Heralded Bell State of Dissipative Qubits Using Classical Light in a Waveguide

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Maximally entangled two-qubit states (Bell states) are of central importance in quantum technologies. We show that heralded generation of a maximally entangled state of two intrinsically open qubits can be realized in a one-dimensional (1D) system through strong coherent driving and continuous monitoring. In contrast to the natural idea that dissipation leads to decoherence and so destroys quantum effects, continuous measurement and strong interference in our 1D system generate a pure state with perfect quantum correlation between the two open qubits. Though the steady state is a trivial product state that has zero coherence or concurrence, we show that, with carefully tuned parameters, a Bell state can be generated in the system's quantum jump trajectories, heralded by a reflected photon. Surprisingly, this maximally entangled state survives the strong coherent state input—a classical state that overwhelms the system. This simple method to generate maximally entangled states using classical coherent light and photon detection may, since our qubits are in a 1D continuum, find application as a building block of quantum networks.

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Quantum entanglement between two qubits is essential for quantum computing and indeed for quantum information processing more generally [1]. Bell states, which are maximally entangled two-qubit states, have perfect quantum correlations and are therefore especially important. The most common way to generate Bell states is to measure a joint property of two components and has been realized in several systems including, for example, trapped atoms, NV centers, quantum dots, and superconducting qubits (for reviews see Refs. [2–4]). Finding a variety of ways of making Bell states, particularly ones that use different resources, is important in advancing quantum information in new directions. Since it is natural to suppose that classical resources decrease the coherence needed for entanglement, it is particularly interesting to produce Bell states using classical resources while reducing the quantum input to a minimum.

A new platform named waveguide QED has recently been realized in which qubits strongly couple to photons confined in a one-dimensional (1D) waveguide [5–9]. This platform has potential applications in integrating quantum components into complex systems, such as quantum networks [10,11]. In this Letter, we introduce a novel way of generating a Bell state of two qubits coupled to a 1D waveguide: *classical* light plus photon detection leads to entanglement generation heralded by a reflected photon. Previous results concerning entanglement in waveguide QED [12–27] have shown through analysis of the concurrence, entangled state population, or scattered wave function that a degree of entanglement between qubits can be generated using the effective interactions mediated by the waveguide. We show that, under continuous monitoring, *maximal entanglement* can be generated using the strong interference of photons in 1D and photon detection. This maximally entangled state is *heralded* by detection of a reflected photon, which makes it attractive for potential applications.

The driving in our system is a strong coherent state—a classical state that overwhelms the whole system. But surprisingly the Bell state survives this classical component. What is more surprising and intriguing is that the steady state of the qubits is a trivial product state, which has no coherence or concurrence. The continuous monitoring unravels this trivial state such that its trajectories are nontrivial. This "magical" unravelling provides a particularly sharp illustration of the significance of the information gained about quantum systems by measurement, which has wide-reaching implications for advancing the understanding of quantum information and open quantum system.

Seemingly trivial steady state.—The system we want to study, shown in Fig. 1, consists of two identical qubits coupled to a 1D waveguide under resonant driving by a coherent state. The input coherent state $|\alpha\rangle$ has frequency k, and the qubits with frequency $\omega_{\rm eg} = k$ and raising (lowering) operators $\sigma_i^{\pm}$ $(i = 1, 2)$ are separated by distance L. After tracing out the waveguide degrees of freedom making the Markov and rotating wave approximations, the two qubits can be described by a master equation of Lindblad form (see, e.g., Refs. [12,13,17,28,29])

$$\frac{d}{dt}\rho = i[\rho, H_{\rm d} + H_{\rm qq}] + \sum_{i,j=1,2} \Gamma_{ij}\left(\sigma_i^- \rho \sigma_j^+ - \frac{1}{2}\{\rho, \sigma_i^+ \sigma_j^-\}\right). \tag{1}$$

The coherent evolution here has two parts, one describing the drive $H_d = g\alpha(\sigma_1^+ + \sigma_2^+ e^{ikL}) + \text{H.c.}$ with coupling strength g, and the other $H_{qq} = \Omega(\sigma_1^+\sigma_2^- + \sigma_2^+\sigma_1^-)$ describing a waveguide-mediated qubit-qubit interaction of strength $\Omega = 2\pi g^2 \sin(kL)$. In the incoherent Lindblad part, the individual decay rate of each qubit is $\Gamma_{11} = \Gamma_{22} \equiv \Gamma = 4\pi g^2$, and the cooperative decay $\Gamma_{12} = \Gamma_{21} = 4\pi g^2 \cos(kL)$ is a waveguide-mediated incoherent coupling. The validity of the rotating wave and Markov approximations requires $\Gamma \ll \omega_{eg}$ and $\Gamma L \ll 1$; thus, $kL \sim 1$ is clearly in the regime of validity.

