Interaction Effects and Charge Quantization in Single-Particle Quantum Dot Emitters

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(Received 8 October 2018; published 29 March 2019)

We discuss a theoretical model of an on-demand single-particle emitter that employs a quantum dot, attached to an integer or fractional quantum Hall edge state. Via an exact mapping of the model onto the spin-boson problem we show that Coulomb interactions between the dot and the chiral quantum Hall edge state, unavoidable in this setting, lead to a destruction of precise charge quantization in the emitted wave packet. Our findings cast doubt on the viability of this setup as a single-particle source of quantized charge pulses. We further show how to use a spin-boson master equation approach to explicitly calculate the current pulse shape in this setup.

DOI: 10.1103/PhysRevLett.122.127701

Introduction.—The venerable field of quantum optics has brought many remarkable technological advances in, e.g., communication and encryption [1]. More fundamentally it has allowed experimental tests of quantum mechanics with unprecedented precision and control. This success would not have been possible without innovations in reliable on-demand single-photon sources. Recently, there has been exciting new experimental activity in creating and studying analogous sources, but with electrons and fractional quasiparticles in quantum Hall edge states [2–10]. The particles emitted by these devices can be entangled using electronic interferometers [11], thereby allowing one to extend the ideas developed in quantum optics to the realm of condensed matter physics. More importantly, the particles' statistics are different from that of photons, and they are more amenable to the studies of interaction effects. Therefore, this experimental setting offers new possibilities in manipulating entangled quasiparticle pairs, and in high-precision experimental studies of correlations in many-body electron systems; see review [12].

One experimental proposal for creating single-particle pulses uses a quantum dot (QD) connected to a quantum Hall edge state [13–15]. An experiment with such an on-demand single-electron source was performed in [16]. Here, the putatively quantized pulses are generated via nonequilibrium driving of the quantum dot. In this paper we study a model of this setup, shown in Fig. 1 with the QD having a single level whose energy can be varied using an applied bias voltage. When this energy rises from below to above the chemical potential a particle can tunnel from the dot into the edge. In the integer quantum Hall effect (IQHE) case a linear voltage ramp generates a single-electron excitation with minimal noise [14]. The presumed advantage of this setup is that quantization of charge on the dot is expected to lead to the quantization of the resulting charge pulse on the edge. In contrast, we find that Coulomb interactions, however weak, between the particles on the dot and the edge destroy this precise charge quantization of the emitted current pulse.

In this Letter we study the model shown in Fig. 1 describing a quantum dot with a time-dependent energy level coupled by tunneling to a chiral QHE edge (integer or fractional). In the integer quantum Hall effect case the energy level on the dot represents an electron, whereas in the fractional (FQHE) case, the energy level may represent either an electron or a fractionally charged quasiparticle. The particle on the dot is allowed to tunnel between the dot and the edge. If the dot contains an electron it may be either inside or outside of the QHE fluid, whereas if it contains a fractionally charged quasiparticle it must be surrounded by FQHE fluid in order to support these fractionalized charges.

As mentioned above, in both the integer and fractional QHE case, interactions renormalize the charge of the pulses, which can be described within the following

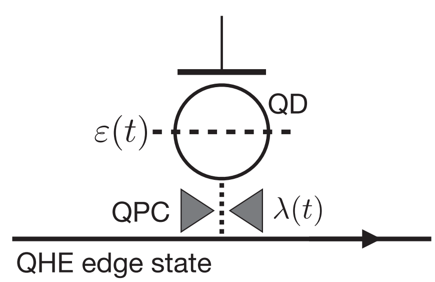

FIG. 1. Schematic picture of the model. A quantum dot is attached to a quantum Hall edge state (integer or fractional) via a quantum point contact. The voltage on the QPC can be used to control the tunneling $\lambda(t)$ between the dot and the edge. We assume that the dot has a single level with energy $\varepsilon(t)$, which is controlled by an applied gate voltage. The level can be occupied with an electron in the IQHE case, or with a quasiparticle in the case of FQHE.

physical picture. Because of repulsive Coulomb interactions the charge on the edge close to the quantum point contact (QPC) gets depleted in the presence of a charge on the dot. Following emission of a particle by the dot, charge fills up the depleted area on the edge, which reduces the net charge flowing downstream from the dot. Therefore, when a particle tunnels from the dot into the edge, while the charge leaving the dot may be quantized, the net charge in the resulting current pulse downstream is always less than the particle charge (for repulsive interactions).

