Absence of Criticality in the Phase Transitions of Open Floquet Systems

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We address the nature of phase transitions in periodically driven systems coupled to a bath. The latter enables a synchronized nonequilibrium Floquet steady state at finite entropy, which we analyze for rapid drives within a nonequilibrium renormalization group (RG) approach. While the infinitely rapidly driven limit exhibits a second-order phase transition, here we reveal that fluctuations turn the transition first order when the driving frequency is finite. This can be traced back to a universal mechanism, which crucially hinges on the competition of degenerate, near critical modes associated with higher Floquet Brillouin zones. The critical exponents of the infinitely rapidly driven system—including a new, independent one—can yet be probed experimentally upon smoothly tuning towards that limit.

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Introduction.—Many-body Floquet systems [1,2]—ensembles of particles subject to periodic driving—have recently triggered enormous research interest, both experimentally and theoretically. For example, very rapid drive can lead to effective conservative dynamics on short enough timescales, as was successfully exploited for Hamiltonian engineering of artificial gauge fields for ultracold atoms [3,4]. When instead the driving frequency Ω is closer to the natural energy scales of the problem, phenomena directly tied to driving can be observed, such as time crystals in atomic [5] and ionic [6] systems. Theoretical research spans the question of equilibration [7–20], the search for novel topological states without equilibrium counterparts [21–25], or driven analogs of many-body localization [26–28].

Specifically when it comes to implementations of periodically driven quantum systems with generic interactions, the ensuing irreversibility can lead to unbounded heating [7,8,10–12,14–16,18,20,29–32]. This represents an important hurdle to experimental implementation of many of the anticipated phenomena. A natural cure is to couple the driven system to a bath, such that the system can reach a Floquet steady state, with observables synchronized to the drive. Often such baths occur quite naturally, such as phonons in solid state superfluids [33–37], quantum dots and optical cavities [38–42], Brownian motors [43–45], spin chains [46–48], or cold atoms in optical lattices [49–51].

A natural and fundamental question in this large class of periodically driven, open quantum systems concerns the effect of the periodic drive on symmetry breaking phase transitions [52]. Previous work has addressed this question in the slowly driven limit, establishing the connection to Kibble-Zurek physics [53,54], as well as intermediate driving frequencies [48,55–57]. The effect of fast, but not infinitely fast driving remained elusive so far.

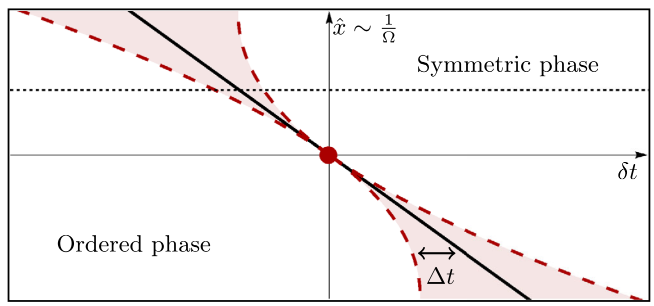

FIG. 1. Schematic phase diagram of the open Floquet system in 3D. δt is the distance from the phase transition in the infinitely rapidly driven system, $\hat{x} \sim \Omega^{-1}$ is the rescaled drive coefficient [cf. Eq. (7)]. The symmetry breaking phase transition occurs at the solid black line. It is second order only at $\Omega^{-1} = 0$ (red dot). Otherwise, fluctuations associated with the periodic drive transform the phase transition to weakly first order. The dashed red lines represent a crossover region between the known $\Omega^{-1} = 0$ scaling regime and one where scaling is frozen out (light red). The black dotted line represents a typical experimental path through the phase diagram.

In this work, we focus on a minimal model for a rapidly periodically driven open quantum system with phase rotation symmetry in three dimensions (3D). We identify a universal mechanism, according to which a seeming second-order phase transition is unavoidably driven first order by fluctuations.

Basic physical picture.—At first sight, the qualitative modification of the critical behavior by a fast scale may appear counterintuitive. It can be rationalized, however, when taking into account the fact that energy is not conserved in open Floquet systems. For any mode with a given frequency, there is a tower of modes with the same frequency but shifted by integer multiples of Ω. This represents the possibility of exchanging energy quanta

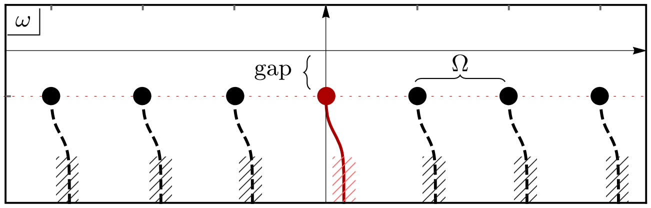

FIG. 2. Location of the poles of the retarded Green function in the complex frequency plane. The absence of energy conservation gives rise to lines of poles spaced by Ω. The imaginary parts of the pole is the damping rate of the corresponding mode. In a Floquet system, all the modes have the same damping rate and reach criticality simultaneously.

