Weyl Invariance of String Theories in Generalized Supergravity Backgrounds

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We revisit Weyl invariance string theories in generalized supergravity backgrounds. A possible counterterm was constructed in a work by Sakamoto, Sakatani, and Yoshida, but it seems to be a point of controversy in some literatures whether or not it is nonlocal. To settle down this issue, we show that the counterterm may be local and exactly cancels out the one-loop trace anomaly in generalized supergravity backgrounds.

Introduction.—A great progress in the recent study of string theory is that the generalized supergravity equations of motion (GSE) [1–3] (historically, GSE were discovered in the study of Yang-Baxter deformations of the $AdS_5 \times S^5$ superstring [4,5], though the bosonic part has already appeared in much older literature [6]) have been derived from the κ-symmetry constraints in the Green-Schwarz (GS) formulation of superstring theories [3]. It is well known that the usual supergravity equations of motion are solutions to the κ-symmetry constraints [7,8], but the discovery of this new supergravity indicates that there might exist more generalized supergravities.

In this Letter, we are concerned with string theory defined on generalized supergravity backgrounds (i.e., solutions to GSE). As a remarkable characteristic of GSE, a nondynamical vector field I is contained. In order to solve the κ-symmetry constraints, it should be a Killing vector, and this Killing condition plays a crucial role in our later discussion. It is instructive to note that this Killing condition was not taken into account in the old literature [6,9], where a prototype of GSE was derived from the one-loop finiteness (or the scale invariance) of string theory. In addition, this extra vector field may be identified with the trace of nongeometric Q flux, and many solutions of GSE can be regarded as T folds [10].

There is an issue with the consistency of string theories in generalized supergravity backgrounds. As a matter of course, at a classical level, there is no problem. Thanks to other work [3], the κ symmetry is ensured in generalized supergravity backgrounds and the GS formulation is consistently available. The issue arises at a quantum level.

Indeed, the Weyl anomaly may appear in string theories on generalized supergravity backgrounds [2,6]. In a recent work [1], Weyl invariance of bosonic string theories on generalized supergravity backgrounds was shown by constructing a possible counterterm {this counterterm was inspired from the embedding of GSE into double field theory (DFT) [1]. For the detail of the notation, see Ref. [1]} as

$$S_{\text{FT}} = \frac{1}{4\pi} \int d^2\sigma \sqrt{-\gamma} R^{(2)} \Phi_*, \qquad \Phi_* \equiv \Phi + I^i \tilde{Y}_i. \quad (1)$$

This is a generalization of the standard coupling to dilaton Φ, the so-called Fradkin-Tseytlin (FT) term [11]: indeed the standard FT term is reproduced when $I^i = 0$. To be more concrete, in generalized supergravity backgrounds, the Weyl anomaly takes the following form

$$\langle T^a{}_a \rangle = -\mathcal{D}_a[(Z_m \gamma^{ab} - I_m \varepsilon^{ab}) \partial_b X^m], \quad (2)$$

which is canceled out introducing the counterterm [Eq. (1)]. Compared to the sigma model action, the counterterm [Eq. (1)] is higher order in α', and it should be regarded as a quantum correction. Note also that the Killing vector I entering the GSE, does not appear in the classical action of string sigma model, but first appears as a quantum correction at a stringy level.

A point of controversy in some literature [12–18] is whether the counterterm [Eq. (1)] is local or not. The integrand depends on the dual coordinate $\tilde{Y}_i$. In computing its contribution to the trace of the energy-momentum tensor $T^a{}_a$, we need to use the equation of motion of the double sigma model [19,20],

$$\partial_a \tilde{Y}_i = g_{\text{in}} \varepsilon^b{}_a \partial_b X^n + B_{\text{in}} \partial_a X^n. \quad (3)$$

This equation implies that $\tilde{Y}_i$ would be a nonlocal function of X^m and one may suspect that the counterterm [Eq. (1)] is

nonlocal as well. However, as we show in this Letter, we can construct a possible local counterterm by taking account of the fact that the two-dimensional Ricci scalar $R^{(2)}$ is locally a total derivative (we really appreciate J. Maldacena for elucidating this point) and I is a Killing vector. That is, the (possible) nonlocality in the integrand of [Eq. (1)] can be removed. This is the main claim in this Letter.