Our results suggest that creating a source of precisely quantized electron or quasiparticle pulses using such a quantum dot setup would require extra fine-tuning. In order to have a nonvanishing tunneling between the dot and the edge they should be placed in proximity, thus inevitably producing Coulomb interactions between the two. While recent pioneering experiments by Glattli *et al.* reported creation of single-electron pulses using a quantum dot setup in the IQHE case [16], our theory suggests that higher-precision measurements should find that this quantization is not exact, and it would be interesting to compare the results of such measurements with our predictions. In the experimental setup of [16], the Coulomb interactions between the dot and the edge will be partially screened by the metallic gate. However, dipole interactions will remain.

In this paper, we first introduce our theoretical model and show how the Hamiltonian of this model can be mapped to the spin-boson problem. This mapping allows us to analyze the effects of Coulomb interactions between the dot and the edge. In the second part of the paper we use a generalized master equation (GME) approach discussed in [17,18] to obtain results for current pulse profiles. We refer to our companion paper for more details on the calculations, where we also compare the results obtained via a spin-boson mapping with the results of perturbative calculations [19].

The model.—We consider a theoretical model of the experimental setup presented in Fig. 1. The model is described by the time-dependent Hamiltonian

$$\hat{H}(t) = \hat{H}_0(t) + \hat{H}_{\rm tun}(t) + \hat{H}_{\rm int}. \tag{1}$$

Here, the first term describes the quantum dot with a single energy level $\varepsilon(t)$ [20], which is controlled by a time-dependent gate voltage, together with the edge state with velocity v, given in the bosonized form

$$\hat{H}_0(t) = \varepsilon(t)\hat{S}_z + \frac{v}{2}\int \frac{dx}{2\pi}(\partial_x\hat{\varphi})^2. \tag{2}$$

Here we introduced spin-1/2 operators describing occupation numbers of the quantum dot, which we treat as a two-level system. The operator $\hat{S}^+(S^-)$ creates (destroys) a particle on the QD. In the case of electron tunnelling $\hat{S}^+$ creates an electron with charge $-e$ on the dot with $e > 0$, whereas in the case of a fractionalized charge tunneling it creates a quasiparticle with charge $-\nu e$ [21]. The presence or absence of a particle on the dot is measured by the operator $\hat{N} = \hat{S}_z + 1/2$. In the following we assume large Zeeman splitting and omit the physics of electron spin on the edge.

The second term in the Hamiltonian (2) describes a chiral edge (for a system of length L, assumed very large, with periodic boundary conditions) of a Laughlin state at filling fraction $\nu = 1/(2n+1)$, and $n = 0, 1, 2, \ldots$ [22]. Here, the bosonic field $\hat{\varphi}$ is given in terms of its eigenmode expansion with momentum $k = 2\pi m/L$, $m \in \mathbb{Z}$ as follows [23],

$$\hat{\varphi}(x) = -\sum_{k>0}\sqrt{\frac{2\pi}{kL}}(\hat{b}_k e^{ikx} + \hat{b}_k^\dagger e^{-ikx})e^{-ka/2}, \tag{3}$$

where a is the short-distance cutoff, and bosonic operators $\hat{b}_k$ obey commutation relations $[\hat{b}_k, \hat{b}_{k'}^\dagger] = \delta_{kk'}$. Here we omit zero modes as well as the corresponding Klein factors, as these do not affect the results in the thermodynamic limit in our setup. We also note that the results do not depend on the cutoff a, after sending it to 0 at the end of the calculations.

The electron and quasiparticle operators in the bosonized form [22–24] are described by the vertex operators

$$\hat{\psi}(x) = \frac{1}{\sqrt{2\pi}}a^{-\frac{\gamma^2}{2}}e^{-i\gamma\hat{\varphi}(x)}, \tag{4}$$

where $\gamma = 1/\sqrt{\nu}$ for electrons, and $\gamma = \sqrt{\nu}$ for quasiparticles. It is convenient to account for these two different possibilities in a unified manner, and in the following by referring to particles we assume electrons or quasiparticles with the corresponding value of γ. Note that the charge of the particle is given by $q = -\gamma\sqrt{\nu}e$.