$n\Omega$ with the driving field—a notion of "high" and "low" energies, or "slow" and "fast" modes, is thus not well defined *a priori*.

Let us first consider the undriven situation for a general open system. The proper object to characterize criticality is the retarded single-particle Green function, Eq. (5). In the frequency and momentum domain it takes the form

$$G_{R;0}(\omega, \boldsymbol{q}) = \frac{1}{\omega - \epsilon_{\boldsymbol{q}} - i\gamma_{\boldsymbol{q}}},$$

where we have absorbed the quasiparticle residue in the definition of the energy $\epsilon_{\boldsymbol{q}}$ and damping rate $\gamma_{\boldsymbol{q}}$. In general, both $\epsilon_{\boldsymbol{q}}$, but also $\gamma_{\boldsymbol{q}}$ are momentum dependent, continuous functions. Within our model, they are given by $\epsilon_{\boldsymbol{q}} + i\gamma_{\boldsymbol{q}} = -Kq^2 - \mu_0$. The poles of $G_{R;0}$ depend on $\boldsymbol{q}$ and thus lie on a line in the complex frequency plane (central solid red line in Fig. 2). The imaginary part of the end point of the line (red dot) represents the system gap—it provides the decay rate for the slowest mode of the system. Tuning to criticality is then achieved by making this gap vanish, which happens when the line of poles touches the real axis. A renormalization procedure is then needed to control the singularities induced by the vanishing of the gap, but is well defined in the undriven open system case: It can be designed to gradually integrate out modes with decreasing q along the lines of poles (red overshadowed range).

The situation is drastically different for a periodically driven open system: Now poles are located not only on one central line, but also on all copies of that line shifted by integer multiples of Ω (dashed lines in Fig. 2), according to Floquet's formalism. In particular, when the system becomes critical, all lines extend jointly towards the real axis. Then, the usual strategy of integrating out high energy scales to find the effective low energy theory has to be carefully adapted: The coarse graining has to take place within each of the lines of poles; but in principle, all the critical poles associated to different lines have to be taken into account. Small scales are therefore integrated out as before, but fast scales remain even at criticality.

We find, however, that the contributions from additional poles are parametrically suppressed for a weak and fast drive. We take advantage of this, and devise an expansion in powers of Ω^{-1}. In addition, we work at one-loop order, which is systematic to first order in powers of $\epsilon = 4 - d$ [58]. Our approach is a double expansion, and systematic at $\mathcal{O}(\Omega^{-1}) \times \mathcal{O}(\epsilon)$.

The fact that phase transitions can be driven from second to first order by strong fluctuations occurs also in other contexts. One class is provided by the Coleman-Weinberg or Halperin-Lubensky-Ma mechanism, where additional gapless modes—such as gauge fields [59,60] or Goldstone modes [61,62]—compete with the critical ones in the vicinity of a phase transition. A second class derives from the Potts model, where the common prerequisite is that a continuous *external* (order parameter) symmetry is explicitly broken down to a nontrivial discrete subgroup [e.g., $U(1) \to Z_3$ in the Potts model [63,64], or similar phenomena in $O(N)$ models [65,66]]. This allows for new operators that may turn out to be relevant. Here we reveal another class, where a continuous *internal* symmetry (time translation invariance) is broken down to a discrete one—while keeping the external phase rotation symmetry $U(1) \simeq O(2)$ fully intact. Since discrete time translation invariance and energy conservation modulo Ω are two sides of the same coin, this provides an alternative, RG based viewpoint on our mechanism.

Open Floquet dynamics.—Microscopically, our system is made of generic interacting particles on a lattice, governed by a Hamiltonian with a bounded energy spectrum, and coupled to an external bath. The periodic time dependence typically occurs in the Hamiltonian $H(t + 2\pi/\Omega) = H(t)$, but it could also enter through periodic excitations of the bath. The dynamics have a $U(1)$ phase rotation symmetry, also respected by the drive. Our focus will be on phase transitions in three-dimensional systems, where the phase rotation symmetry is broken spontaneously. In the absence of drive, these are continuous, and correspond to critical points where the order parameter has strong large-scale fluctuations that overwhelm the microscopic degrees of freedom. We therefore employ an effective semiclassical, mesoscopic Landau-Ginzburg-type model, where only the dynamics of the complex order parameter ϕ is taken into account quantitatively [67]. The ensuing stochastic dynamics is governed by the Langevin equation,

$$i\partial_t \phi = [K\nabla^2 - \mu - g|\phi|^2]\phi + \xi. \tag{1}$$

ξ is a Gaussian white noise, which has correlation $\langle \xi(t, \boldsymbol{x})\xi^*(t', \boldsymbol{x}')\rangle = 2\gamma\delta(t - t')\delta(\boldsymbol{x} - \boldsymbol{x}')$, with $\gamma > 0$, and vanishes on average.