In the strong driving limit $\alpha \gg g$ (driving power $\gg \Gamma$), by letting $d\rho/dt = 0$ we obtain a trivial steady state in which the density matrix is an identity matrix. We consider $kL \neq n\pi$ where n is an integer, in which case the steady state $\rho_\infty = (|ee\rangle\langle ee| + |eg\rangle\langle eg| + |ge\rangle\langle ge| + |gg\rangle\langle gg|)/4$ is an identity matrix in the space spanned by $\{|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle\}$. [For $kL = n\pi$ where n is an even (odd) integer, the steady state starting from the ground state is an identity matrix in the space spanned by $\{|ee\rangle, |gg\rangle, |S\rangle(|A\rangle)\}$ where the symmetric and antisymmetric states are $|S\rangle(|A\rangle) \equiv (|eg\rangle \pm |ge\rangle)/\sqrt{2}$.] This density matrix can be written simply as $\rho_\infty = (\mathbb{1}_1 \otimes \mathbb{1}_2)/4$ where $\mathbb{1}_i$ is the identity matrix in the Hilbert space of ith qubit. Therefore, the steady state has *no entanglement* (concurrence $\mathcal{C} = 0$ [30]) since it can be written as a product state and *no coherence* since there is no off-diagonal element. The qubit-qubit interaction mediated by the waveguide usually exploited to generate entanglement (see, e.g., Ref. [13]) is completely washed out by the classical driving and dissipation. However, the system's trajectories can be nontrivial, as we now show.

Entanglement within trajectories.—Our description in terms of a master equation is similar to that used for open quantum systems [31]. In that context, the interaction between system and environment typically generates entanglement between them, and then a trace over the environmental degrees of freedom yields a mixed state for the system. During the partial trace, some information is lost as attested by the nonzero von Neumann entropy of the mixed state. However, under continuous monitoring, a mixed state can be unraveled as an ensemble of pure states (quantum trajectories) [32–34]. Unlike the mixed state, this ensemble gives a complete description of the open quantum system under continuous monitoring.

Within the quantum trajectory description, mixed state entanglement can be defined without ambiguity as the average of pure state entanglement as follows [35]. Denote the ensemble of trajectories by $\{\sqrt{p_i}|\psi_i\rangle\}$, where p_i is the probability of trajectory $|\psi_i\rangle$ being detected, and form $\rho = \sum_i p_i|\psi_i\rangle\langle\psi_i|$. If we divide the open system into subsystems A and B, the entanglement between A and B within the ith trajectory is defined through the usual von Neumann entropy as $S_i = -\text{Tr}(\rho_i^A \log_2 \rho_i^A)$ with $\rho_i^A = \text{Tr}_B(|\psi_i\rangle\langle\psi_i|)$. The entanglement in the ensemble is defined naturally as the average, $\bar{S} \equiv \sum_i p_i S_i$.

It has been shown that measuring different quantities leads to different amounts of entanglement by unraveling with different ensembles of trajectories [35–39]. For example, the trivial steady state above, $\rho_\infty = (\mathbb{1}_1 \otimes \mathbb{1}_2)/4$, can be unraveled nontrivially as either the ensemble $\{\frac{1}{2}|\Phi^+\rangle, \frac{1}{2}|\Phi^-\rangle, \frac{1}{2}|\Psi^+\rangle, \frac{1}{2}|\Psi^-\rangle\}$ or $\{\frac{1}{2}|gg\rangle, \frac{1}{2}|ee\rangle, \frac{1}{2}|\Psi^+\rangle, \frac{1}{2}|\Psi^-\rangle\}$, where $|\Phi^\pm\rangle = (|gg\rangle \pm |ee\rangle)/\sqrt{2}$ and $|\Psi^\pm\rangle = (|ge\rangle \pm |eg\rangle)/\sqrt{2}$ are the four conventional Bell bases. The former ensemble yields $\bar{S} = 1$ while the latter gives $\bar{S} = 1/2$ even though they both produce the seemingly trivial mixed state ρ_∞.