The second term in the Hamiltonian (1) describes the coupling of the dot to the edge via a QPC with, in general, time-dependent tunneling amplitude $\lambda(t)$, which can be produced by varying the QPC gate voltage

$$\hat{H}_{\rm tun}(t) = \lambda(t)\hat{\psi}^\dagger(0)\hat{S}^- + \text{H.c.} \tag{5}$$

Finally, we model the Coulomb interactions between the dot and the edge as

$$\hat{H}_{\rm int} = -\gamma\frac{g}{2\pi}\partial_x\hat{\varphi}(0)\hat{S}_z, \tag{6}$$

where we used the bosonized form of the charge density operator on the edge $\hat{\rho}(x) = +e\sqrt{\nu}\partial_x\hat{\varphi}/2\pi$, with $g > 0$ being the interaction strength. In this model, the Coulomb interaction is assumed to be a delta function acting at a single point $x = 0$ on the edge. In the case of the Coulomb interaction being spread over a finite region we can still use the above form, where the coupling $g_{\rm eff}$ can be determined from the interaction form, as discussed in Supplemental Material [25].

Mapping to the spin-boson problem.—One can map (1) to the well-known spin-boson model using the unitary transformation suggested by Furusaki and Matveev [26] (see also Supplemental Material [25]). Following these authors we define an operator $\hat{U}_1 = \exp[-i\gamma\hat{\varphi}(0)\hat{S}_z]$. Under a unitary transformation $\hat{\tilde{H}} = \hat{U}_1{}^\dagger \hat{H}\,\hat{U}_1$ the Hamiltonian assumes the spin-boson form that, omitting an unimportant constant, is given by

$$\hat{\tilde{H}} = \varepsilon(t)\hat{S}_z + \frac{v}{2}\int \frac{dx}{2\pi}(\partial_x\hat{\varphi})^2 + \lambda(t)\sqrt{\frac{2}{\pi}}a^{-(\gamma^2/2)}\hat{S}_x + v\tilde{\gamma}\hat{S}_z\partial_x\hat{\varphi}(0). \tag{7}$$

In this representation the effect of the Coulomb interactions amounts to a rescaling of γ such that

$$\tilde{\gamma} = \gamma\left(1 - \frac{g}{2\pi v}\right). \tag{8}$$

After introducing a shorthand notation for the coupling strengths as

$$\Delta(t) = \lambda(t)\sqrt{\frac{2}{\pi}}a^{-(\gamma^2/2)}, \qquad \eta_k = v\tilde{\gamma}\sqrt{\frac{2\pi k}{L}}e^{-ka/2}, \tag{9}$$

we arrive at familiar expression for the spin-boson Hamiltonian cf., [27],

$$\hat{\tilde{H}} = \varepsilon(t)\hat{S}_z + \Delta(t)\hat{S}_x + \sum_{k>0}\omega_k\hat{b}_k^\dagger\hat{b}_k - i\hat{S}_z\sum_{k>0}\eta_k(\hat{b}_k - \hat{b}_k^\dagger), \tag{10}$$

where $\omega_k = vk$. It is worth noting that the transformation between Hamiltonians of Eq. (1) and Eq. (10) is exact.

The first two terms of the Hamiltonian in Eq. (10) represent a spin 1/2 in the presence of a time-dependent magnetic field $B(t) = \varepsilon(t)\hat{e}_z + \Delta(t)\hat{e}_x$. The last two terms describe the Hamiltonian of a bosonic heat bath together with the spin-boson coupling. The spectral function of the spin-boson model is defined in the standard way using the following equation,

$$J(\omega) = \pi\sum_{k>0}\eta_k^2\delta(\omega - \omega_k) = 2\pi\alpha\omega\Theta(\omega)e^{-\omega a/v}, \tag{11}$$

where $\Theta(\omega)$ is the Heaviside theta function. This corresponds to a heat bath with Ohmic dissipation, and dimensionless coupling $\alpha = \tilde{\gamma}^2/2$. We estimate for experiments similar to [28,29] that $g/2\pi v = 0.04$ and hence $\alpha = 0.15$ for the $\nu = 1/3$ state. See Supplemental Material [25] for more details.