The couplings K, μ, g are complex valued. Their real parts account for the coherent dynamics inherited from the underlying Hamiltonian, and the coupling to the bath is responsible for their imaginary parts [71,72]. These determine the phase structure of the system's stationary state. In particular, a second-order phase transition accompanied with the spontaneous breakdown of phase rotation symmetry occurs in the undriven system when $\mathrm{Im}(\mu)$ is lowered below its critical value. In our case, all couplings are time dependent with period $2\pi/\Omega$, as a consequence of the microscopic drive. For definiteness, we choose a monochromatic drive

$$\begin{aligned} \mu &= \mu_0 + \mu_1 e^{-i\Omega t} + \mu_{-1} e^{i\Omega t}, \\ g &= g_0 + g_1 e^{-i\Omega t} + g_{-1} e^{i\Omega t}, \end{aligned} \tag{2}$$

with γ and K constants [73].

An effectively time independent, yet driven-dissipative model emerges not only when $\mu_{\pm 1} = g_{\pm 1} = 0$, but also in the limit of infinitely fast driving $\Omega \to \infty$. This limit is appropriate for typical settings in quantum optics, or quantum optical many-body systems [71,72,75]. In that case, the driving scale is approximated as infinitely fast $\Omega^{-1} = 0$ (rotating wave approximation). This problem exhibits a true second-order phase transition, but a modified criticality compared to equilibrium due to the microscopic breaking of detailed balance [76,77].

Here we focus on weakly and rapidly driven Floquet systems, where the driving frequency Ω is large, but still of comparable order to the other energy scales of the problem. Technically, we incorporate the leading rotating wave corrections $\mathcal{O}(\Omega^{-1})$ into the analysis of the near-critical driven open many-body problem.

Action and symmetries.—We rewrite the stochastic dynamics of Eq. (1) in terms of a dynamical functional integral [72,78], using the effective action $\Gamma[\Phi]$,

$$e^{i\Gamma[\Phi]} = \int D\varphi e^{i(S[\Phi+\varphi] + \int_{t,x} \varphi \delta\Gamma[\Phi]/\delta\Phi)}, \tag{3}$$

which includes all the field fluctuations, and provides the correlation and response functions. Equation (1) translates to the mesoscopic action

$$S = \int_{t,\boldsymbol{x}} \Phi^\dagger \begin{pmatrix} 0 & G_A^{-1} \\ G_R^{-1} & P_K \end{pmatrix} \Phi + (g\tilde{\phi}^* \phi|\phi|^2 + \text{c.c.}), \tag{4}$$

with $G_R^{-1} = i\partial_t - K\nabla^2 + \mu$, $G_A^{-1} = i\partial_t - K^*\nabla^2 + \mu^*$, and $P_K = i\gamma$. $\Phi = (\phi, \tilde{\phi})$ contains the order parameter ϕ, as well as the "response" or "quantum" field $\tilde{\phi}$ that is inherent to the dynamical functional formalism. The following symmetry considerations will guide our understanding: (i) Discrete time translations: Continuous time translations are implemented by $\Phi(t) \to \Phi(t + \Delta t)$ for arbitrary Δt. A drive with frequency Ω, breaks this continuous symmetry down to a discrete one, $\Delta t = 2\pi n/\Omega$ with n integer. The continuous symmetry is restored in the undriven limit $\mu_{\pm 1} = g_{\pm 1} = 0$, but also in the infinitely rapidly driven limit $\Omega^{-1} = 0$, where the rotating wave approximation is applicable. Conversely, its explicit breaking allows for the presence of additional dimensionful couplings $\mu_{\pm 1}$ and $g_{\pm 1}$. These are not compatible with the undriven dynamical ϕ^4 theory, and will lead to a new relevant direction at the Wilson-Fisher (WF) fixed point. (ii) Absence of detailed balance: Thermodynamic equilibrium can be formulated in terms of a dynamical symmetry, whose presence is equivalent to the obedience of thermal fluctuation-dissipation relations, i.e., detailed balance [79–81]. Out of equilibrium, this symmetry is generically lost. It can, however, formally be recovered by fine-tuning the drive and the dissipation. In our case, this would amount to having the ratios of all pairs of complex couplings to be both real and time independent (see Supplemental Material III [74]). Whenever this unnatural fine-tuning is not realized, we will encounter the effect described in this work. In this sense, it is generic, or universal, for periodically driven, open quantum systems.

