_c$, which can be mapped into the Su-Schriefer-Heeger (SSH) model [48]. Equation (11) implies that nontrivial topological phases exist if $0 < \gamma_p < 4t_d$. Here we can analytically find expressions for the edge state wave functions which can be written in terms of the localization length [49], $\xi^{-1} = -\ln|1 - [(\gamma_p)/(2t_d)]|$. In the following we assume the limit $N \gg \xi \gg 1$, to simplify the discussion, such that the edge singular vector is $u_j^{(N)} = \sqrt{2/\xi}e^{-j/\xi}$, with singular value $s_N = 2\gamma_p e^{-N/\xi}$, leading to

$$\alpha_j^{ss} = -\frac{1}{\gamma_p \xi} e^{N/\xi} e^{-(N+1-j)/\xi} \sum_l e^{-l/\xi} \epsilon'_l. \tag{12}$$

The signal is thus directionally amplified by coherently driving the left end of the chain such that photon density accumulates at the opposite end.

Stability phase diagram.—We address now the stability of the steady state. First, we define fluctuations $\bar{a}_j$ by the relation $a_j = \alpha_j^{ss} + \bar{a}_j$, and consider the correlation matrix in the steady state, $M_{jl} = \langle \bar{a}_j^\dagger \bar{a}_l \rangle$, which evolves like

$$\dot{M}_{jl} = \sum_{j'} H_{jj'}^* M_{j'l} + \sum_{l'} H_{ll'} M_{jl'} + 2\Gamma_{lj}^{(p)}. \tag{13}$$

Together with Eq. (2), Eq. (13) provides us with a complete characterization of the steady state. M_{jl} converges to a noninfinite value provided that $\Re(\lambda_n) < 0$. Whereas the steady state of the coherences can be analyzed by means of the SVD, see Eq. (3), fluctuations are directly governed by eigenvalues of the non-Hermitian matrix H.

Our one-dimensional example (9) can be exactly diagonalized [50] and we can seek topologically nontrivial stable regimes. The eigenvalues of H with periodic

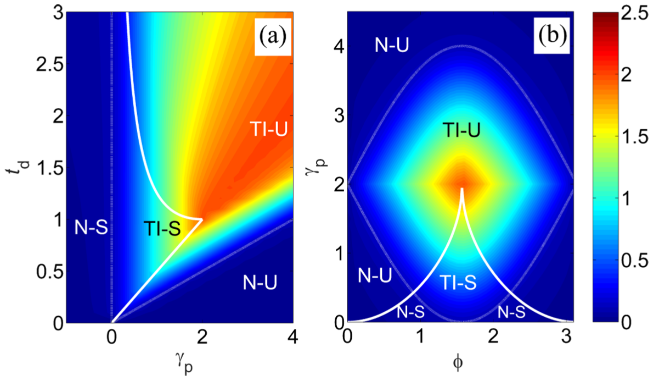

FIG. 2. Singular value gap, $\Delta_s = s_{N-1} - s_N$, for the model defined by Eq. (9), $N = 100$. Dashed lines separate topological insulator (TI) and normal (N) phases. The intersection, TI-S, is the region of stable nontrivial topological phases. Continuous lines separate stable (S) and unstable (U) phases. (a) $\phi = \pi/2$, $t_c = 1$ and different values of t_d and γ_p. (b) $t_d = t_c = 1$ and different values of ϕ and γ_p.

boundary conditions are $\lambda(k) = \gamma_p - 2t_d + 2t_d\cos(k) + 2it_c\cos(k+\phi)$. Condition $\Re[\lambda(k)] < 0$ requires that $\gamma_p < 0$, which is not compatible with the existence of a nontrivial topological phase. However the situation radically changes when we consider open boundary conditions. Here an exact solution is also available, leading to a set of $n = 1, \ldots, N$ eigenvalues of H,

$$\lambda_n = \gamma_p - 2t_d + 2\sqrt{(it_c e^{i\phi} - t_d)(it_c e^{-i\phi} - t_d)}\cos\left(\frac{n\pi}{N+1}\right). \quad (14)$$

Assume that $t_c = t_d$, then stable solutions exist if

$$\gamma_p < 2t_d(1 - \sqrt{\cos\phi}). \quad (15)$$

An overlapping region between the stable and nontrivial topological regimes defined by both conditions Eqs. (15) and (11) can be found as long as $|\cos(\phi)| < (-1+\sqrt{5})/2 \approx 0.62$. Our model thus requires a threshold minimum value of ϕ for stable nontrivial topological phases to exist. Numerical calculations in a wide range of parameters confirm the existence of stable topologically nontrivial phases in the steady-state phase diagram, see Fig. 2.

Topological protection.—We check the robustness of topological amplification by adding a diagonal disorder term to the Hermitian coupling matrix G, $\delta G_{j,l} = \delta\omega_j \delta_{j,l}$, where $\delta\omega_j$ are Gaussian random variables with standard deviation σ. We calculate the gap $\Delta_s = s_{N-1} - s_N$ for increasing number of lattices sites (Fig. 3), and find that the topological phase is robust for a wide range of σ.