Waveguide mediated collective jumps.—Returning to our system, we suppose that photon counting measurements are performed at both ends of the waveguide, as shown in Fig. 1. As shown in our previous work [29], the photon detections at the left and right end can be described as discrete changes (quantum jumps) of quantum trajectories through the jump operators J_L^- and J_R^- defined as

$$J_L^- \equiv \sqrt{2\pi}g(\sigma_1^- + \sigma_2^- e^{ikL}),$$
$$J_R^- \equiv \sqrt{2\pi}g(\sigma_1^- + \sigma_2^- e^{-ikL}) + i\frac{\alpha}{\sqrt{2\pi}}. \tag{2}$$

Note that J_R^- incorporates interference between the driving field α and the qubit emission. The master equation for the two qubits, Eq. (1), can be rewritten in an equivalent form as

$$\frac{d}{dt}\rho = i[\rho, H_h] + \sum_{i=R,L} J_i^- \rho J_i^+ - \frac{1}{2}\{\rho, J_i^+ J_i^-\}, \tag{3}$$

where $H_h = H_{qq} + \frac{1}{2}g\alpha(\sigma_1^+ + \sigma_2^+ e^{ikL}) + \text{H.c.}$ [29]. Based on the jump operator [Eq. (2)] that corresponds to photon detection, quantum jump formalism [34] then yields quantum trajectories described by the stochastic Schrödinger equation (SSE)

$$d|\psi(t)\rangle = \sum_{i=L,R} dN_i(t)\left(\frac{J_i^-}{\sqrt{\langle J_i^+ J_i^-\rangle}} - 1\right)|\psi(t)\rangle + \left(\frac{(1 - idtH_{\text{eff}})}{|(1 - idtH_{\text{eff}})|\psi(t)\rangle|} - 1\right)|\psi(t)\rangle, \tag{4}$$

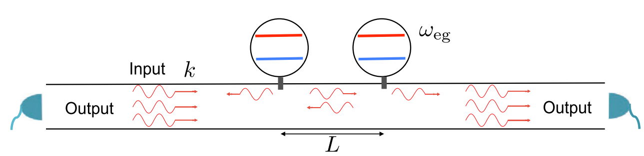

FIG. 1. Schematic of two qubits coupled to a 1D waveguide. We have a right-going coherent state as input from the left end. Transmitted and reflected photons are measured using photon counting detection at the right and left end, respectively.

where $dN_{\mathrm{i}}(t) = 0, 1$ describes the stochastic process of a photon being detected with probability $\langle dN_i(t)\rangle = dt\langle\psi(t)|J_{\mathrm{i}}^{+}J_{\mathrm{i}}^{-}|\psi(t)\rangle$, dt is the time step, and $H_{\mathrm{eff}} \equiv H_{\mathrm{h}} - \frac{1}{2}\sum_{i=\mathrm{R,L}} J_i^{+}J_i^{-}$ is the non-Hermitian effective Hamiltonian describing the segments of continuous evolution.

It is intriguing that the left jump operator here, $J_{\mathrm{L}}^{-} \sim (\sigma_1^{-} + e^{ikL}\sigma_2^{-})$, can produce a jump $J_{\mathrm{L}}^{-}|ee\rangle \to (|ge\rangle + e^{ikL}|eg\rangle)$ that yields a maximally entangled state. This derives from the fact that detection of a reflected photon necessarily comes from a coherent superposition of the emission from both qubits, i.e., $|ee\rangle \to |ge\rangle$ and $|ee\rangle \to |eg\rangle$. This route to entanglement generation is in the same spirit as the scheme proposed in Ref. [40]. Note the following two requirements. (i) To realize this jump process, the jump must start from $|ee\rangle$ or superpositions of $|ee\rangle$ and eigenstates of J_{L}^{-} with vanishing eigenvalues. (ii) To make this maximally entangled state available for exploitation, it must not be destroyed for some time by the dynamics, such as the continuous evolution or subsequent jumps. We now show that when $kL = (n + 1/2)\pi$ and the driving $|\alpha\rangle$ is strong, these two requirements can be met.