Single-particle Green functions and critical poles.—The Wigner representation [14,82–85] of the single-particle Green functions $G_n(\omega)$, is the double Fourier transform of the real-time Green functions $G(t, t')$ (See Supplemental Material I A [74]). The discrete time-translation invariance is encoded in the index n. The retarded Wigner Green function $G_{R;n}(\omega)$ is composed of an infinite sum of poles located on lines in the complex plane (see Fig. 2 and Supplemental Material I B [74]). The residues of the poles of $G_{R;n}(\omega)$ are organized in a power series in $\mu_{\pm 1}/\Omega$. This means that a systematic expansion of the loop corrections in powers of Ω^{-1} is obtained by expanding the Green functions in powers of $\mu_{\pm 1}/\Omega$ before the frequency integrations are performed. To order 1 in $\mu_{\pm 1}/\Omega$, we find

$$\begin{aligned} G_{R;0}(\omega, \boldsymbol{q}) &= h_0^R(\omega, \boldsymbol{q}), h_0^R(\omega, \boldsymbol{q}) = (\omega + K\boldsymbol{q}^2 + \mu_0)^{-1}, \\ G_{R;n\neq 0}(\omega, \boldsymbol{q}) &= -\mu_n h_0^R\left(\omega - \frac{n\Omega}{2}, \boldsymbol{q}\right) h_0^R\left(\omega + \frac{n\Omega}{2}, \boldsymbol{q}\right). \end{aligned} \tag{5}$$

$h_0^R(\omega, \boldsymbol{q})$ describes the fundamental pole in the single-particle Green functions. We emphasize that this expansion still captures the correct pole structure and their location, which is fixed by the Floquet formalism. We see that the Green functions involve poles separated by integer multiples of Ω, that all become critical as the gap closes $\mathrm{Im}(\mu_0) \to 0$.

Perturbation theory.—As anticipated above, care must be taken when renormalizing the problem, due to the absence of a direct meaning of high and low energies. More practically, this forces us to keep the various poles on equal footing. This imposes a summation over the Floquet-Brillouin zone (FBZ) label n in the diagrammatics. The point is illustrated in the one-loop correction to the self-energy at zero frequency and momentum,

$$\Delta\mu_0 = 2i\sum_n \int_{\omega,\boldsymbol{q}} g_n G_{K;-n}(\omega,\boldsymbol{q}). \tag{6}$$

Using $G_K = -G_R P_K G_A$ and inserting the expansion Eq. (5), we can perform the frequency integration and expand it to $\mathcal{O}(\Omega^{-1})$

$$\Delta\mu_0 = \gamma \int_{\boldsymbol{q}} \frac{1}{|\mathrm{Im}(Kq^2+\mu_0)|}(g_0 + ix),$$
$$x = \frac{i}{\Omega}\sum_{n\neq 0} \frac{g_{-n}(\mu_n - \mu_{-n}^*)}{n} \equiv \sum_{n\neq 0}\tilde{g}_n. \tag{7}$$

This shows explicitly the appearance of divergences from the $n=0$ term, describing processes exclusively within the zeroth FBZ but also from $n\neq 0$, which describe scattering between different FBZs enabled by the drive. Along the frequency integral of Eq. (6), each pole contributes with the same degree of divergence. For a monochromatic drive however, we find $\mathrm{Res}(\omega_n) \sim (\mu_{\pm 1}/\Omega)^n$, which leads to a suppression of terms involving higher FBZs. Thus, all the FBZs contribute to the critical physics through these divergences, but interactions between different FBZs are parametrically small in Ω^{-1}.

RG analysis.—Equipped with the understanding of parametrically small but equally divergent contributions from the coupling to higher FBZs at leading order in Ω^{-1}, we proceed to the resummation of these divergences in an RG analysis to study their impact on the critical behavior. We first fix the canonical power counting: We transform spatial and temporal coordinates as $\hat{\boldsymbol{q}} = \boldsymbol{q}/k$ and $\hat{\omega} = \omega/[\mathrm{Im}(K)k^2]$. The couplings are then rescaled as

$$\hat{\mu}_n = k^{-2}\frac{\mu_n}{\mathrm{Im}(K)}, \qquad \hat{g}_n = k^{d-4}\frac{\gamma g_n}{4\mathrm{Im}(K)^2}, \tag{8}$$

(with $\tilde{g}_n$ being rescaled as g_n). To keep the argument of the oscillatory functions dimensionless, we also rescale $\hat{\Omega} = \Omega/[\mathrm{Im}(K)k^2]$.