Weyl invariance of bosonic string.—Let us first recall the basics on Weyl invariance of bosonic string theory in $D = 26$ dimensions,

$$S_b = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-\gamma}(g_{mn}\gamma^{ab} - B_{mn}\varepsilon^{ab})\partial_a X^m \partial_b X^n.$$

The Weyl anomaly of this system takes the form,

$$2\alpha' \langle T^a{}_a \rangle = (\beta^g_{mn}\gamma^{ab} - \beta^B_{mn}\varepsilon^{ab})\partial_a X^m \partial_b X^n. \quad (4)$$

Here, the β functions at the one-loop level have been computed (e.g., in Ref. [6]) as

$$\beta^g_{mn} = \alpha'\left(R_{mn} - \frac{1}{4}H_{mpq}H_n{}^{pq}\right),$$

$$\beta^B_{mn} = \alpha'\left(-\frac{1}{2}D^k H_{kmn}\right), \quad (5)$$

where D_m and R_{mn} are the covariant derivative and the Ricci tensor associated with the spacetime metric g_{mn} and $H_{mnp} \equiv 3\partial_{[m}B_{np]}$. For the Weyl invariance of the worldsheet theory, it is not necessary to require $\beta^g_{mn} = \beta^B_{mn} = 0$. As long as they take the form

$$\beta^g_{mn} = -2\alpha' D_m \partial_n \Phi, \qquad \beta^B_{mn} = -\alpha' \partial_k \Phi H^k{}_{mn}, \quad (6)$$

the Weyl anomaly has a simple form

$$\langle T^a{}_a \rangle \overset{\text{e.o.m.}}{\sim} -\mathcal{D}^a \partial_a \Phi, \quad (7)$$

under the equations of motion. Here, $\mathcal{D}_a$ is the covariant derivative associated with γ_{ab} and $\overset{\text{e.o.m.}}{\sim}$ represents the equality up to the equations of motion. This anomaly can be canceled out by adding the FT term [11],

$$S_{\text{FT}} = \frac{1}{4\pi} \int d^2\sigma \sqrt{-\gamma} R^{(2)} \Phi. \quad (8)$$

Therefore, as long as the target space satisfies Eq. (6), namely the supergravity equations of motion,

$$R_{mn} - \frac{1}{4}H_{mpq}H_n{}^{pq} + 2D_m\partial_n\Phi = 0,$$

$$-\frac{1}{2}D^k H_{kmn} + \partial_k \Phi H^k{}_{mn} = 0, \quad (9)$$

the Weyl invariance is ensured. As shown in Ref. [21], Eq. (9) implies that

$$\beta^\Phi \equiv R + 4D^m\partial_m\Phi - 4|\partial\Phi|^2 - \frac{1}{12}H_{mnp}H^{mnp} \quad (10)$$

is constant, and by choosing $\beta^\Phi = 0$, we obtain the usual dilaton equation of motion.

The main observation of this Letter is that the requirement [Eq. (6)] is a sufficient condition for the Weyl invariance but is not necessary.

Local counterterm for GSE.—Let us consider a milder requirement,

$$\beta^g_{mn} = -2\alpha' D_{(m}Z_{n)},$$

$$\beta^B_{mn} = -\alpha'(Z^k H_{kmn} + 2D_{[m}I_{n]}), \quad (11)$$

where I_m and Z_m are certain vector fields in the target space, which are functions of $X^m(\sigma)$. The condition [Eq. (11)] reduces to Eq. (6) when $Z_m = \partial_m\Phi$ and $I^m = 0$.

