Spinor-Helicity Formalism for Massless Fields in AdS$_4$

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In this Letter, we suggest a natural spinor-helicity formalism for massless fields in four-dimensional anti–de Sitter space (AdS$_4$). It is based on the standard realization of the AdS$_4$ isometry algebra $so(3,2)$ in terms of differential operators acting on $sl(2,\mathbb{C})$ spinor variables. We start by deriving the anti–de Sitter counterpart of plane waves in flat space and then use them to evaluate simple scattering amplitudes. Finally, based on symmetry arguments, we classify all three-point amplitudes involving massless spinning fields. As in flat space, we find that the spinor-helicity formalism allows us to construct additional consistent interactions as compared to approaches employing Lorentz tensors.

Introduction.—The spinor-helicity formalism by now has established itself as the most efficient framework for representing on-shell scattering amplitudes of massless particles in the 4D Minkowski space; see, e.g., [1,2] for reviews. The success of this formalism in the original setup has motivated various extensions—to other dimensions [3–7] and to massive particles [8–10]. At the same time, literature on the spinor-helicity formalism in curved space remains very limited. In [11], a version of the spinor-helicity formalism for massless fields in four-dimensional de Sitter space (dS$_4$) was proposed. Despite its virtues, in some aspects, it departs from the spinor-helicity formalism in flat space; e.g., it loses manifest Lorentz covariance.

In this Letter, we make an alternative proposal for the spinor-helicity formalism in four-dimensional anti–de Sitter space (AdS$_4$), which has all features of the flat space formalism and, in particular, reduces to the latter for conformal theories [12]. In this respect, it is worth mentioning the twistor approach [13], which is naturally adapted to describing fields in conformally flat spaces. Upon specializing to anti–de Sitter (AdS) backgrounds, the twistor approach can be used to obtain certain representations of massless scattering amplitudes in AdS$_4$ [14–20]. Our approach provides a different perspective on these results because it allows us to compute amplitudes directly from the space-time action and does not rely on a twistor-space description of the massless theories in question [21].

Besides having obvious motivations—e.g., development of tools that could facilitate computations of holographic/inflationary correlators and simplify their analytic structure—

we are also interested in gaining a better understanding of higher-spin interactions in flat and curved backgrounds and clarifying their relation. In particular, as was emphasized recently [25–27], in flat space, the spinor-helicity formalism and the light-cone approach admit additional cubic higher-spin vertices as compared to those built of Lorentz tensors. Moreover, these additional vertices are crucial for consistency of the higher-spin interactions [28,29] and are present in chiral higher-spin theories [30–32]; see also [33] for a related earlier result. Until recently, the fate of additional interactions in AdS was not clear. In [34], the expectation that they also exist in AdS$_4$ was confirmed in the light-cone approach. Below, we classify all consistent three-point amplitudes for massless particles in AdS$_4$ using the spinor-helicity formalism and find agreement with [34].

Spinor-helicity and flat space.—The basic fact about massless representations in the 4D Minkowski space is that they are labelled by two quantum numbers—helicity h and momentum p. Using the isomorphism $so(3,1) \sim sl(2,\mathbb{C})$, we have $p_\mu = -\frac{1}{2}(\sigma_\mu)^{\dot\alpha\alpha}\lambda_\alpha\bar\lambda_{\dot\alpha}$. For $h \geq 0$, the associated state can be represented by a potential

$$\varphi^h_{\nu_1\ldots\nu_h} = \varepsilon^+_{\nu_1}\ldots\varepsilon^+_{\nu_h}e^{ipx}, \tag{1}$$

where ε^+_ν is a polarization vector defined by

$$\varepsilon^+_\nu = \frac{(\sigma_\nu)^{\dot\alpha\alpha}\mu_\alpha\bar\lambda_{\dot\alpha}}{\mu^\beta\lambda_\beta}. \tag{2}$$

Here, μ is an auxiliary spinor and the ambiguity of its choice reflects gauge ambiguity. Alternatively, states can be represented by gauge invariant field strengths. For Eq. (1), the field strength reads

$$F^h_{\dot\alpha_1\ldots\dot\alpha_{2h}} = \bar\lambda_{\dot\alpha_1}\ldots\bar\lambda_{\dot\alpha_{2h}}e^{ipx}. \tag{3}$$

Extension to $h < 0$ and to fermions is straightforward. Once plane wave solutions Eq. (1) are available, one can

evaluate amplitudes using the Feynman rules in any theory of massless fields.