In order to assess the relevance of these couplings at the interacting WF fixed point established at $\Omega^{-1}=0$ [77], we include fluctuations into our RG analysis. To this end, we work at leading order in the $\epsilon = 4-d$ expansion, which requires us to include one-loop corrections. The RG flow equations for μ and g take the form of a coupled set of differential equations for the dependence of the Fourier modes μ_n and g_n, on the running cutoff scale k (see Supplemental Material II [74]). To order Ω^{-1}, the RG flow equations of $\hat{\mu}_0$ and $\hat{g}_0$ are

$$k\partial_k \hat{g}_0 = -\epsilon\hat{g}_0 + \frac{10S_d}{|1+\hat{\mu}_0|(1+\hat{\mu}_0)}\hat{g}_0\left(\hat{g}_0 + \sum_m \hat{\tilde{g}}_{m\neq 0}\right),$$
$$k\partial_k\hat{\mu}_0 = -2\hat{\mu}_0 - \frac{4S_d}{|1+\hat{\mu}_0|}\left(\hat{g}_0 + \sum_{m\neq 0}\hat{\tilde{g}}_m\right), \tag{9}$$

with $S_d = 2\pi^{d/2}/[(d/2-1)!(2\pi)^d]$. The drive parameter is $\hat{x} = \sum_m \hat{\tilde{g}}_m$. Here and in the following we have simplified our system to make the computation more transparent: We choose K, μ, and g to be purely imaginary. Physically, this anticipates the decoherence that occurs in the vicinity of the phase transition, where all coherent dynamics fades away under coarse graining [77]. We have extracted a factor i from μ_0, g_0, and K. The couplings were renamed as $\mu_0 = i\mu_0'$, $g_0 = ig_0'$ and $K = iK'$ with μ_0', g_0' and K' real. We omit the primes to simplify the notation.

In principle, additional variables must be taken into account to compute the RG flow of $\hat{x}$, since it depends on all the harmonics of μ and g [see Eq. (7)]. However, as we show in Supplemental Material II A [74], the loop corrections to the flow of x can be neglected at $\mathcal{O}(\Omega^{-1})\times\mathcal{O}(\epsilon)$, giving rise to simple dimensional running

$$k\partial_k\hat{x} = -\epsilon\hat{x}. \tag{10}$$

The RG flow Eqs. (9) and (10) provide a generalization of the well-known, time translation invariant, RG flow. Indeed, the WF fixed point emerges when $\hat{x}=0$ (and $\hat{\mu}_0 = \mu^*$ and $\hat{g}_0 = g^*$). Our analysis reveals that the periodic drive gives rise to a new relevant coupling. In the absence of continuous time translation invariance, the critical point is thus bicritical: Two fine-tunings are necessary to reach it, and to reveal its critical scaling properties. Thus, when tuning across the symmetry breaking phase transition at finite Ω^{-1} (along the dotted line of Fig. 1) the additional relevant direction provides a finite correlation length. Moreover, in the absence of drive and far away from the critical point, the system is either in a disordered or an ordered phase. This property is robust for a finite, rapid drive since the Green functions are gapped in these phases (cf. Fig. 2), and perturbation theory converges [86]. This gives rise to a symmetry breaking phase transition without asymptotic criticality, which must be interpreted as a fluctuation induced first order transition.

The linear stability analysis of the RG flow equations close to the WF fixed point provides three quantitative predictions: (i) New scaling exponent: We find three critical exponents: $-2+2\epsilon/5 = -1/\nu$, ϵ and a new independent exponent $-\epsilon = -1/\nu_d$. The first two are known from the equilibrium system, with the first being negative and corresponding to the relevant direction. When the system is infinitely rapidly driven, it is tuned to criticality by tuning μ and/or g such that $\delta t = A(\delta g + 4\pi^2\delta\mu)$ vanishes (with $A>0$ a nonuniversal constant, $\delta\mu = \hat{\mu}_0 - \mu_0^*$ and $\delta g = \hat{g}_0 - g_0^*$). Then the correlation length diverges as $\xi \sim \delta t^{-\nu}$. In the presence of a drive however ($\hat{x}\neq 0$), the correlation length never diverges. δt can be tuned to maximize it (or, in RG terms, bring the flow as close as possible to the WF fixed point), but ξ ultimately crosses over to a finite value that scales as $\xi \sim \hat{x}^{-\nu_d}$. (ii) Shift of the phase transition: The location of the phase transition is shifted in a nonuniversal although drive-dependant way. The macroscopic phase is ultimately determined by the sign of $\Delta t = \delta t + A\hat{x}$. See Fig. 1 and Supplemental Material VI [74] for additional details. (iii) Observability of scaling: The above scaling analysis can be refined by replacing δt by Δt. $\hat{x}$ and Δt control the crossover between the two scaling regimes. For $|\Delta t| \gg |\hat{x}|^{\nu_d/\nu}$, the undriven relevant coupling dominates and the correlation length scales as $\xi \sim \Delta t^{-\nu}$. When $|\Delta t| \ll |\hat{x}|^{\nu_d/\nu}$ the correlation length saturates to $\xi \sim \hat{x}^{-\nu_d}$. This crossover is represented as red dashed lines in Fig. 1. The correlation length scales with Δt outside of the light red area and it saturates as the dashed red lines are crossed. In particular, this implies that the new critical exponent ν_d can be observed by varying Ω.