Suppose here that I_m and Z_m satisfy

$$\pounds_I g_{mn} = 0, \qquad I^p H_{pmn} + 2\partial_{[m}Z_{n]} = 0,$$

$$\pounds_I \Phi = 0, \qquad Z_m I^m = 0. \quad (12)$$

In this case, the string sigma model has a conserved current associated with the global symmetry $X^m \to X^m + \epsilon I^m$, where ϵ is an infinitesimal constant. Then the on-shell conserved Noether current is given by

$$J^a \equiv [I^m(g_{mn}\gamma^{ab} - B_{mn}\varepsilon^{ab}) - \tilde{I}_n\varepsilon^{ab}]\partial_b X^n, \quad (13)$$

where the 1-form $\tilde{I}_m$ is defined through

$$Z_m = \partial_m\Phi + I^n B_{nm} + \tilde{I}_m. \quad (14)$$

When the β functions take the forms of Eq. (11), the Weyl anomaly [Eq. (4)] becomes

$$\langle T^a{}_a \rangle \overset{\text{e.o.m.}}{\sim} -\mathcal{D}_a[(Z_m\gamma^{ab} - I_m\varepsilon^{ab})\partial_b X^m]. \quad (15)$$

Then, there is a rigid scale invariance [6], but it had been believed that the Weyl invariance would be broken because the counterterm [Eq. (8)] cannot cancel out the anomaly [Eq. (15)]. However, we will construct a *modified* local counterterm so as to cancel out Eq. (15).

Recall that the Lagrangian of the two-dimensional gravity is locally a total derivative,

$$\sqrt{-\gamma}R^{(2)} = \partial_a\alpha^a, \quad (16)$$

where α^a is a vector density that should transform as

$$\delta_\xi \alpha^a = \pounds_\xi \alpha^a = \xi^b \partial_b \alpha^a - \alpha^b \partial_b \xi^a + \partial_b \xi^b \alpha^a, \quad (17)$$

under diffeomorphisms on the worldsheet. We then introduce the following counterterm (since α^a is defined only locally, the integral itself here should be defined more carefully depending on topologies of the string worldsheet)

$$S_{\rm FT}^{(I,Z)} = -\frac{1}{4\pi}\int d^2\sigma \alpha^a (Z_m \partial_a X^m - I_m \varepsilon_a{}^b \partial_b X^m). \quad (18)$$

Note that this reduces to the FT term [Eq. (8)] when $I^m = 0$ and $Z_m = \partial_m\Phi$. Supposing that Z_m and I_m are independent of γ_{ab}, the contribution of the counterterm [Eq. (18)] to the Weyl anomaly becomes

$$\begin{aligned}
\langle T\rangle_{\rm FT} &= \frac{4\pi}{\sqrt{-\gamma}}\gamma^{ab}\frac{\delta S_{\rm FT}^{(I,Z)}}{\delta\gamma^{ab}}\\
&= \mathcal{D}_a[(Z_m\gamma^{ab} - I_m\varepsilon^{ab})\partial_b X^m]\\
&\quad - \varphi^a{}_a\mathcal{D}_c[(I_m\gamma^{cd} - Z_m\varepsilon^{cd})\partial_d X^m]. \quad (19)
\end{aligned}$$

Here, suggested by the identity in two dimensions,

$$\delta(\sqrt{-\gamma}R^{(2)}) = \partial_c[\sqrt{-\gamma}(\gamma^{ca}\mathcal{D}^b\delta\gamma_{ab} - \gamma^{ab}\mathcal{D}^c\delta\gamma_{ab})], \quad (20)$$

we have used the variation

$$\begin{aligned}
\delta\alpha^c &= \sqrt{-\gamma}(\gamma^{ca}\mathcal{D}^b\delta\gamma_{ab} - \gamma^{ab}\mathcal{D}^c\delta\gamma_{ab})\\
&\quad + \epsilon^{cd}\partial_d(\varphi^{ab}\delta\gamma_{ab}), \quad (21)
\end{aligned}$$

where φ^{ab} is a symmetric tensor made of the fundamental fields and their derivatives. In fact, the divergence in the last term of Eq. (19) vanishes by using the on-shell conservation law of a Noether current

$$\mathcal{D}_c[(I_m\gamma^{cd} - Z_m\varepsilon^{cd})\partial_d X^m] = \mathcal{D}_c J^c \overset{\rm e.o.m.}{\sim} 0, \quad (22)$$

and we obtain

$$\langle T\rangle_{\rm FT} \overset{\rm e.o.m.}{\sim} \mathcal{D}_a[(Z_m\gamma^{ab} - I_m\varepsilon^{ab})\partial_b X^m]. \quad (23)$$

Thus, this can exactly cancel out the anomaly [Eq. (15)].