The amplitudes are strongly constrained by the Poincaré covariance. These constraints allow us to fix three-point amplitudes up to a coupling constant [35] to be

$$\mathcal{A}_I(h_1, h_2, h_3) = [12]^{d_{12,3}}[23]^{d_{23,1}}[31]^{d_{31,2}}\delta^4(p),$$
$$\mathcal{A}_{II}(h_1, h_2, h_3) = \langle 12 \rangle^{-d_{12,3}}\langle 23 \rangle^{-d_{23,1}}\langle 31 \rangle^{-d_{31,2}}\delta^4(p). \quad (4)$$

Here, $[ij] \equiv \epsilon_{\dot\alpha\dot\beta}\bar\lambda_i^{\dot\alpha}\bar\lambda_j^{\dot\beta}$, $\langle ij \rangle \equiv \epsilon_{\alpha\beta}\lambda_1^\alpha\lambda_2^\beta$, and $d_{ij,k} \equiv h_i + h_j - h_k$; and $p \equiv \sum_i p_i$ is the total momentum. To make Eq. (4) nontrivial, one assumes that the momenta are complex; hence, λ and $\bar\lambda$ are not complex conjugate to each other. Then, $\mathcal{A}_I$ ($\mathcal{A}_{II}$) is singular for $\sum_i h_i < 0$ ($\sum_i h_i > 0$) in the limit of real momenta and should be dropped as physically irrelevant.

AdS$_4$ and plane waves.—Massless representations of the AdS$_4$ isometry algebra $so(3,2)$ can be obtained by deforming the flat space translation generator as follows [36]:

$$\mathcal{P}_{\alpha\dot\alpha} = \lambda_\alpha\bar\lambda_{\dot\alpha} - R^{-2}\partial/\partial\lambda^\alpha\partial/\partial\bar\lambda^{\dot\alpha}, \quad (5)$$

where R is the AdS radius. This realization of massless representations is often referred to as the twisted adjoint representation [39]. Similarly to what happens in flat space, all algebra generators commute with the helicity operator $2H \equiv \bar\lambda^{\dot\alpha}\bar\partial_{\dot\alpha} - \lambda^\alpha\partial_\alpha$, which allows us to split the representation space into representations of definite helicity.

For our further purposes, it will be convenient to choose coordinates in AdS that make Lorentz symmetry manifest. Starting from the ambient space description of AdS as a hyperboloid $X^M X_M = -R^2$, $M = 0, 1, \ldots, 4$, and making the stereographic projection from $X^M = (0, \ldots, 0, -R)$, followed by the appropriate rescaling, we arrive at intrinsic coordinates x^μ, $\mu = 0, 1, 2, 3$, with the metric

$$ds^2 = \left(1 - \frac{x^2}{4R^2}\right)^{-2}\eta_{\mu\nu}dx^\mu dx^\nu. \quad (6)$$

The AdS boundary in these coordinates is given by $x^2 = 4R^2$, and the patch $x^2 < 4R^2$ ($x^2 > 4R^2$) corresponds to $X^4 > -R$ ($X^4 > -R$) in ambient coordinates. We will refer to these patches as the inner and outer patches, whereas their union will be referred to as the global AdS. Finally, we note that the inversion $x^\mu \leftrightarrow x^\mu(4R^2/x^2)$ acts as the reflection with respect to the origin in ambient space.

The AdS isometries act on bulk fields by Lie derivatives along Killing vectors. In our analysis, the Lorentz symmetry will be manifest, and so we only specify Killing vectors associated with deformed translations. They act on scalar fields by

$$P_a = -i\left(1 + \frac{x^2}{4R^2}\right)\delta_a^\mu\frac{\partial}{\partial x^\mu} + i\frac{x_a}{2R^2}x^\mu\frac{\partial}{\partial x^\mu}. \quad (7)$$

To deal with spinning fields in terms of spinors, we introduce a local Lorentz frame by means of the frame field

$$e_\mu^a = \left(1 - \frac{x^2}{4R^2}\right)^{-1}\delta_\mu^a, \quad (8)$$

where $a = 0, 1, 2, 3$. It can be used to convert tensor fields from the coordinate basis to the local Lorentz basis, e.g., $A^a = e_\mu^a A^\mu$. The frame field e_μ^a transforms as a 1-form with respect to diffeomorphisms. It is not hard to check that diffeomorphisms along Eq. (7) do not leave e_μ^a invariant. One can, however, complement them with compensating local Lorentz transformations so that the frame field becomes invariant. These compensating local Lorentz transformations then act on all fields carrying local Lorentz indices according to their index structure. In particular, for local Lorentz spinors, we have