Hybridizing jumps and state diffusion.—In the strong driving limit $\alpha/g \to \infty$, each right jump leads to an infinitesimal change of the wave function, since the right jump operator J_{R}^{-} is dominated by the constant term. However, within a time step dt there will be infinitely many right jumps due to the large photon flux given by the strong coherent state. Therefore, the quantum trajectory will be continuous, as in classic homodyne detection [34] when left jumps are absent and the photon current is measured. Then, the number of right jumps detected in a time step, denoted $dN_{\mathrm{R}}(t)$, can be written as

$$dN_{\mathrm{R}}(t) = \langle dN_{\mathrm{R}}(t)\rangle + \frac{|\alpha|}{\sqrt{2\pi}}d\xi(t), \tag{5}$$

where $d\xi(t)$ is stochastic noise. Since the coherent state dominates the signal detected, Gaussian noise with $\langle d\xi(t)\rangle = 0$ and $\langle d\xi(t)^2\rangle = dt$ is a good approximation. By expanding in $1/|\alpha|$, the SSE Eq. (4) is simplified to (for details, see Supplemental Material [41])

$$d|\tilde{\psi}(t)\rangle = dt\left(-i(g\alpha c^{+} + g\alpha^{*}c^{-} + H_{qq}) - ie^{-i\theta}\pi g^2\langle(ie^{i\theta}c^{+} - ie^{-i\theta}c^{-})\rangle c^{-} - \pi g^2 c^{+}c^{-} - \frac{1}{2}J_{\mathrm{L}}^{+}J_{\mathrm{L}}^{-}\right)|\tilde{\psi}(t)\rangle$$
$$+ d\xi(t)(-ie^{-i\theta}\sqrt{2\pi}gc^{-})|\tilde{\psi}(t)\rangle + dN_{\mathrm{L}}(t)\left(\frac{J_{\mathrm{L}}^{-}}{\sqrt{\langle J_{\mathrm{L}}^{+}J_{\mathrm{L}}^{-}\rangle}} - 1\right)|\tilde{\psi}(t)\rangle, \tag{6}$$

where $|\tilde{\psi}\rangle$ is an unnormalized wave function, $\alpha = |\alpha|e^{i\theta}$, $\langle\cdot\rangle = \langle\psi|\cdot|\psi\rangle$, and $c^{\pm} \equiv (\sigma_1^{\pm} + e^{\pm ikL}\sigma_2^{\pm})$ is the operator part of J_{R}^{-} such that $J_{\mathrm{R}}^{-} = \sqrt{2\pi}gc^{-} + i\alpha/\sqrt{2\pi}$. If the left jumps are dropped, note that this SSE becomes a quantum state diffusion equation with fluctuations given by a Weiner process $d\xi(t)$.

Heralded Bell state.—We wish to focus on the case $kL = (n + 1/2)\pi$, where n is an even (odd) integer, and define two maximally entangled states $|\pm i\rangle \equiv (|ge\rangle \pm i|eg\rangle)/\sqrt{2}$ (Bell states). Then, the operator c^{-} (J_{L}^{-}) is a lowering operator in the space spanned by $\{|ee\rangle, |-i\rangle, |gg\rangle\}$ while J_{L}^{-} (c^{-}) is a lowering operator in the space spanned by $\{|ee\rangle, |+i\rangle, |gg\rangle\}$. In the following, we let $kL = \pi/2$; i.e., the qubit separation is a quarter wavelength. For other even n, the conclusions are the same; for odd n, they hold upon switching the roles of $|\pm i\rangle$.

The energy level diagram for $kL = \pi/2$ is shown in Fig. 2(a). The quantum diffusion process given by the operator $c^{\pm}$ causes $|gg\rangle \leftrightarrow |-i\rangle \leftrightarrow |ee\rangle$, and the left jump process causes $|ee\rangle \to |+i\rangle \to |gg\rangle$. Thus, the two maximally entangled states $|\pm i\rangle$ are dynamically separated. The ground state of the qubits $|gg\rangle$ will be driven to the excited state $|ee\rangle$, from which there is a finite probability for a left jump. In that case, the two qubits jump to the maximally entangled state $|+i\rangle$, while at the same time a left-going (reflected) photon is detected. The qubits will stay in $|+i\rangle$ until a second left jump occurs, taking the qubits back to $|gg\rangle$. The whole process then repeats. Thus, there are repeated windows of maximally entangled state $|+i\rangle$, whose lifetime is $1/\langle +i|J_{\mathrm{L}}^{+}J_{\mathrm{L}}^{-}|+i\rangle = 1/\Gamma$, each heralded by a reflected photon.