Actually, the requirement [Eq. (11)] was proposed as the condition for the one-loop finiteness of string sigma model [6]. Now, we found that the Weyl symmetry can also be preserved upon introducing the above counterterm; hence one may anticipate that string theory should be consistently defined with the relaxed condition [Eq. (11)]. In the following, we will explain the condition [Eq. (11)] in terms of supergravity.

Generalized supergravity e.o.m.—From Eqs. (5) and (11), the condition for the Weyl invariance can be expressed as modified supergravity equations of motion,

$$R_{mn} - \frac{1}{4}H_{mpq}H_n{}^{pq} + 2D_{(m}Z_{n)} = 0,$$

$$-\frac{1}{2}D^k H_{kmn} + Z^k H_{kmn} + 2D_{[m}I_{n]} = 0. \quad (24)$$

In fact, these are GSE for g_{mn} and B_{mn} originally proposed in Ref. [2] and later derived in Ref. [3] from the requirement for the κ invariance of the GS type IIB superstring theory on an arbitrary background. There, the conditions [Eq. (12)] are also required for the κ invariance, and then equations of motion [Eq. (24)] lead to the following generalized dilaton equation of motion:

$$R - \frac{1}{12}|H|^2 + 4D_m Z^m - 4(|I|^2 + |Z|^2) = 0. \quad (25)$$

Equations of motion in Eqs. (24) and (25) define the NS-NS sector of the generalized supergravity. See Refs. [1–3] for the modified equations of motion for the Ramond-Ramond fields. In particular, when $Z_m = \partial_m\Phi$ and $I^m = 0$, these reduce to the conventional supergravity equations of motion.

In general, from the condition [Eq. (12)], we can choose a particular gauge where the 1-form $\tilde{I}_m$ in Eq. (14) vanishes [1,2]. Therefore, in the generalized supergravity, the generalization is characterized only by the vector field I^m. Note also that due to the presence of a Killing vector, any solution to GSE may be regarded as a nine-dimensional background via compactification on a circle.

In earlier works, many solutions to GSE have been obtained from the q deformation [22], homogeneous Yang-Baxter deformations [10,23–25], and non-Abelian T duality [10,15,26] (see also Ref. [13]), while it was not clarified whether these solutions are consistent string backgrounds at a quantum level or not. However, the cancellation of the Weyl anomaly that we have explicitly shown here would be an important step towards clarifying the quantum consistency (the solution obtained from the q deformation includes an imaginary Ramond-Ramond field, and would not be a consistent string background).

As presented in Refs. [1,27], we can regard solutions to GSE as solutions in DFT [28–31], which is a manifestly T-duality covariant formulation of supergravity. For the solutions of DFT, by using adapted coordinates where the Killing vector I^m is constant, we find that the dilaton has a linear dependence on the dual coordinate $\tilde{x}_m$ [1]. Moreover, if we perform a formal T duality (a formal T duality means the factorized T duality along a nonisometry direction x^z, which maps the coordinate x^z into the dual coordinate $\tilde{x}_z$. Such a transformation is a symmetry of the equations of motion of DFT) along the I^m direction, an arbitrary solution to GSE is mapped to a solution of the conventional supergravity that has a linear coordinate dependence in the dilaton [1,2,32].

Constructions of local α^a.—So far, we have not presented an explicit form of the vector density α^a. Let us

explain here two ways to construct α^a. Naïvely, from the defining relation,

$$\sqrt{-\gamma}R^{(2)} = \partial_a \alpha^a, \tag{26}$$

one might expect that α^a can be expressed consistently in terms of the metric γ_{ab}. However, it is not the case as it is clearly explained in Refs. [33,34]. To construct α^a in terms of the metric γ_{ab}, the general covariance on the worldsheet should be broken. On the other hand, similarly to the approach of Ref. [33], if we introduce a zweibein $e_{\bar{a}}{}^a$ on the worldsheet ($\bar{a}$ and $\bar{b}$ are the flat indices), we find that

$$\alpha^a = -2\sqrt{-\gamma}\, e_{\bar{a}}{}^a \omega_{\bar{b}}{}^{\bar{b}\bar{a}} \tag{27}$$

satisfies Eq. (26), where $\omega_{\bar{a}}{}^{\bar{b}\bar{c}}$ is the spin connection. In this case, despite α^a being manifestly covariant under diffeomorphisms, it is not covariant under the local Lorentz symmetry. In the following, we will introduce two possible manners to construct covariant expressions of α^a.

