

## Transition to Turbulence in Particle Laden Flows

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Suspended particles can alter the properties of fluids and in particular also affect the transition from laminar to turbulent flow. An earlier study [Matas *et al.*, *Phys. Rev. Lett.* **90**, 014501 (2003)] reported how the subcritical (i.e., hysteretic) transition to turbulent puffs is affected by the addition of particles. Here we show that in addition to this known transition, with increasing concentration a supercritical (i.e., continuous) transition to a globally fluctuating state is found. At the same time the Newtonian-type transition to puffs is delayed to larger Reynolds numbers. At even higher concentration only the globally fluctuating state is found. The dynamics of particle laden flows are hence determined by two competing instabilities that give rise to three flow regimes: Newtonian-type turbulence at low, a particle induced globally fluctuating state at high, and a coexistence state at intermediate concentrations.

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Particle laden flows are ubiquitous in nature and applications, including slurry flows, sediment transport, blood flow, and pollutant dispersion in atmospheric flows. Particle-fluid interactions strongly affect the dynamics especially when the particles are sufficiently large, i.e., larger than the smallest scales of the flow, and the inertial effects become important [1]. In particular this also influences the transition from laminar to turbulent flow and the nature of turbulence. However, at present, the phenomenon of laminar-turbulent transition for particle laden flows is poorly understood, especially when compared to flow of a single phase Newtonian fluid.

In case of a Newtonian fluid in a pipe, the laminar flow is linearly stable for all Reynolds numbers ( $Re = \rho UD/\mu$ ) [2,3], yet turbulence can be triggered in the presence of finite amplitude perturbations if  $Re$  is sufficiently large [4,5]. At the lowest Reynolds numbers where turbulence is first encountered it only occurs in localized patches called puffs, which are spatially separated by laminar flow [6]. The coexistence of laminar and turbulent states, i.e., spatiotemporal intermittency, and the dependence of the transition point on the strength of perturbations (and hence implying hysteresis) are characteristics for transition in Newtonian, single phase flows.

Adding particles to the fluid can significantly alter this scenario due to particle-fluid and particle-particle interactions [7]. Matas *et al.* [8] investigated the effect of neutrally buoyant inertial particles on the laminar-turbulence transition in a pipe flow. They presented the critical Reynolds number  $Re_c$  at which puffs were first detected for varying particle concentrations and sizes and showed that, for sufficiently large particles,  $Re_c$  varied nonmonotonically with particle concentration. With the initial increase in particle concentration, transition was triggered at a progressively lower  $Re_c$ . However, upon further increase, the trend unexpectedly reverses and  $Re_c$  starts to increase.

More recently, Yu *et al.* [9] reported similar nonmonotonic behavior in a numerical study. They also noted that it was difficult to rigorously judge whether the flow is laminar or turbulent, as velocity fluctuations increased smoothly with  $Re$ . In another numerical study of neutrally buoyant spherical particles in a channel flow, Lashgari *et al.* [10] showed the existence of three different flow regimes: a “laminarlike” regime that occurs at low-concentrations and low  $Re$ , a “turbulentlike” regime at low concentrations and high  $Re$ , and a “shear-thickening” regime at high concentrations and high  $Re$ . In the latter regime, the wall friction increased with  $Re$  due to particle induced stresses, while turbulent transport was weakly affected. Because of this, they speculated that at a high enough particle concentration the transition to turbulence might not only be delayed, as reported by Matas *et al.* [8], but could be completely suppressed. Additionally, similar to Yu *et al.* [9], they also noted that the velocity fluctuations increased smoothly with  $Re$  at high particle concentrations. Newtonian-type turbulence, in contrast, is accompanied by a sharp increase in velocity and pressure fluctuations.