$$(\delta_L P_{\alpha\dot\alpha}\bar\lambda)^{\dot\beta} = \frac{i}{4R^2}(\delta_{\dot\alpha}^{\dot\beta}x_{\alpha\dot\gamma} + \epsilon^{\dot\beta\dot\delta}x_{\alpha\dot\delta}\epsilon_{\dot\gamma\dot\alpha})\bar\lambda^{\dot\gamma},$$

$$(\delta_L P_{\alpha\dot\alpha}\lambda)^\beta = \frac{i}{4R^2}(\delta_\alpha^\beta x_{\gamma\dot\alpha} + \epsilon^{\beta\delta}x_{\delta\dot\alpha}\epsilon_{\gamma\alpha})\lambda^\gamma. \quad (9)$$

All spinor indices that we will encounter below refer to the local Lorentz basis.

Now, we will find the AdS counterpart of the flat plane wave solutions [40]. As in flat space, this is necessary to give the amplitudes we are going to find later a familiar field-theoretic interpretation. The plane waves will be derived based on a consideration that they should serve as intertwining kernels between two representations—the spinor-helicity representation and the space-time representation. We will focus on plane waves for field strengths because these are gauge invariant and do not require any auxiliary objects, such as reference spinors. Then, the Lorentz invariance requires that the indices of field strengths can only be carried by $\lambda_\alpha, \bar\lambda_{\dot\alpha}$, $(x\lambda)_{\dot\alpha} \equiv x_{\alpha\dot\alpha}\lambda^\dot\alpha$ or $(x\bar\lambda)_\alpha \equiv x_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha}$. All the remaining spinor indices should be covariantly contracted, which implies that plane waves may also depend on two scalars: $x_{\alpha\dot\alpha}\lambda^\alpha\bar\lambda^{\dot\alpha}$ and $x_{\alpha\dot\alpha}x^{\dot\alpha\alpha}$. Finally, we require that the action of the deformed translations on the plane wave in representation (5) agrees with that in space-time Eq. (7), supplemented with compensating Lorentz transformations Eq. (9). This results in a differential equation that fixes the functional dependence of plane waves on $x_{\alpha\dot\alpha}\lambda^\alpha\bar\lambda^{\dot\alpha}$ and $x_{\alpha\dot\alpha}x^{\dot\alpha\alpha}$. For $h \geq 0$, it has four linearly independent solutions

$$F_{\dot\alpha_1 \ldots \dot\alpha_{2h}}^{r|i} = \bar\lambda_{\dot\alpha_1} \ldots \bar\lambda_{\dot\alpha_{2h}}\left(1 - \frac{x^2}{4R^2}\right)_+^{1+h}e^{ipx},$$

$$F_{\dot\alpha_1 \ldots \dot\alpha_{2h}}^{r|o} = \bar\lambda_{\dot\alpha_1} \ldots \bar\lambda_{\dot\alpha_{2h}}\left(1 - \frac{x^2}{4R^2}\right)_-^{1+h}e^{ipx},$$

$$F_{\alpha_1 \ldots \alpha_{2h}}^{s|i} = \frac{(x\bar\lambda)_{\alpha_1} \ldots (x\bar\lambda)_{\alpha_{2h}}}{(x^2)^h}\left(1 - \frac{4R^2}{x^2}\right)_+^{1+h}e^{ipx\frac{4R^2}{x^2}},$$

$$F_{\alpha_1 \ldots \alpha_{2h}}^{s|o} = \frac{(x\bar\lambda)_{\alpha_1} \ldots (x\bar\lambda)_{\alpha_{2h}}}{(x^2)^h}\left(1 - \frac{4R^2}{x^2}\right)_-^{1+h}e^{ipx\frac{4R^2}{x^2}}, \quad (10)$$

where $x_+ \equiv x\theta(x)$ and $x_- \equiv -x\theta(-x)$. Analogously, solutions can be constructed for $h < 0$.