A construction with Noether current: The first approach is based on the approach explained in Sec. II. B of Ref. [34]. In two dimensions, if there exists a normalized vector field n^a ($\gamma_{ab}n^a n^b = \pm 1 \equiv \sigma$), we can show

$$\sqrt{-\gamma}R^{(2)} = 2\sigma\partial_a[\sqrt{-\gamma}(n^b\mathcal{D}_b n^a - n^a\mathcal{D}_b n^b)]. \tag{28}$$

In string theories on generalized supergravity backgrounds, there exists a natural vector field on the worldsheet, which is the Noether current J^a in Eq. (13). Supposing J^a is not a null vector on the worldsheet, the vector field n^a can be defined as $n^a \equiv (J^a/\sqrt{\sigma\gamma_{cd}J^c J^d})$. Then α^a is defined as

$$\alpha^a \equiv 2\sigma\sqrt{-\gamma}(n^b\mathcal{D}_b n^a - n^a\mathcal{D}_b n^b), \tag{29}$$

which is manifestly covariant and a local function of the fundamental fields. Moreover, by taking a variation of this α^a in terms of γ_{ab}, where the Noether current transforms as

$$\delta(\sqrt{-\gamma}J^a) = \delta(\sqrt{-\gamma}\gamma^{ab})\partial_b X^m I_m, \tag{30}$$

after a tedious computation, we find the desired variation formula [Eq. (21)] with φ^{ab} given by

$$\varphi^{ab} = \sigma\left(n^c\varepsilon_c{}^{(a}n^{b)} + \frac{2\varepsilon^{(a}{}_{(c}\delta^{b)}_{d)}}{\sqrt{\sigma\gamma_{gh}J^g J^h}}\mathcal{D}^c X^m I_m n^d\right). \tag{31}$$

Therefore, this fully determines the variation of α^a, for which the Weyl anomaly is canceled out in generalized supergravity backgrounds.

A construction from a gauged sigma model: As the second approach, we shall introduce some auxiliary fields to construct α^a. For simplicity, we take a gauge $\tilde{I}_m = 0$ here.

Let us consider the action of a gauged sigma model

$$S' = -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-\gamma}[(g_{mn}\gamma^{ab} - B_{mn}\varepsilon^{ab})D_a X^m D_b X^n - \tilde{Z}\varepsilon^{ab}F_{ab}], \tag{32}$$

where $D_a X^m \equiv \partial_a X^m - I^m A_a$, $F_{ab} \equiv \partial_a A_b - \partial_b A_a$, and $I \equiv I^m \partial_m$ satisfies the Killing equations. This model has a local symmetry,

$$X^m \to X^m + I^m v, \qquad A_a \to A_a + \partial_a v. \tag{33}$$

This action can reproduce the bosonic string action S_b after integrating out the auxiliary field $\tilde{Z}$. In order to cancel out the one-loop Weyl anomaly, we have to add the following local term to S':

$$S_c \equiv \frac{1}{4\pi}\int d^2\sigma\sqrt{-\gamma}R^{(2)}(\Phi + \tilde{Z}), \tag{34}$$

which is a higher order in α'. The contribution to the trace of the energy-momentum tensor coming from S_c is

$$\langle T\rangle_c = \frac{4\pi}{\sqrt{-\gamma}}\gamma^{ab}\frac{\delta S_c}{\delta\gamma^{ab}} \overset{\text{e.o.m.}}{\sim} \mathcal{D}^a(\partial_a\Phi + \partial_a\tilde{Z}). \tag{35}$$