Several studies addressing dilute polymeric flows have also reported smoothly increasing fluctuations in velocity and pressure [11,12]. Here, the transition to turbulence occurred without hysteresis or intermittency. They also noted that the ordinary Newtonian turbulence was suppressed and replaced by a different kind of disordered motion called elastoinertial turbulence. Hence, it is possible that also in the case of particle laden flows, the smoothly increasing velocity and pressure fluctuations observed in numerics and experiments [8–10,13] indicate that the nature of transition, and perhaps the turbulent state itself, is altered by the presence of particles.

In the following we present an experimental study of the laminar-turbulent transition in suspensions of neutrally buoyant spherical particles in pipe flows. We show that

at high particle concentrations, an instability occurs at low Reynolds numbers that is continuous and lacks the spatiotemporal characteristics of transition in Newtonian fluids. As shown, this particle driven instability is distinct from the subcritical transition. While it dominates the dynamics at high concentrations, at intermediate values both instabilities coexist giving rise to a mixed state featuring puffs superimposed on a uniformly fluctuating flow.

The experimental setup consists of a straight, horizontal glass pipe of circular cross section with diameter  $D = 4$  mm and a total length of  $500D$ . Measurements were performed  $300D$  downstream from the inlet. Here, the pressure was measured over a length of  $L = 120D$  using a differential pressure sensor. Just downstream of this, another differential pressure sensor is used over a length of  $5D$  to measure fluctuating quantities. A continuous perturbation is used, in the form of a pin, 0.9 mm in diameter, located  $15D$  downstream of the inlet. The fluid used was a 21.65% glycerin-water solution matching the density  $\rho = 1.051 \pm 0.01$  g cm $^{-1}$  of the suspended polystyrene spheres of diameter  $d = 0.20 \pm 0.05$  mm. Consequently, the pipe-to-particle diameter ratio  $D/d \approx 20$ . The suspension was driven by a piston to ensure a constant volumetric flow rate.

For all the plots shown in the Letter we have used suspension Reynolds number  $Re_s = \rho UD / \mu_{\text{eff}}$ , where  $U$  is bulk velocity and  $\mu_{\text{eff}}$  is the effective dynamic viscosity of the suspension.  $\mu_{\text{eff}}$  is determined for each concentration  $\Phi_v$  by collapsing measured pressure drop values onto the Hagen-Poiseuille curve when the flow is in the laminar state. The viscosity of suspensions of spherical particles is known to depend on  $\Phi_v$  and shear rate, but for  $\Phi_v \lesssim 25\%$  the behavior is approximately Newtonian and independent of the shear rate [7]. This is confirmed in Fig. 1 by the excellent collapse of the friction factor  $f = 2\Delta PD / (L\rho U^2)$  on the laminar line with a constant  $\mu_{\text{eff}}$  used for each concentration.  $\Delta P$  is the pressure drop across the measurement length  $L$ . While qualitative differences in the friction factor scaling are apparent from Fig. 1, the different flow regimes can be more accurately identified from pressure fluctuations measured over the shorter distance and as shown in Fig. 2(a).

The pressure fluctuations  $p'$  are normalized by the standard deviation due to background noise  $p'_o$ . For  $\Phi_v = 0$ , fluctuations increase steeply at the onset of turbulence. Here the flow intermittently changes between laminar and turbulent regions which causes the high fluctuation levels. As  $Re_s$  is increased, the turbulent fraction rises until the flow is fully turbulent. This behavior is similar for concentrations up to 5% although the fluctuation peak becomes less pronounced and moves to lower  $Re_s$ . However for concentrations larger than 5%, weak but uniform fluctuations are observed which increase steadily with  $Re_s$ , atypical of Newtonian transition. For intermediate concentrations ( $5\% \lesssim \Phi_v \lesssim 12.5\%$ ), in