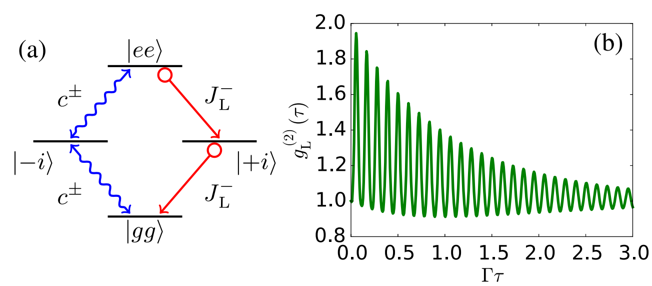

FIG. 2. (a) Energy level diagram for $kL = \pi/2$. Red and blue arrows represent left jumps and driving, respectively, $|\pm i\rangle \equiv (|ge\rangle \pm i|eg\rangle)/\sqrt{2}$, J_{L}^{-} is the left jump operator, and $c^{\pm}$ comes from the right jump operator J_{R}^{-}. The effective qubit-qubit interaction H_{qq} is suppressed by the strong driving. (b) Second order correlation function for the reflected photons calculated from input-output theory. (Parameters: $kL = \pi/2$, $\alpha = 100$.)

An example trajectory is shown in Fig. 3(a) for $\alpha = 100$. There are clearly time windows of maximal entanglement, whose birth and death are heralded by the detection of reflected photons. The populations of the energy levels show that the qubits are in the $|+i\rangle$ state in the maximal entanglement windows and are dynamically decoupled from the other three levels in these windows. The small deviations from maximal entanglement that can be seen are due to the effective qubit-qubit interaction term $H_{\rm qq}$ that exchanges excitations between the two qubits and so leads to the process $|+i\rangle \leftrightarrow |-i\rangle$. This term ($\sim g^2$) is suppressed by the strong driving term ($\sim g|\alpha|$) as shown in the Supplemental Material [41], which is the reason why strong driving is needed. Outside the windows of maximal entanglement, the dynamics is dominated by Rabi oscillations in a three-level system with fluctuations coming from the Weiner process.

This special dynamics is encoded in the behavior of the second-order correlation function $g_{\rm L}^{(2)}(\tau)$ of the reflected light, shown in Fig. 2(b). $g_{\rm L}^{(2)}$ starts at 1 and then oscillates at the Rabi frequency with an envelope that decays in a time of order Γ^{-1}. It is bounded by 2 and reaches maximal points when $|gg\rangle$ is driven to $|ee\rangle$ (see Supplemental Material [41] for details).

When parameters are detuned from their ideal values (either k or L), the dynamics becomes more complicated than shown in Fig. 2(a), with for instance a (weak) direct connection between the left and right sides. For small detuning, the dynamics will be qualitatively similar; we leave a quantitative study of these features to future work.

Imperfect photon detection.—To understand the role and importance of the information gained by observing a quantum system, we introduce information loss through imperfect photon detection. The effect of such loss is modeled using the jump operators $\sqrt{\eta_i}J_i^-$, where $i = \mathrm{R,L}$ and $\eta_i < 1$ is the efficiency of photon detection [34]. Then the SSE (6) becomes a stochastic master equation (SME) (for details see the Supplemental Material [41]),

$$d\tilde{\rho}_s(t) = dt\left(i[\tilde{\rho}_s, H_{qq} + g\alpha^* c^- + \mathrm{H.c.}] + (1-\eta_{\rm L})J_{\rm L}^-\tilde{\rho}_s J_{\rm L}^+ + 2\pi g^2 c^-\tilde{\rho}_s c^+ - \frac{1}{2}\{\tilde{\rho}_s, J_{\rm L}^+J_{\rm L}^- + 2\pi g^2 c^+c^-\}\right)$$
$$+ d\xi(t)\sqrt{\eta_{\rm R}}(-ie^{-i\theta}\sqrt{2\pi}gc^-\tilde{\rho}_s + \mathrm{H.c.}) + dN_{\rm L}(t)\left(\frac{J_{\rm L}^-\tilde{\rho}_s J_{\rm L}^+}{\mathrm{Tr}[J_{\rm L}^-\tilde{\rho}_s J_{\rm L}^+]} - \tilde{\rho}_s\right), \qquad (7)$$