Current.—Now let us turn to a discussion of the main subject of this paper, the behavior of the current under a nonequilibrium drive of the QD. First, it is useful to obtain general exact results for the current, while we postpone the discussion of the numerical approach to the next section. The Hamiltonian (7) can be refermionized using a unitary transformation with the operator $\hat{U}_2 = \exp[i\tilde{\gamma}\,\hat{\varphi}(0)\hat{S}_z]$, which brings it into a noninteracting form with the new value of $\tilde{\gamma}$; i.e., the Hamiltonian is of the form of Eq. (1) except the last term is absent. The equations of motion generated by this Hamiltonian can be used to relate the currents on the edge and on the QD,

$$\frac{v}{2\pi}[\partial_x\hat{\varphi}(+0) - \partial_x\hat{\varphi}(-0)] = \tilde{\gamma}\frac{d\hat{N}}{dt}. \tag{12}$$

Using equations of motion for $\hat{\varphi}(x,t)$ away from $x = 0$, we obtain an expression for the current on the edge at $x > 0$,

$$\hat{I}(x,t) = -\tilde{q}\frac{d\hat{N}(t - x/v)}{dt}, \tag{13}$$

where $\hat{I} = v\hat{\rho}$ is the current operator, and $\tilde{q} = (\tilde{\gamma}/\gamma)q$.

One would expect from charge conservation that the proportionality constant should be equal to the charge of the particle q. Remarkably, in the interacting case the charge gets renormalized by a factor $\tilde{\gamma}/\gamma$ that is less than 1 for repulsive interactions. In other words, in the presence of interactions, one cannot obtain a precisely quantized charge pulse.

Master equation approach.—The mapping to the spin-boson Hamiltonian is particularly useful, since it enables one to use powerful numerical techniques developed for this well-studied problem. For $\alpha < 1/2$ one could also use the stochastic Schrödinger equation method [30]. However, in this Letter we adopt the generalized master equation approach, which makes possible calculations for arbitrary times provided α is small. This allows the calculation of the current resulting from nonequilibrium driving of the quantum dot.

The starting point of the calculations is the derivation of the path-integral solution for the time evolution of the reduced density matrix for the spin 1/2 using the Feynman-Vernon influence functional approach; see [17]. This is done by exactly tracing out the heat-bath degrees of freedom. From the path-integral solution one then derives the GME describing the time evolution of $\langle\hat{S}_z\rangle$ [17,18],

$$\frac{d}{dt}\langle\hat{S}_z(t)\rangle = \int_0^t d\tau\left[\frac{1}{2}K^a(t,\tau) - K^s(t,\tau)\langle\hat{S}_z(\tau)\rangle\right]. \tag{14}$$

Here the integral kernels $K^{(a,s)}(t,\tau)$ can be obtained in terms of a series expansion in $\Delta(t)$ for arbitrary α. However, each factor of $\Delta(t)$ in this expansion comes with the integration over time; hence we have to truncate the series in our numerical calculations in the case when α is not small. Remarkably, to linear order in α it is possible to sum up the entire series expansion in $\Delta(t)$ analytically [18] and obtain expressions for $K^{a,s}(t,\tau)$ that are exact in $\Delta(t)$. This truncation of the master equation is useful for $\alpha = \tilde{\gamma}^2/2 \ll 1$. We summarize the derivation of the GME and the definitions of the kernels in Supplemental Material [25].

In the top panel of Fig. 2 we present the results for the current at constant bias voltage applied to the dot, $\varepsilon(t) = \varepsilon_0$ for $t > 0$. The dot is taken to be occupied at $t = 0$ corresponding to $\varepsilon(t)$ large and negative for $t < 0$. This models the step in the first half-period of a square-wave bias. The time dependence of the tunneling strength is $\lambda(t) = \lambda\Theta(t)$. Here we use the exact analytical expression obtained in [30,31] for the time evolution, which is valid at $\alpha \ll 1$; see details in Supplemental Material [25]. We find that the current is a highly oscillatory function of time after the voltage ramp and decays exponentially at long times. In the inset we show behavior of $N(t)$ as a function of time for the same step-function protocol. Notice that the total charge leaving the dot converges to q in the long time limit, which, according to Eq. (13), corresponds to a downstream current pulse of charge $\tilde{q}$.