Conclusion.—There is an interesting "duality" of our scenario to the paradigmatic Kibble-Zurek phenomenology [87,88]. Both the equilibrium limit of an undriven system $\Omega = 0$, and the infinitely rapidly driven limit $\Omega^{-1} = 0$, afford time-independent descriptions, and exhibit symmetry breaking continuous phase transitions. Here we have shown that asymptotic scaling is cutoff at any finite Ω^{-1}. The Kibble-Zurek phenomenon occurs in the opposite limit of a slow driving: The nonequilibrium conditions are encoded in a slow quench of the couplings. Then the quench rate is analogous to x; it stops the correlation length from diverging. Although the underlying mechanisms are very different, in both cases the critical physics is masked and observable only upon smoothly approaching the extreme limiting cases. We reserve the exploration of this connection to future work.

Another intriguing direction of research concerns the applicability of our results to possible phase transitions in long-lived transient states of Floquet systems not coupled to external baths [10–12,15,16,18,20].

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[1] A. Eckardt, Rev. Mod. Phys. **89**, 011004 (2017).
[2] R. Moessner and S. L. Sondhi, Nat. Phys. **13**, 424 (2017).
[3] J. Struck, M. Weinberg, C. Ölschläger, P. Windpassinger, J. Simonet, K. Sengstock, R. Hoppner, P. Hauke, A. Eckardt, M. Lewenstein, and L. Mathey, Nat. Phys. **9**, 738 (2013).
[4] N. Goldman and J. Dalibard, Phys. Rev. X **4**, 031027 (2014).
[5] S. Choi, J. Choi, R. Landig, G. Kucsko, H. Zhou, J. Isoya, F. Jelezko, S. Onoda, H. Sumiya, V. Khemani, C. von Keyserlingk, N. Y. Yao, E. Demler, and M. D. Lukin, Nature (London) **543**, 221 (2017).
[6] J. Zhang, P. W. Hess, A. Kyprianidis, P. Becker, A. Lee, J. Smith, G. Pagano, I.-D. Potirniche, A. C. Potter, A. Vishwanath, N. Y. Yao, and C. Monroe, Nature (London) **543**, 217 (2017).
[7] L. D'Alessio and M. Rigol, Phys. Rev. X **4**, 041048 (2014).
[8] A. Lazarides, A. Das, and R. Moessner, Phys. Rev. E **90**, 012110 (2014).
[9] T. Shirai, T. Mori, and S. Miyashita, Phys. Rev. E **91**, 030101 (2015).
[10] A. Chandran and S. L. Sondhi, Phys. Rev. B **93**, 174305 (2016).
[11] E. Canovi, M. Kollar, and M. Eckstein, Phys. Rev. E **93**, 012130 (2016).
[12] M. Bukov, S. Gopalakrishnan, M. Knap, and E. Demler, Phys. Rev. Lett. **115**, 205301 (2015).
[13] V. Khemani, A. Lazarides, R. Moessner, and S. L. Sondhi, Phys. Rev. Lett. **116**, 250401 (2016).
[14] M. Genske and A. Rosch, Phys. Rev. A **92**, 062108 (2015).
[15] T. Kuwahara, T. Mori, and K. Saito, Ann. Phys. (Amsterdam) **367**, 96 (2016).
[16] T. Mori, T. Kuwahara, and K. Saito, Phys. Rev. Lett. **116**, 120401 (2016).
[17] T. Shirai, J. Thingna, T. Mori, S. Denisov, P. Hänggi, and S. Miyashita, New J. Phys. **18**, 053008 (2016).
[18] S. A. Weidinger and M. Knap, Sci. Rep. **7**, 45382 (2017).
[19] T. Shirai, T. Mori, and S. Miyashita, Eur. Phys. J. Spec. Top. **227**, 323 (2018).
[20] O. Howell, P. Weinberg, D. Sels, A. Polkovnikov, and M. Bukov, Phys. Rev. Lett. **122**, 010602 (2019).
[21] T. Kitagawa, E. Berg, M. Rudner, and E. Demler, Phys. Rev. B **82**, 235114 (2010).
[22] N. H. Lindner, G. Refael, and V. Galitski, Nat. Phys. **7**, 490 (2011).
[23] J. Cayssol, B. Dóra, F. Simon, and R. Moessner, Phys. Status Solidi RRL **7**, 101 (2013).
[24] M. S. Rudner, N. H. Lindner, E. Berg, and M. Levin, Phys. Rev. X **3**, 031005 (2013).
[25] T. Karzig, C.-E. Bardyn, N. H. Lindner, and G. Refael, Phys. Rev. X **5**, 031001 (2015).
[26] L. D'Alessio and A. Polkovnikov, Ann. Phys. (Amsterdam) **333**, 19 (2013).
[27] A. Lazarides, A. Das, and R. Moessner, Phys. Rev. Lett. **115**, 030402 (2015).
[28] P. Ponte, A. Chandran, Z. Papić, and D. A. Abanin, Ann. Phys. (Amsterdam) **353**, 196 (2015).
[29] M. Bukov, M. Heyl, D. A. Huse, and A. Polkovnikov, Phys. Rev. B **93**, 155132 (2016).
[30] L. W. Clark, A. Gaj, L. Feng, and C. Chin, Nature (London) **551**, 356 (2017).
[31] E. Kandelaki and M. S. Rudner, Phys. Rev. Lett. **121**, 036801 (2018).
[32] K. Shibata, A. Torii, H. Shibayama, Y. Eto, H. Saito, and T. Hirano, Phys. Rev. A **99**, 013622 (2019).