for trajectories of mixed states $\tilde{\rho}_s$ [42] due to loss of information about the system. The probability of photon detection now becomes $\langle dN_{\rm L}\rangle = \eta_{\rm L} dt \mathrm{Tr}[\rho_s J_{\rm L}^+ J_{\rm L}^-]$ in terms of the normalized density matrix $\rho_s = \tilde{\rho}_s/\mathrm{Tr}[\tilde{\rho}_s]$. Other information loss mechanisms, such as the coupling of the qubits to channels other than the waveguide, can be taken into account by simply adding additional Lindbladian dissipators to Eq. (7); however, this will produce no qualitative change in our results and so is left to the interested reader.

We quantify the entanglement for each mixed trajectory using the entanglement of formation $S_{\rm F}$ [30]. To define $S_{\rm F}$, consider a "purification" of a mixed state, by which is meant a pure state of the system plus environment that yields the known mixed state through partial trace. The entanglement entropy of a purification is simply that of the two qubits, $\bar{S}$, conditioned on measurement of the environment (photon detection here). The entanglement of formation $S_{\rm F}$ is the minimum entanglement entropy for all possible purifications of a mixed state, and so gives a lower bound on the entanglement contained in a mixed trajectory. A subtle point should be emphasized here: information gained about a quantum system constrains possible purifications and therefore gives a different lower bound. For our system (assume $\eta_i = \eta$ for now), for example, if $\eta = 0$, i.e., no photons are measured so no information is gained, Eq. (7) becomes Eq. (3) whose steady state is $(\mathbb{1}_1 \otimes \mathbb{1}_2)/4$ and $S_F = 0$. As η increases, more information is gained and the number of possible purifications decreases. When $\eta = 1$, Eq. (7) becomes Eq. (6), which becomes the only way to purify given the physical setup.

An example trajectory for $\eta = 0.95$ is shown in Fig. 3(b). As can be seen, the information loss leads to very different behavior. In the first window, the entanglement S_F and the $|+i\rangle$ population do not jump up to 1 as for perfect photon detection. This is because there is a possibility that photons have been emitted without being detected, as shown by the term $(1-\eta_{\rm L})J_{\rm L}^-\tilde{\rho}_s J_{\rm L}^+$ in Eq. (7), which makes the trajectory be in the space spanned by all four energy levels. When a photon is detected, the trajectory is projected to a space spanned by $\{|+i\rangle, |gg\rangle\}$ through processes $|ee\rangle \to |+i\rangle$ and $|+i\rangle \to |gg\rangle$. In the third window, although the qubits jump to $|+i\rangle$, its population keeps decreasing with time. This is because of the undetected decaying process $|+i\rangle \to |gg\rangle$.

Only one detector needed.—Even though the scheme proposed here is not robust against photon detection loss at the left end, it works independently of the photon detection efficiency at the right end. It can be seen in Eq. (7) that, as long as $\eta_{\rm L} = 1$, the continuous part describes time evolution of a mixed state in the space