In the bottom panel of Fig. 2 we present our numerical results using the GME for the current on the quantum dot after a linear voltage ramp with rate ξ, so that $\varepsilon(t) = \xi(t - t_0)$. However, in this case in contrast to a step pulse, not all the charge leaves the quantum dot during the ramp; instead the occupation number of the QD at late times saturates to $\exp(-\pi\Delta^2/2\xi)$. This is rather unexpected because $\varepsilon(t)$ becomes very large at late times. A similar observation was made previously in the context of the spin-boson problem [30,32,33]. This behavior is distinctly nonadiabatic since the equilibrium occupation of the QD at large bias must vanish. At late times, the current produced by the linear ramp exhibits Rabi oscillations with an instantaneous frequency set by $\varepsilon(t)$ [34].

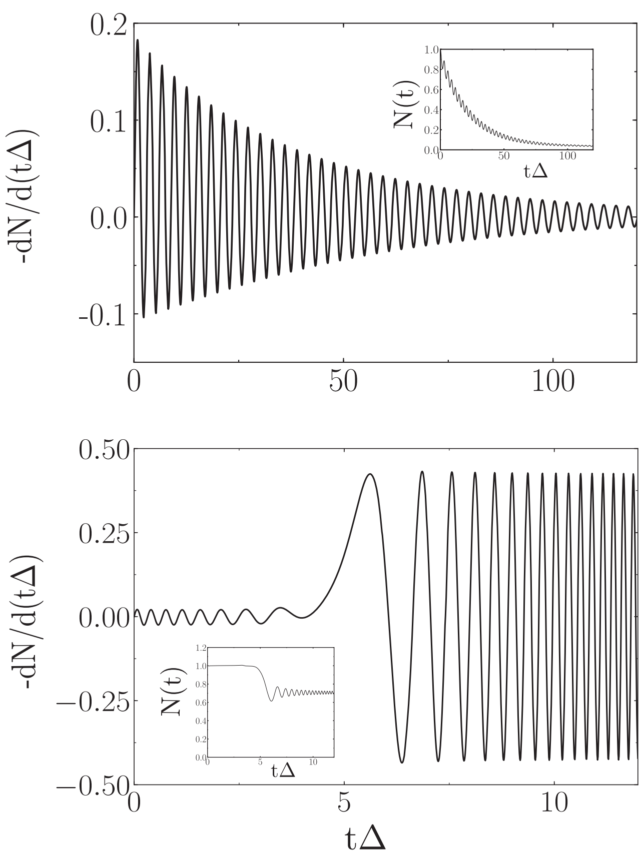

FIG. 2. Results of the numerical solution of the generalized master equation for the time dependence of $-dN/d(t\Delta)$, which is related to the current on the edge via Eq. (13). In both figures we turn on the tunneling $\lambda(t)$ at the QPC at $t = 0$, provided that the dot is filled, and the edge is in equilibrium at $t < 0$. (Top) Time evolution of the current after a steplike pulse, see text, which leads to discharging of the dot at long times. In the calculations we use parameters $a = 0.005v\Delta^{-1}$, $\alpha = 0.05$, $\varepsilon_0 = 2\Delta$. (Bottom) Time evolution of the current after a linear ramp $\varepsilon(t) = \xi(t - t_0)$ with parameters $a = 0.005v\Delta^{-1}$, $\alpha = 0.01$, $\xi = 4\Delta^2$, $t_0 = 5\Delta^{-1}$. See insets for the corresponding time dependence of $N(t)$.

In the experimental setting, including effects such as phonons, the remaining charge on the dot is eventually expected to leave the QD at long times, producing a charge pulse downstream with charge $\tilde{q}$. However, if the current is measured over a timescale shorter than these processes, then our results provide another constraint to quantization of charge pulses in the linear voltage ramp protocol.

Discussion.—In this paper we studied a theoretical model of a single-particle emitter of charge pulses that uses a quantum dot coupled to a quantum Hall edge state. We showed that it is not possible to obtain precise quantization of these pulses due to Coulomb interactions between the dot and the edge. The interactions effectively add a capacitance to the system, and the charge stored on this capacitor is released in addition to the charge on the dot in the emission process, thus reducing the charge in the outgoing pulse on the edge. Coulomb interactions are unavoidable in the QD setup, and hence we argue that it is perhaps not the most promising route for creating precisely quantized charge pulses. It would be interesting to compare our theoretical predictions with higher precision measurements of charge in single-particle emitters using a quantum dot, such as in [16].