Physical implementations.—Our ideas can be implemented by using schemes for nonreciprocal transport and directional amplification [10,12,35,51–55]. Complex

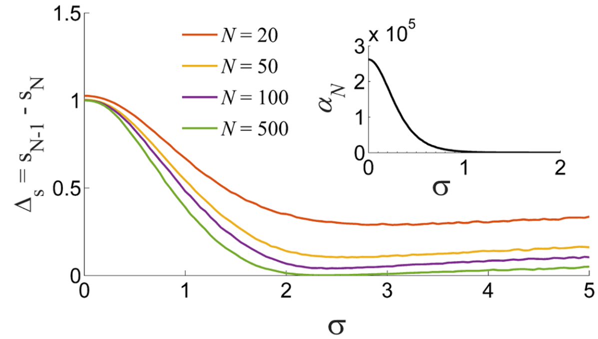

FIG. 3. Gap between the second and lowest singular value of H given by Eq. (9) with $t_c = t_d = \gamma_p = 1$, as a function of disorder strength, averaged over $10^5/N$ realizations of disorder. Inset: Field amplitude at site $j = N$ induced by a coherent drive at $j = 1$ with $\epsilon_1 = 1$ in a chain with $N = 100$.

photon tunneling can be induced by using periodic modulations in circuit QED [14,56–59] or even vibronic lattices [24,25]. This approach allows for smaller system sizes than, e.g., the use of magnetic fields.

We propose an implementation motivated by the superconducting circuit setup presented in Ref. [16]. We describe below the main ideas—technical details can be found in Ref. [60].

The main system is a chain of N cavities (mode operators a_j) coupled to an auxiliary chain (modes b_j). Cavities are arranged in the ladder configuration of Fig. 4. The noninteracting cavity system is described by

$$H_0 = \sum_{j=1}^{N} \omega_j a_j^\dagger a_j + \sum_{j=1}^{N+1} \omega_j b_j^\dagger b_j. \quad (16)$$

Nearby cavity frequencies are separated by $\Delta\omega$, such that $\omega_j = \omega + \Delta\omega(j-1)$. The frequency gradient is used to break time-reversal symmetry. Both main and auxiliary cavities are subjected to photon leakage with rates κ_a and κ_b, respectively.

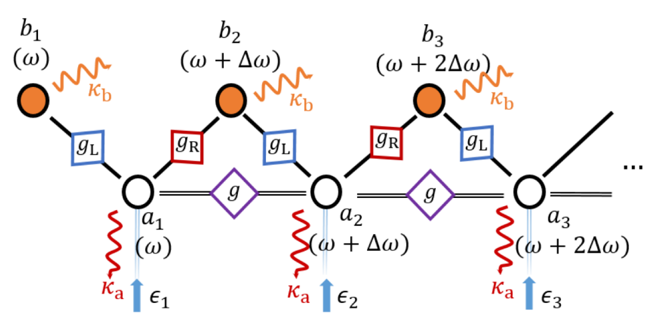

FIG. 4. Scheme for a physical implementation of the 1D topological amplifier (9). White circles: main local photonic modes. Orange circles: auxiliary fast decaying modes for reservoir engineering.

Complex coherent couplings are induced by periodic modulations,

$$H_c(t) = g(t)\sum_{j=1}^{N}(a_j + a_j^\dagger)(a_{j+1} + a_{j+1}^\dagger), \qquad (17)$$

with $g(t) = g_0 \cos(\Delta\omega t + \phi_d)$. In the interaction picture with respect to H_0 and in a rotating wave approximation (RWA) valid if $\Delta\omega \gg g_0$, we obtain the photon tunneling terms of Eq. (9) with $t_c = g_0/2$ and $\phi = \phi_d$.

The auxiliary cavities will provide us with collective incoherent pumping. We consider the couplings

$$H_{\mathrm{aux}}(t) = \sum_{j=1}^{N} g_{\mathrm{R},j}(t)(a_j + a_j^\dagger)(b_{j+1} + b_{j+1}^\dagger) + \sum_{j=1}^{N} g_{\mathrm{L},j}(t)(a_j + a_j^\dagger)(b_j + b_j^\dagger), \qquad (18)$$

with periodic modulations $g_{\mathrm{R},j}(t) = \bar{g}_0 \cos[(\omega_j + \omega_{j+1})t]$, $g_{\mathrm{L},j}(t) = \bar{g}_0 \cos(2\omega_j t)$. In the interaction picture with respect to H_0, and in a RWA valid if $\bar{g}_0 \ll \omega_j$, we get

$$H'_{\mathrm{aux}} \underset{\mathrm{RWA}}{=} \frac{\bar{g}_0}{2}\sum_{j=1}^{N}[a_j(b_j + b_{j+1}) + \mathrm{H.c.}]. \qquad (19)$$

Consider now that auxiliary b modes are very fast decaying ($\bar{g}_0 \ll \kappa_b$), such that we can adiabatically eliminate them [61] and get dissipative couplings $\Gamma_{jl} = (2t_d - \kappa_a)\delta_{jl} + t_d\delta_{l,j+1} + t_d\delta_{l,j-1}$, with $t_d = (\bar{g}_0)^2/(4\kappa_b)$ (see Ref. [60] for details). We thus obtain a collective incoherent pumping induced by virtual photons being emitted into the auxiliary cavities. Finally, we can obtain the dissipative coupling matrix in the form of Eq. (9) by defining the net pumping rate $\gamma_p = 2t_d - \kappa_a$, such that $\gamma_p = 0$ is precisely the value at which gain and loss are balanced.

Topologically phases can be detected, for example, by adding a coherent drive at one edge and measuring the coherences $\langle a_j \rangle$ in the steady state by homodyne detection. Exponential amplification would signal the existence of a singular edge state.

Conclusions and outlook.—This work presents a connection between directional amplification and topological insulator theory. Our approach leads to a classification of topological phases of non-Hermitian matrices that is directly connected to applications of photonic lattices as amplifiers. We have presented an example that could be implemented with circuit QED setups. In the future we aim to investigate many-body effects [62], lasing phases [63], disorder and/or long range interactions and couplings [64].

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