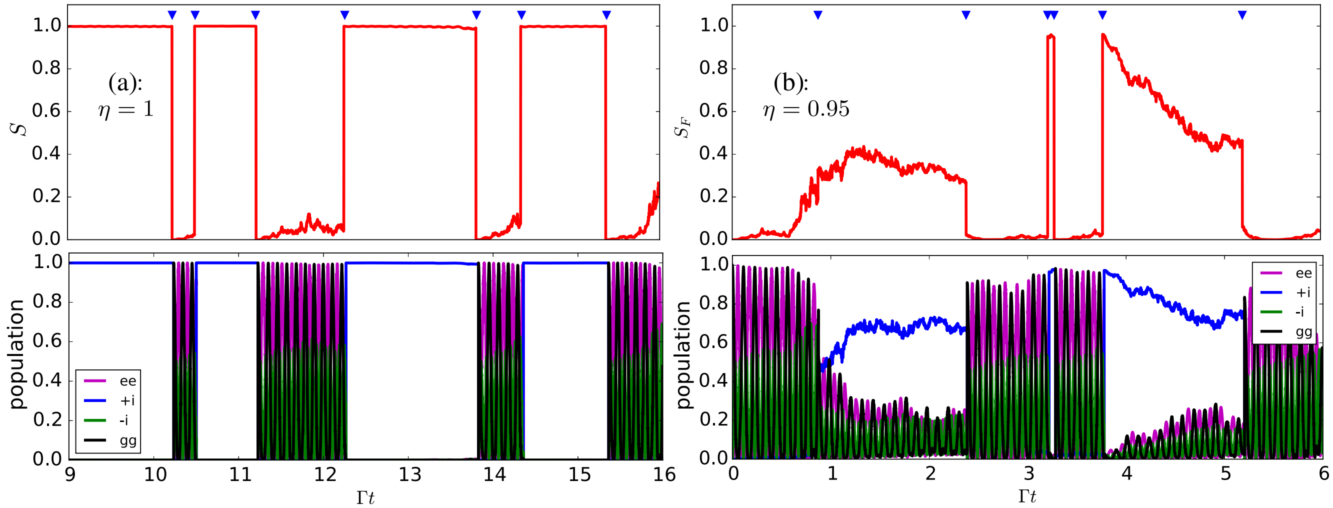

FIG. 3. Example trajectories of entanglement (first row) and populations (second row) for (a) perfect photon detection and (b) lossy photon detection with efficiency $\eta_{\mathrm{i=L,R}} = \eta = 0.95$. The entanglement for pure states in (a) and mixed states in (b) is quantified using the von Neumann entropy S and the entanglement of formation S_F, respectively. The times at which quantum jumps occur are marked with blue triangles. Longer trajectories, from $\Gamma\tau = 0$ to 20, are shown in the Supplemental Material [41]. (Parameters: $kL = \pi/2$, $\alpha = 100$, qubits initially in the ground state $|gg\rangle$.)

spanned by $\{|ee\rangle, |-i\rangle, |gg\rangle\}$ and the jump part still describes detection of reflected photons, which project the $|ee\rangle\langle ee|$ component onto a pure state $|+i\rangle$ as shown in Fig. 2(a) [41]. That is, the scheme still works even without photon detection at the right end ($\eta_{\mathrm{R}} = 0$).

Conclusion and outlook.—In summary, we have shown that for two qubits coupled to a waveguide separated by $(n/2 + 1/4)$ wavelengths, a heralded Bell state can be generated using classical driving and photon counting detection. Although the steady state is a trivial product state, the continuous monitoring unravels the master equation such that a Bell state is dynamically decoupled from the other three levels during the continuous part of the evolution. Discrete jumps, heralded by detections of reflected photons, project the wave function onto the Bell state. This physical example that nonentangled mixed states can have entangled trajectories calls for careful usage of commonly used entanglement measures, such as concurrence, especially when measurement is present. Since the qubits are already in the continuum and coupled to itinerant photons, the method presented here will have particular application in integrating quantum components into complex systems [10,11].

The importance of the information gained by observing a quantum system is shown by introducing information loss caused by imperfect photon detections. A small information loss causes the quantum entanglement to behave very differently. This implies that methods to stabilize the Bell state, such as bath engineering [43], are needed in applications.

In this Letter, the Markov approximation has been applied, which is valid when the qubit separation is not too large. It will be interesting to explore in the future the effects caused by time delayed feedback in the non-Markovian regime [16,17,44–50], which is important for the generation of remote entanglement between qubits.

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[41] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.122.140502 for (i) derivation of the stochastic Schrödinger equation [Eq. (6)], (ii) analysis of the qubit-qubit interaction (iii) analysis of the second-order correlation function shown in Fig. 2(b), (iv) derivation of the stochastic master equation [Eq. (7)], (v) analysis of the single detector case, and (vi) some trajectories to complement those shown in Fig. 3.
[42] Note that for perfect photon detection ($\eta = 1$), when unnormalized, our notation $\tilde{\rho_s} = c(t)|\tilde{\psi}\rangle\langle\tilde{\psi}|$, where $c(t) \neq 1$ due to their different normalization factors. After normalization, $\rho_s = |\psi\rangle\langle\psi|$.

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