[33] H. Dehghani, T. Oka, and A. Mitra, Phys. Rev. B **90**, 195429 (2014).
[34] M. Knap, M. Babadi, G. Refael, I. Martin, and E. Demler, Phys. Rev. B **94**, 214504 (2016).
[35] M. Babadi, M. Knap, I. Martin, G. Refael, and E. Demler, Phys. Rev. B **96**, 014512 (2017).
[36] Y. Murakami, N. Tsuji, M. Eckstein, and P. Werner, Phys. Rev. B **96**, 045125 (2017).
[37] K. I. Seetharam, C.-E. Bardyn, N. H. Lindner, M. S. Rudner, and G. Refael, Phys. Rev. B **99**, 014307 (2019).
[38] X. Xu, M. Gullans, and J. M. Taylor, Phys. Rev. A **91**, 013818 (2015).
[39] R. Chitra and O. Zilberberg, Phys. Rev. A **92**, 023815 (2015).
[40] M.-A. Lemonde, N. Didier, and A. A. Clerk, Nat. Commun. **7**, 11338 (2016).
[41] J. Stehlik, Y.-Y. Liu, C. Eichler, T. R. Hartke, X. Mi, M. J. Gullans, J. M. Taylor, and J. R. Petta, Phys. Rev. X **6**, 041027 (2016).
[42] Z. Gong, R. Hamazaki, and M. Ueda, Phys. Rev. Lett. **120**, 040404 (2018).
[43] P. Hänggi and F. Marchesoni, Rev. Mod. Phys. **81**, 387 (2009).
[44] T. Salger, S. Kling, S. Denisov, A. V. Ponomarev, P. Hänggi, and M. Weitz, Phys. Rev. Lett. **110**, 135302 (2013).
[45] S. Denisov, S. Flach, and P. Hänggi, Phys. Rep. **538**, 77 (2014).
[46] K. I. Seetharam, C.-E. Bardyn, N. H. Lindner, M. S. Rudner, and G. Refael, Phys. Rev. X **5**, 041050 (2015).
[47] A. Lazarides and R. Moessner, Phys. Rev. B **95**, 195135 (2017).
[48] A. Lerose, J. Marino, A. Gambassi, and A. Silva, arXiv:1803.04490.
[49] J. Li, A. K. Harter, J. Liu, L. de Melo, Y. N. Joglekar, and L. Luo, arXiv:1608.05061.
[50] K. Iwahori and N. Kawakami, Phys. Rev. A **95**, 043621 (2017).
[51] T. Tomita, S. Nakajima, I. Danshita, Y. Takasu, and Y. Takahashi, Sci. Adv. **3**, e1701513 (2017).
[52] S. De Sarkar, R. Sensarma, and K. Sengupta, J. Phys. Condens. Matter **26**, 325602 (2014).
[53] G. Nikoghosyan, R. Nigmatullin, and M. B. Plenio, Phys. Rev. Lett. **116**, 080601 (2016).
[54] B. Feng, S. Yin, and F. Zhong, Phys. Rev. B **94**, 144103 (2016).
[55] G. Korniss, C. J. White, P. A. Rikvold, and M. A. Novotny, Phys. Rev. E **63**, 016120 (2000).
[56] H. Fujisaka, H. Tutu, and P. A. Rikvold, Phys. Rev. E **63**, 036109 (2001).
[57] G. M. Buendía and P. A. Rikvold, Phys. Rev. E **78**, 051108 (2008).
[58] The applicability of the ϵ expansion is a property of the WF fixed point. This expansion works because the fixed-point coupling vanishes as $d \to 4$, i.e., $g^* \sim \epsilon$. Here we investigate near-equilibrium critical physics in the sense that the relevant RG fixed point is the same as at equilibrium. Therefore the ϵ expansion remains valid here.
[59] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).
[60] B. I. Halperin, T. C. Lubensky, and S.-K. Ma, Phys. Rev. Lett. **32**, 292 (1974).
[61] M. E. Fisher and D. R. Nelson, Phys. Rev. Lett. **32**, 1350 (1974).
[62] D. R. Nelson, J. M. Kosterlitz, and M. E. Fisher, Phys. Rev. Lett. **33**, 813 (1974).
[63] G. R. Golner, Phys. Rev. B **8**, 3419 (1973).
[64] F. Y. Wu, Rev. Mod. Phys. **54**, 235 (1982).
[65] J. M. Carmona, A. Pelissetto, and E. Vicari, Phys. Rev. B **61**, 15136 (2000).
[66] A. Aharony, J. Stat. Phys. **110**, 659 (2003).