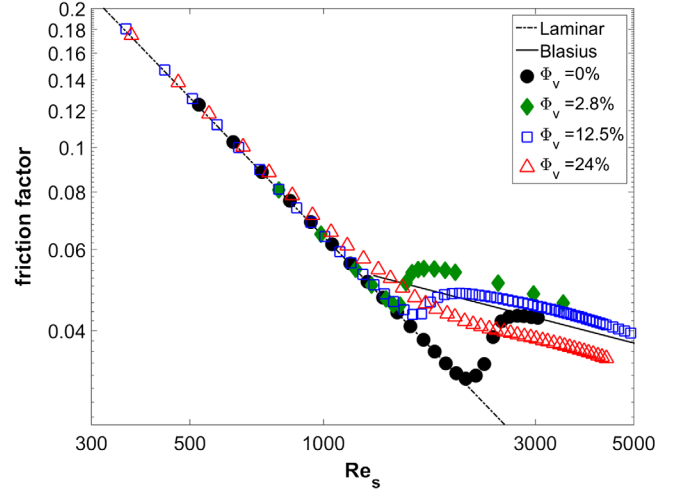


FIG. 1. Friction factor as a function of suspension Reynolds number. Experiments were carried out in the presence of the continuous perturbation at the pipe entrance.

addition, signatures of localized pufflike structures are also found as the  $Re_s$  is further increased sufficiently beyond the onset of the weakly fluctuating state. The Reynolds number for the onset of puffs in this regime increases with concentration. This shows that the critical Reynolds

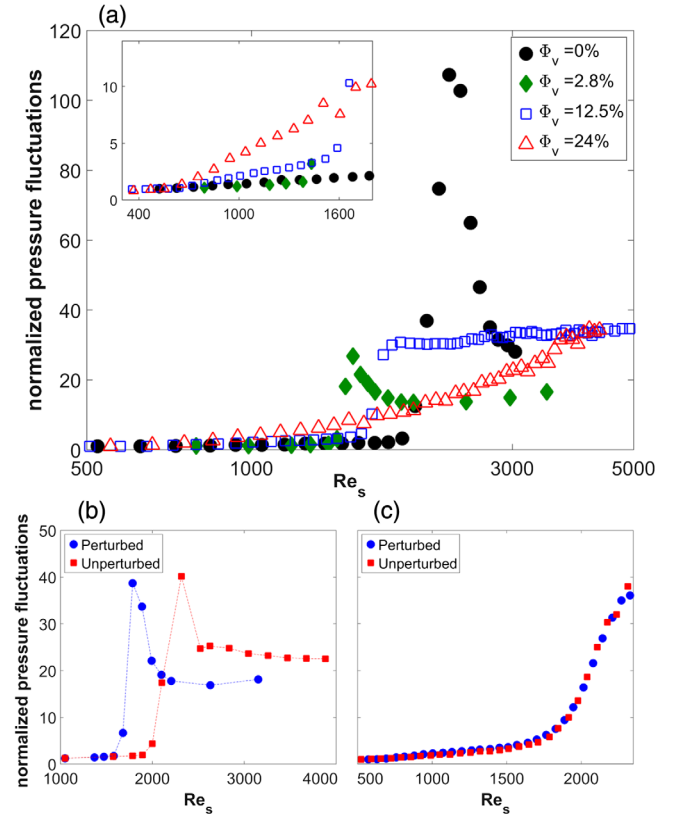


FIG. 2. Normalized pressure fluctuations as a function of suspension Reynolds number, (a) for different concentrations in presence of the perturbation, (b)  $\Phi_v = 0.6\%$  showing hysteresis, and (c)  $\Phi_v = 16\%$  showing no hysteresis.

number where puffs first appear varies nonmonotonically with particle concentration, with the transition point first decreasing and then increasing again. This observation is in agreement with Matas *et al.* [8] and Yu *et al.* [9]. For concentrations higher than 12.5%, no spatiotemporal intermittent pufflike structures could be found at any  $Re_s$ . Here, the transition occurs gradually and continuously via a uniformly fluctuating state, and with increasing  $Re_s$  fluctuation levels increase evenly throughout space. These fluctuations can be observed for  $Re_s$  as low as 800, which is far below the lowest  $Re_s$  where turbulence is first observed for Newtonian, single phase pipe flow.