This raises the question of how to mitigate the destruction of charge quantization if one wants to obtain a single-particle source with precisely quantized charge pulses. In the quantum dot setup described above, one will want to screen the Coulomb interaction as much as possible in order to minimize the effect; however it can never be eliminated completely.

Coulomb interactions do not plague proposals where there is no quantum dot but instead a voltage is applied directly to the edge. This makes them perhaps a more promising route to the realization of single-particle sources, although the applied voltage pulses must be fine-tuned to a Lorentzian profile [35–37]. This setup has been studied by Martin *et al.* [38–41].

It is also possible to consider a pump geometry [42–45]. In this case we must necessarily transfer exactly one quantized charge over one period. However the effect of the interactions is to spread the current over two pulses. There will be a first pulse on the edge as the dot is charged due to the Coulomb repulsion. Then there will be a second pulse when the charge jumps from the dot onto the edge. This second pulse will not carry the full quantized charge due to the depletion of the edge.

From the theoretical perspective we showed how a mapping to the spin-boson problem, and generalized master equation solution, can be used to efficiently simulate this interesting class of experimentally relevant nonequilibrium interacting quantum systems.

We thank C. Glattli for insightful discussions and suggestions. We are grateful to J. Keeling, G. Fux, L. Brendon, and M. Moskalets for helpful comments on an earlier draft of the paper. This work was supported by EPSRC Grant No. EP/N01930X/1. D. K. was supported by EPSRC Grant No. EP/M007928/2. Statement of compliance with EPSRC policy framework on research data: This publication is theoretical work that does not require supporting research data.

[1] S. Haroche, Rev. Mod. Phys. **85**, 1083 (2013).

[2] F. Parmentier, E. Bocquillon, A. Mahé, J.-M. Berroir, D. Glattli, B. Plaçais, G. Fève, A. Cavanna, and Y. Jin, in *Proceedings of 21st International Conference on Noise and Fluctuations (ICNF), 2011* (IEEE, New York, 2011), pp. 94–99.

[3] C. Grenier, R. Hervé, E. Bocquillon, F. D. Parmentier, B. Plaçais, J.-M. Berroir, G. Fève, and P. Degiovanni, New J. Phys. **13**, 093007 (2011).

[4] F. D. Parmentier, E. Bocquillon, J.-M. Berroir, D. C. Glattli, B. Plaçais, G. Fève, M. Albert, C. Flindt, and M. Büttiker, Phys. Rev. B **85**, 165438 (2012).

[5] E. Bocquillon, F. D. Parmentier, C. Grenier, J.-M. Berroir, P. Degiovanni, D. C. Glattli, B. Plaçais, A. Cavanna, Y. Jin, and G. Fève, Phys. Rev. Lett. **108**, 196803 (2012).

[6] M. Moskalets and M. Büttiker, Phys. Rev. B **80**, 081302 (2009).

[7] V. Kashcheyevs and J. Timoshenko, Phys. Rev. Lett. **109**, 216801 (2012).

[8] M. Moskalets, G. Haack, and M. Büttiker, Phys. Rev. B **87**, 125429 (2013).

[9] M. Moskalets, Low Temp. Phys. **43**, 865 (2017).

[10] M. Kapfer, P. Roulleau, M. Santin, I. Farrer, D. A. Ritchie, and D. C. Glattli, Science **363**, 846 (2019).

[11] Y. Ji, Y. Chung, D. Sprinzak, M. Heiblum, D. Mahalu, and H. Shtrikman, Nature (London) **422**, 415 (2003).

[12] C. Bäuerle, D. C. Glattli, T. Meunier, F. Portier, P. Roche, P. Roulleau, S. Takada, and X. Waintal, Rep. Prog. Phys. **81**, 056503 (2018).

[13] D. C. Glattli and P. S. Roulleau, Phys. Status Solidi B **254**, 1600650 (2017).

[14] J. Keeling, A. V. Shytov, and L. S. Levitov, Phys. Rev. Lett. **101**, 196404 (2008).

[15] D. Ferraro, J. Rech, T. Jonckheere, and T. Martin, Phys. Rev. B **91**, 205409 (2015).

[16] G. Fève, A. Mahé, J.-M. Berroir, T. Kontos, B. Placais, D. Glattli, A. Cavanna, B. Etienne, and Y. Jin, Science **316**, 1169 (2007).

[17] M. Grifoni, E. Paladino, and U. Weiss, Eur. Phys. J. B **10**, 719 (1999).