These solutions have the following properties: Plane waves $F^{r|i}$ ($F^{r|o}$) are supported on the inner (outer) patch that is for $x^2 < 4R^2$ ($x^2 > 4R^2$). The inversion maps $F^{r|i} \leftrightarrow F^{s|o}$ and $F^{s|i} \leftrightarrow F^{r|o}$. Solutions $F^{s|i}$ ($F^{s|o}$) are supported on $0 < x^2 < 4R^2$ ($x^2 < 0$ and $x^2 > 4R^2$). One can also consider the following linear combinations [42]:

$$F^{r|g}_{\dot{\alpha}_1 \ldots \dot{\alpha}_{2h}} = \bar{\lambda}_{\dot{\alpha}_1} \ldots \bar{\lambda}_{\dot{\alpha}_{2h}} \left(1 - \frac{x^2}{4R^2}\right)^{1+h} e^{ipx},$$

$$F^{s|g}_{\alpha_1 \ldots \alpha_{2h}} = \frac{(x\bar{\lambda})_{\alpha_1} \ldots (x\bar{\lambda})_{\alpha_{2h}}}{(x^2)^h} \left(1 - \frac{4R^2}{x^2}\right)^{1+h} e^{ipx(4R^2/x^2)}, \quad (11)$$

which are supported on the global AdS patch. Note that both $F^{r|g}$ and $F^{r|i}$ reduce to familiar flat plane waves in the flat space limit $R \to \infty$. Referring to the behavior of solutions at $x \to 0$, we will call $F^{r|g}$, $F^{r|i}$, and $F^{r|o}$ regular solutions, whereas $F^{s|g}$, $F^{s|i}$, and $F^{s|o}$ will be called singular [45].

At this point, one may wonder whether splitting of the plane wave solutions into patches as in Eq. (10) is physically meaningful and whether it is enough to consider only solutions supported on the global patch Eq. (11). We do not have much to say about this, except that splitting Eq. (10) is mathematically consistent with the symmetry arguments that we employed to derive these solutions. It is also worth remarking that, for fermionic fields, global

solutions Eq. (11) feature square roots leading to ambiguities of the analytic continuation across the interfaces between the patches. Any such continuation is consistent with the symmetry arguments discussed above.

Finally, we would like to comment on the role of conformal symmetry in this discussion. Massless fields in 4D are conformally invariant [46]; however, their description in terms of potentials breaks conformal invariance, except for the spin one case. Given that AdS and flat spaces are conformally equivalent, this means that at least the regular solution in Eq. (11) could have been obtained by applying the appropriate conformal transformation on a flat space plane wave solution. Putting it differently, our labelling of AdS plane waves is consistent with the flat space one modulo conformal transformations. Conformal invariance also allows us to conclude that the spin-1 potential is given by flat formula (1). A thorough investigation of potentials will be given elsewhere.

AdS_4 *scattering amplitudes.*—In AdS, one can define tree-level scattering amplitudes as the classical action evaluated on the solutions to the linearized equations of motion. Below, we will evaluate some simple amplitudes using plane wave solutions we have just obtained. We will focus on the scattering of regular plane waves because they have a smooth flat limit and a clearer connection to the familiar flat space amplitudes [47].

In the following, we will encounter integrals [49]:

$$\mathcal{I}^{r|i}_\lambda \equiv \int d^4 x \left(1 - \frac{x^2}{4R^2}\right)^\lambda_+ e^{ipx} = 2^{\lambda+6}\Gamma(\lambda+1)\pi i R^4 \left(e^{-i\pi[\lambda-(1/2)]} \frac{K_{\lambda+2}(-2iR(p^2+i0)^{1/2})}{(-2iR(p^2+i0)^{1/2})^{\lambda+2}} - \text{c.c.}\right),$$

$$\mathcal{I}^{r|o}_\lambda \equiv \int d^4 x \left(1 - \frac{x^2}{4R^2}\right)^\lambda_- e^{ipx} = 2^{\lambda+6}\Gamma(\lambda+1)\pi i R^4 \left(e^{i(\pi/2)} \frac{K_{\lambda+2}(-2iR(p^2+i0)^{1/2})}{(-2iR(p^2+i0)^{1/2})^{\lambda+2}} - \text{c.c.}\right), \quad (12)$$

where c.c. denotes the complex conjugate, and K is the modified Bessel function of the second kind. These formulas should be understood in the sense of distributions and are valid for real λ, except negative integers, where $\mathcal{I}^{r|i}_\lambda$ and $\mathcal{I}^{r|o}_\lambda$ diverge. In the following, we will only need $\mathcal{I}^{r|i}_\lambda$ and $\mathcal{I}^{r|o}_\lambda$ for integer values of $\lambda = n$. We find it convenient to use the notation

$$\mathcal{I}^{r|i}_n = \left(1 + \frac{\Box_p}{4R^2}\right)^n_+ \delta^4(p), \qquad \mathcal{I}^{r|o}_n = \left(1 + \frac{\Box_p}{4R^2}\right)^n_- \delta^4(p),$$

$$\mathcal{I}^{r|g}_n = \mathcal{I}^{r|i}_n + (-1)^n \mathcal{I}^{r|o}_n = \left(1 + \frac{\Box_p}{4R^2}\right)^n \delta^4(p), \quad (13)$$

which can be regarded as a result of a formal evaluation of the Fourier transform according to the rule $x^2 \to -\Box_p$. Representation (13) makes the distributional nature of amplitudes manifest and the flat space limit more intuitive.