To probe the dependence of the transition point on perturbation levels and to determine if it is hysteretic, we compared measurements made with and without the continuous perturbation. As can be seen in Fig. 2(b), transition is hysteretic at low concentrations as in the presence of the perturbation, turbulence appears earlier than in the unperturbed case. In contrast, at high concentrations [see Fig. 2(c)], transition occurs at a specific Reynolds number regardless of whether the fluid is perturbed or not. The transition in this case is continuous and pressure fluctuations feature neither an abrupt jump nor an initial overshoot (these being characteristic of spatiotemporal intermittency). The insensitivity of the transition to the finite amplitude perturbations, the smooth and continuous increase in fluctuations with increasing  $Re_s$ , and uniformly fluctuating flow suggest that this type of transition (lower branch shown by red circle) in Fig. 3) may correspond to a linear instability of the laminar base flow.

Figure 3 depicts transition thresholds  $Re_c$  for the two different types of instabilities observed. The first branch, “Newtonian-like,” is a result of a finite amplitude perturbation and varies nonmonotonically with  $\Phi_v$ . The second branch, potentially corresponding to a linear instability, which we denote as “particle-induced branch,” is only detected for  $\Phi_v \gtrsim 5\%$  and decreases monotonically with increasing concentration. However, overall, the transition threshold (either finite amplitude or linear instability) decreases monotonically with  $Re_s$ .

Based on the existence of the two instabilities, we propose three different regimes. The “Newtonian-like turbulence” regime exists for  $\Phi_v < 5\%$ . The transition is abrupt, intermittent, and extremely sensitive to inlet conditions. In this regime  $Re_c$  decreases with increasing particle concentration. For  $5\% \lesssim \Phi_v \lesssim 12.5\%$ , a “mixed” regime is found where both branches exist. First, fluctuations appear globally and increase in intensity as  $Re_s$  is increased. With further increase in  $Re_s$ , a secondary transition is eventually encountered and spatially intermittent turbulent puffs appear atop the fluctuating background flow [see mixed state in Fig. 3(b)]. However, no signs of hysteresis were detected. Here  $Re_c$  for the Newtonian-like branch increases with increase in  $\Phi_v$ . Interestingly, the trend change of the Newtonian-like branch occurs at about