The equations of motion for A_a and $\tilde{Z}$ give

$$\partial_a\tilde{Z} = \varepsilon^b{}_a J_b - |I|^2\varepsilon^b{}_a A_b, \qquad \varepsilon^{ab}F_{ab} = -\alpha'R^{(2)}, \tag{36}$$

where J_a is the Noether current defined in Eq. (13). Since the field strength F_{ab} vanishes to the leading order in α', by using the local symmetry [Eq. (33)], we can find a gauge where the order $\mathcal{O}(\alpha'^0)$ term vanishes

$$A_a = 0 + \alpha'\mathcal{A}_a, \qquad \varepsilon^{ab}(\partial_a\mathcal{A}_b - \partial_b\mathcal{A}_a) = -R^{(2)}. \tag{37}$$

Here, $\mathcal{A}_a$ is a quantity of order $\mathcal{O}(\alpha'^0)$. Then the trace [Eq. (35)] is evaluated as

$$\langle T\rangle_c \overset{\text{e.o.m.}}{\sim} \mathcal{D}^a(\partial_a\Phi + \varepsilon^b{}_a J_b) + \mathcal{O}(\alpha')$$
$$= \mathcal{D}_a[(Z_m\gamma^{ab} - I_m\varepsilon^{ab})\partial_b X^m] + \mathcal{O}(\alpha'). \tag{38}$$

This completely cancels out the one-loop Weyl anomaly [Eq. (15)], which comes from S'.

After eliminating $\tilde{Z}$, the action $S' + S_c$ becomes

$$S' + S_c = S_b + \frac{1}{4\pi}\int d^2\sigma\sqrt{-\gamma}[R^{(2)}\Phi + \epsilon_{ab}(-2\epsilon^{ac}\mathcal{A}_c)J^b - \alpha'|I|^2\gamma^{ab}\mathcal{A}_a\mathcal{A}_b]. \tag{39}$$

As it is clear from Eq. (37), the gauge field $\mathcal{A}_a$ may be regarded as the desired α^a via $\alpha^a = -2\epsilon^{ab}\mathcal{A}_b$. Then, by neglecting the higher order term in α', the resulting

expression is precisely the same as the standard sigma model action including our local counterterm $S_{\mathrm{FT}}^{(I,Z)}$ (18).

Note that the second term in the action [Eq. (39)] is the same as Eq. (5.13) of Ref. [16]. There, it was obtained by rewriting the nonlocal piece of the effective action S_{nonlocal} of Ref. [12] through the identifications of I_m and Z_m with some quantities in the Yang-Baxter sigma model. In Ref. [12], the nonlocal action S_{nonlocal} appeared in the process of non-Abelian T duality, and it played an important role to show the tracelessness of T_{ab}. However, according to the nonlocal nature of the effective action, by truncating the nonlinear term by hand, it was concluded in Ref. [12] that the string model (called the B' model) is scale invariant but not Weyl invariant. On the other hand, the action [Eq. (39)] or our local counterterm [Eq. (18)] with α^a defined as Eq. (29) is local and free from the Weyl anomaly.

Conclusion and Discussion.—We have constructed a local counterterm [Eq. (18)] that cancels out the Weyl anomaly of bosonic string theory defined in generalized supergravity backgrounds, without introducing a manifestly T-duality-covariant formulation of string theory. This result supports the Weyl invariance of string theory in generalized supergravity backgrounds. In order to claim the quantum consistency of string theory in generalized supergravity backgrounds, it may be necessary to study some aspects of the associated CFT picture in more detail (e.g., higher genus cases), but the first nontrivial test has been passed. Here, we have considered the bosonic string theory, but the same counterterm should work in the RNS superstring theory as well.

Our result indicates new possibilities of string theory in more general backgrounds. In fact, if we appropriately choose the parameters of the nine-dimensional gauged supergravity [35,36] and perform a formal T duality along the ten-dimensional direction, we can obtain the GSE [37]. In DFT or its extension, the exceptional field theory, we can construct various deformed supergravities that are similar to GSE by performing the formal T dualities and S dualities [38]. It is important to study the consistency of string theories defined on solutions of these deformed supergravities. A reasonable conjecture is that as long as the target space satisfies the equations of motion of the exceptional field theory, the string theory could be defined consistently. We hope to come back on this interesting topic in our future researches.

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