[18] L. Hartmann, I. Goychuk, M. Grifoni, and P. Hänggi, Phys. Rev. E **61**, R4687 (2000).

[19] G. Wagner, D. X. Nguyen, D. Kovrizhin, and S. H. Simon (to be published).

[20] In the experimental setup of Ref. [16], the QD has a set of equally spaced energy levels. If the energy level spacing is much larger than the applied bias $\varepsilon(t)$ and the energy scale associated with tunnelling, then we can assume that only the energy level closest to the chemical potential contributes to the dynamics.

[21] We note that spin operators commute with $\hat{\psi}(x)$ instead of satisfying electron or quasiparticle statistics. However, this does not affect our calculations of the current. See also our companion paper.

[22] X.-G. Wen, Int. J. Mod. Phys. B **06**, 1711 (1992).

[23] J. von Delft and H. Schoeller, Ann. Phys. (Berlin) **7**, 225 (1998).

[24] R. A. J. van Elburg and K. Schoutens, Phys. Rev. B **58**, 15704 (1998).

[25] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.122.127701 for estimation of the parameter g from the experimental setup. We also present the contribution of multi-pole interactions as well as the derivation of the master equation that we used in the main text.

[26] A. Furusaki and K. A. Matveev, Phys. Rev. Lett. **88**, 226404 (2002).

[27] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).

[28] P. Roulleau, F. Portier, D. C. Glattli, P. Roche, A. Cavanna, G. Faini, U. Gennser, and D. Mailly, Phys. Rev. B **76**, 161309 (2007).

[29] H. le Sueur, C. Altimiras, U. Gennser, A. Cavanna, D. Mailly, and F. Pierre, Phys. Rev. Lett. **105**, 056803 (2010).

[30] P. P. Orth, A. Imambekov, and K. Le Hur, Phys. Rev. B **87**, 014305 (2013).

[31] R. Görlich, M. Sassetti, and U. Weiss, Europhys. Lett. **10**, 507 (1989).

[32] M. Wubs, K. Saito, S. Kohler, P. Hänggi, and Y. Kayanuma, Phys. Rev. Lett. **97**, 200404 (2006).

[33] K. Saito, M. Wubs, S. Kohler, Y. Kayanuma, and P. Hänggi, Phys. Rev. B **75**, 214308 (2007).

[34] Oscillations in the current, when the energy of the dot is well below the chemical potential of the edge (i.e., before the onset of the voltage ramp), are an artefact of an abrupt switching-on of the tunnelling $\lambda(t)$ at $t = 0$.

[35] L. S. Levitov, H. Lee, and G. B. Lesovik, J. Math. Phys. (N.Y.) **37**, 4845 (1996).

[36] T. Jullien, P. Roulleau, B. Roche, A. Cavanna, Y. Jin, and D. C. Glattli, Nature (London) **514**, 603 (2014).

[37] J. Dubois, T. Jullien, F. Portier, P. Roche, A. Cavanna, Y. Jin, W. Wegscheider, P. Roulleau, and D. C. Glattli, Nature (London) **502**, 659 (2013).

[38] L. Vannucci, F. Ronetti, D. Ferraro, J. Rech, T. Jonckheere, T. Martin, and M. Sassetti, J. Phys. Conf. Ser. **969**, 012143 (2018).

[39] L. Vannucci, F. Ronetti, J. Rech, D. Ferraro, T. Jonckheere, T. Martin, and M. Sassetti, Phys. Rev. B **95**, 245415 (2017).

[40] D. Chevallier, J. Rech, T. Jonckheere, C. Wahl, and T. Martin, Phys. Rev. B **82**, 155318 (2010).

[41] F. Ronetti, L. Vannucci, D. Ferraro, T. Jonckheere, J. Rech, T. Martin, and M. Sassetti, Phys. Rev. B **98**, 075401 (2018).

[42] S. H. Simon, Phys. Rev. B **61**, R16327 (2000).

[43] I. L. Aleiner and A. V. Andreev, Phys. Rev. Lett. **81**, 1286 (1998).

[44] P. Sharma and C. Chamon, Phys. Rev. Lett. **87**, 096401 (2001).

[45] M. Switkes, C. M. Marcus, K. Campman, and A. C. Gossard, Science **283**, 1905 (1999).