Note that, for non-negative n, the right hand side for $\mathcal{I}^{r|g}_n$ in Eq. (13) is a well-defined distribution. It can be shown that this result is consistent with representation (12); see [49].

In these terms, the n-point amplitudes for a scalar self-interaction vertex $L = [1/(n!)]\sqrt{-g}\varphi^n$ are given by

$$\mathcal{A}^{r|i}_n = \mathcal{I}^{r|i}_{n-4}, \qquad \mathcal{A}^{r|o}_n = \mathcal{I}^{r|o}_{n-4}, \qquad \mathcal{A}^{r|g}_n = \mathcal{I}^{r|g}_{n-4}, \quad (14)$$

depending on the AdS patch we are using. For $n = 3$, the amplitude is divergent, which is consistent with the standard holographic analysis [50], where the three-point Witten diagram for $\Delta = 1$ scalars also gives a divergent result.

Similarly, we can evaluate more general vertices involving the field strengths of spinning fields. For example, for $L = \frac{1}{2}\sqrt{-g}\varphi F^{\dot{\alpha}_1 \dot{\alpha}_2} F_{\dot{\alpha}_1 \dot{\alpha}_2}$, for different patches, we find

$$\mathcal{A}^{r|i}_3 = [23]^2 \mathcal{I}^{r|i}_1, \qquad \mathcal{A}^{r|o}_3 = [23]^2 \mathcal{I}^{r|o}_1,$$

$$\text{and} \quad \mathcal{A}^{r|g}_3 = [23]^2 \mathcal{I}^{r|g}_1. \quad (15)$$

Amplitudes of the form $\mathcal{A}_3^{r|g}$ have been previously derived in the twistor literature [14–20].

Finally, considering the Yang-Mills vertex, as a consequence of conformal invariance, we find exactly the same amplitude as in flat space, except that now, we also have its variants associated with different patches. In fact, conformal invariance of the Yang-Mills action implies that the same conclusion holds for all tree-level spinor-helicity amplitudes in AdS.

Having studied some simple examples, we will now move to the case of general spinning three-point amplitudes. In contrast to the previous analysis, in which we computed amplitudes using their field-theoretic definition, in the following, the amplitudes will be found by requiring correct transformation properties—that is, solely from representation theory considerations. As in flat space, the Lorentz covariance is manifest and is achieved by combining spinors into spinor products. Moreover, once helicities on external lines are fixed, this imposes constraints on the homogeneity degrees of spinors. For amplitudes being genuine functions of spinor products, this leads to an ansatz

$$\mathcal{A}(h_1, h_2, h_3) = [12]^{d_{12,3}}[23]^{d_{23,1}}[31]^{d_{31,2}} f(x, y, z), \quad (16)$$

where $x \equiv [12]\langle 12 \rangle$, $y \equiv [23]\langle 23 \rangle$, and $z \equiv [31]\langle 31 \rangle$. It only remains to impose correct transformation properties with respect to deformed translations

$$(\mathcal{P}_{\alpha\dot\alpha}^1 + \mathcal{P}_{\alpha\dot\alpha}^2 + \mathcal{P}_{\alpha\dot\alpha}^3)\mathcal{A}(h_1, h_2, h_3) = 0. \quad (17)$$

This gives a system of differential equations on $f(x, y, z)$. It can be shown that, when at least one helicity is nonzero, one has four linearly independent solutions [51]:

$$\mathcal{A}_{\text{I}} = [12]^{d_{12,3}}[23]^{d_{23,1}}[31]^{d_{31,2}}\mathcal{I}_{\sum h-1}^{r|i},$$

$$\mathcal{A}_{\text{II}} = [12]^{d_{12,3}}[23]^{d_{23,1}}[31]^{d_{31,2}}\mathcal{I}_{\sum h-1}^{r|o},$$

$$\mathcal{A}_{\text{III}} = \langle 12 \rangle^{-d_{12,3}}\langle 23 \rangle^{-d_{23,1}}\langle 31 \rangle^{-d_{31,2}}\mathcal{I}_{-\sum h-1}^{r|i},$$

$$\text{and} \quad \mathcal{A}_{\text{IV}} = \langle 12 \rangle^{-d_{12,3}}\langle 23 \rangle^{-d_{23,1}}\langle 31 \rangle^{-d_{31,2}}\mathcal{I}_{-\sum h-1}^{r|o}, \quad (18)$$

where $\mathcal{I}$ are given by Eq. (12). When all helicities are vanishing, f_{I} coincides with f_{III} and f_{II} coincides with f_{IV}.