[67] In principle, the phenomenon of Bose selection [68–70] may arise. If this is the case, then a single order parameter would not be sufficient. We do not believe that this is relevant to our problem however. Indeed, we describe the onset of Bose condensation, while Bose selection happens deep in the symmetry broken phase. In particular, our results are also valid in the symmetric phase. Moreover, the infinitely rapidly driven limit relates to the undriven limit, in that time-translation symmetry invariance is effectively restored, and where there is usual single mode condensation. Finally, although Ref. [70] reproduces an experiment with a small interaction, Refs. [68] describe an ideal Bose gas. Here we describe the strongly correlated critical regime where interactions play a crucial role.
[68] D. Vorberg, W. Wustmann, R. Ketzmerick, and A. Eckardt, Phys. Rev. Lett. **111**, 240405 (2013).
[69] D. Vorberg, W. Wustmann, H. Schomerus, R. Ketzmerick, and A. Eckardt, Phys. Rev. E **92**, 062119 (2015).
[70] D. Vorberg, R. Ketzmerick, and A. Eckardt, Phys. Rev. A **97**, 063621 (2018).
[71] I. Carusotto and C. Ciuti, Rev. Mod. Phys. **85**, 299 (2013).
[72] L. M. Sieberer, M. Buchhold, and S. Diehl, Rep. Prog. Phys. **79**, 096001 (2016).
[73] We have checked (cf. Supplemental Material IV [74]) that the inclusion of a time dependence in these variables does not change our end result.
[74] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.122.110602 for details on the definitions and behavior of the Green functions, the derivation of the RG flow equations, details on the absence (or presence) of detailed balance, the physical interpretation of the drive coefficient, the effect of an eventual far-from-equilibrium fixed point and the linear stability analysis of the RG fixed point.
[75] A. J. Daley, Adv. Phys. **63**, 77 (2014).
[76] U. C. Täuber and S. Diehl, Phys. Rev. X **4**, 021010 (2014).
[77] L. M. Sieberer, S. D. Huber, E. Altman, and S. Diehl, Phys. Rev. Lett. **110**, 195301 (2013).
[78] A. Kamenev, *Field Theory of Non-Equilibrium Systems* (Cambridge University Press, Cambridge, England, 2011).
[79] U. Täuber, *Critical Dynamics: A Field Theory Approach to Equilibrium and Non-Equilibrium Scaling Behavior* (Cambridge University Press, Cambridge, England, 2014).
[80] L. M. Sieberer, A. Chiocchetta, A. Gambassi, U. C. Täuber, and S. Diehl, Phys. Rev. B **92**, 134307 (2015).
[81] C. Aron, G. Biroli, and L. F. Cugliandolo, SciPost Phys. **4**, 008 (2018).
[82] L. Arrachea, Phys. Rev. B **72**, 125349 (2005).

[83] B. H. Wu and J. C. Cao, J. Phys. Condens. Matter **20**, 085224 (2008).
[84] G. Stefanucci, S. Kurth, A. Rubio, and E. K. U. Gross, Phys. Rev. B **77**, 075339 (2008).
[85] N. Tsuji, T. Oka, and H. Aoki, Phys. Rev. B **78**, 235124 (2008).
[86] The phase with broken $U(1)$ symmetry however exhibits gapless Goldstone modes. In $d > 2$ dimensions, the associated IR fluctuations are phase space suppressed and do not destroy the ordered phase of an undriven system. Because the divergences emerging from the drive are copies of the single undriven divergence, we do not expect the presence of a drive to alter this behavior qualitatively.
[87] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).
[88] W. H. Zurek, Nature (London) **317**, 505 (1985).