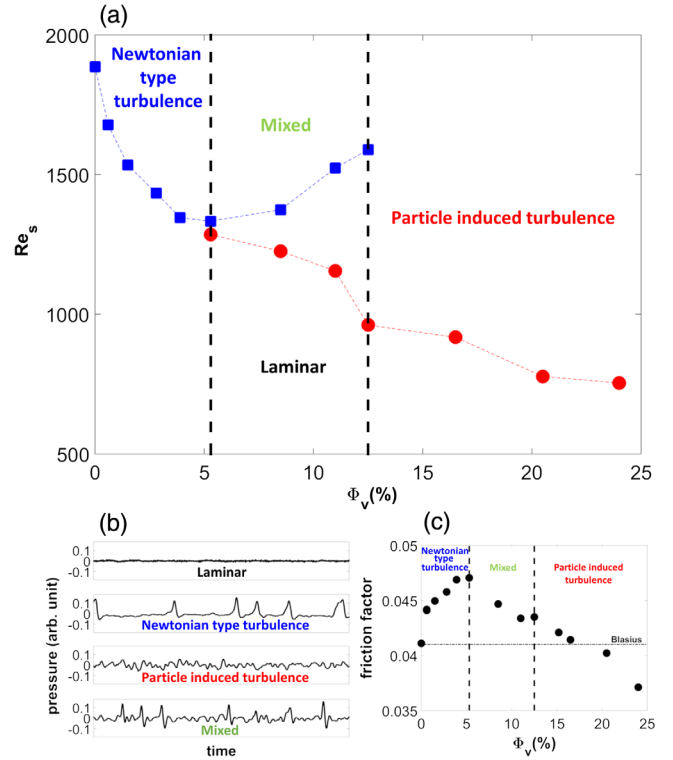


FIG. 3. (a) Transition scenario as a function of particle concentration: Newtonian-like (blue square), Particle-induced (red circle). (b) Time series of pressure fluctuations for laminar flow, Newtonian-type turbulence, particle induced turbulence, and coexisting Newtonian- and particle turbulence (mixed). (c) Friction factor at a fixed Reynolds number of  $Re_s = 3500$  as a function of particle concentration. Experiments were done in the presence of the continuous perturbation.

the same concentration at which the first signs of the particle-induced instability are observed in the flow and could therefore be the cause of the said trend change. For  $\Phi_v > 12.5\%$ , we encounter the “particle-induced turbulence” regime where laminar flow gradually becomes turbulent with increasing  $Re_s$ . The flow is neither intermittent nor hysteretic and turbulent fluctuations can be seen for successively lower  $Re_s$  as we increase the concentration.

To examine how the drag is modified by the presence of particles, we revisit Fig. 1. For zero concentration,  $f$  starts to increase at the onset and reaches the Blasius curve when the flow is fully turbulent. Interestingly, for concentrations  $0 < \Phi_v \lesssim 5\%$ ,  $f$  for fully turbulent flow is higher than the Blasius friction factor even though, as noted before in Fig. 2(a), the pressure fluctuations are significantly lower. The increase in  $f$  as compared to the Blasius scaling cannot be explained by viscous stresses as the enhanced viscosity due to particles is already taken into account in the definition of Reynolds number. However, the increase could be due to additional stresses induced by particles when the flow is turbulent [10,18]. Overall the friction

factor in the fully turbulent regime depends nonmonotonically on concentration. To further elaborate this,  $f$  is plotted in Fig. 3(c) as a function of particle concentration for  $Re_s = 3500$ , where the flow is fully turbulent for all concentrations. First,  $f$  increases with  $\Phi_v$ , reaching a maximum for around 5% and then the trend reverses with further increase in  $\Phi_v$ . This trend reversal occurs around the same concentration when we first encounter signs of particle-induced turbulence. Furthermore, for  $\Phi_v > 16\%$ ,  $f$  falls below the Blasius value, implying reduced drag as compared to a viscosity matched single phase fluid. This drag reduction is observed only in the particle-induced turbulence regime.

In summary, we have uncovered a distinct instability of the laminar base flow, previously unknown for particle laden flows, due to which the laminar-turbulent transition scenario is altered from that for single phase flows. Here, transition occurs without spatiotemporal intermittency and hysteresis, and turbulence arises continuously from laminar flow. Upon a further increase in  $Re$ , puffs will appear in addition to the particle induced fluctuations, giving rise to a mixed state. At high concentrations only particle induced turbulence is found and no secondary instability to a mixed state can be detected. In this high concentration limit, particle induced turbulence exerts a lower drag as compared to ordinary turbulence.

Recently, the study of Hogendoorn and Poelma [19] has been brought to our attention. As in the present study the paper reports that the nature of the transition changes from subcritical puffs at low concentrations to a supercritical transition and a uniformly fluctuating state at high concentrations. However, in this work the coexistence regime and the delayed puff transition to a mixed state at intermediate concentrations was not mentioned. In particular the statement that in contrast to Matas *et al.* [8]  $Re_c$  does not increase for higher volume fractions does not acknowledge that there are two separate instabilities and that at intermediate concentrations puffs are indeed delayed as reported by Matas *et al.* [8]. Hence while Hogendoorn and Poelma [19] detected the first instability that arises for a given concentration (subcritical at low and supercritical at high), Matas *et al.* [8] detected the puff instability. As

clarified in the present study the dynamics of particle laden flows are not simply governed by a single threshold curve but instead by two distinct instabilities.

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