Classification (18) is different from Eq. (4) only in two respects. The first is that the $so(3, 2)$ covariance turns out to be consistent with splitting the global AdS into two patches, with each being associated with its own amplitude. This explains why we get four solutions in Eq. (18) instead of two solutions in flat space. The second difference is that the flat space momentum-conserving delta functions in AdS are replaced with one of the $\mathcal{I}$ Eq. (12), depending on the patch one is interested in. Based on the flat limit, where $\mathcal{A}_{\text{I}}$ and $\mathcal{A}_{\text{II}}$ ($\mathcal{A}_{\text{III}}$ and $\mathcal{A}_{\text{IV}}$) for $\sum_i h_i < 0$ ($\sum_i h_i > 0$) are

singular for real momenta, we argue that they should also be dropped in AdS as physically irrelevant. It is worth mentioning that these amplitudes are divergent; see discussion below Eq. (12). The same refers to all amplitudes with $\sum_i h_i = 0$. Finally, we remark that $\mathcal{A}_{\text{I}}$ ($\mathcal{A}_{\text{II}}$) for $\sum h = 1$ ($\sum h = -1$) in flat space Eq. (4) are conformally invariant [52]. This explains why these are equal to the associated amplitudes in global AdS Eq. (18).

Amplitudes with three singular plane waves using the inversion reduce to amplitudes in which all plane waves are regular. Amplitudes in which regular and singular plane waves are mixed require a separate analysis. If these are genuine functions, they should be given by linear combinations of Eq. (18). Another potential possibility is that they are given by distributions. In this respect, it is worth noting that, by considering an ansatz for a distribution supported on $p = 0$ and requiring Eq. (17), we again end up with Eq. (18), where the $\mathcal{I}$ appear in representation (13).

Conclusions.—In the present Letter, we suggested a natural generalization of the spinor-helicity formalism to AdS$_4$. We started by generalizing the familiar flat space plane wave solutions to AdS and then used them to evaluate some simple three-point amplitudes. We also classified all consistent spinning three-point amplitudes by requiring correct transformation properties. We found that, as in flat space, for three generic spins, by picking different signs of helicities, one can construct four different parity-invariant amplitudes. At the same time, approaches that involve Lorentz tensors result only in two consistent parity-invariant structures, both on the bulk [53–58] and boundary [59,60] sides. This phenomenon directly generalizes an analogous one in flat space and is consistent with a recent analysis in the light-cone gauge [34].

The amplitudes that we computed were defined as the classical action evaluated on the particular basis of solutions to the linearized equations of motion. This definition is related to the holographic one—in which amplitudes are identified with boundary correlators and computed in the bulk by Witten diagrams [61–63]—by a mere change of a basis for the states appearing on external lines. Unlike bulk-to-boundary propagators, the plane wave solutions that we employed do not have a boundary limit that would allow us to associate them with localized boundary sources. Instead, they have a transparent flat limit, which also makes the flat limit of the spinor-helicity amplitudes more intuitive. In this respect, our plane waves serve as the properly focused scattering states necessary to access flat space physics from holography; see, e.g., [64–66]. An explicit transformation relating the two bases will be given elsewhere.

An obvious future direction is to extend these results to higher-point amplitudes and see how various bulk scattering processes manifest themselves in an amplitude's analytic structure; see, e.g., [67–69] for related work. Optimistically, clear understanding of the analytic structure of AdS spinor-helicity amplitudes may lead to the development of the on-shell methods, which are as efficient, as in flat space.

Finally, our construction may be useful in shedding light on how higher-spin no-go theorems (see [70] for review) can be circumvented in flat space. Thus far, it is known how to construct higher-spin theories in flat space only in the chiral sector [30–32], whereas their parity-invariant completions are obstructed by nonlocalities. At the same time, higher-spin theories in AdS have solid support from holography [71,72]. We believe that the connection between higher-spin theories in flat and in AdS spaces does exist, and both sides will benefit from its clarification.

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