

Observation of Topological Edge Modes in a Quasiperiodic Acoustic Waveguide

David J. Apigo,^{1,*} Wenting Cheng,¹ Kyle F. Dobiszewski,² Emil Prodan,^{3,†} and Camelia Prodan^{1,‡}

¹*Department of Physics, New Jersey Institute of Technology, Newark, 07102 New Jersey, USA*

²*Albert Dormans Honors College, New Jersey Institute of Technology, Newark, 07102 New Jersey, USA*

³*Department of Physics, Yeshiva University, New York, 10016 New York, USA*

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Topological boundary and interface modes are generated in an acoustic waveguide by simple quasiperiodic patterning of the walls. The procedure opens many topological gaps in the resonant spectrum and qualitative as well as quantitative assessments of their topological character are supplied. In particular, computations of the bulk invariant for the continuum wave equation are performed. The experimental measurements reproduce the theoretical predictions with high fidelity. In particular, acoustic modes with high Q factors localized in the middle of a breathable waveguide are engineered by a simple patterning of the walls.

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The ideas based on topological concepts [1,2] have revolutionized the field of condensed matter physics and led to the discovery of topological insulators and superconductors. The latter have been classified at the end of the previous decade [3–6] and a table of strong topological phases has been conjectured. One of their common characteristics is the emergence of disorder-immune boundary modes whenever a sample is halved. Physics akin to that of topological condensed matter systems has also been predicted in classical wave-supporting materials [7,8] and many examples of topological metamaterials have been reported in the literature [9–22].

At the same time, it has been pointed out that the periodic table of topological systems is highly enhanced if more complex systems are considered, such as the quasiperiodic or quasicrystalline ones [23–26]. In Ref. [27], K -theoretic arguments [28,29] were applied for quasiperiodically coupled discrete mechanical resonators. The finding was that, if these are single-mode resonators, then every gap in the bulk resonant spectrum is topological, in the sense that it will be completely filled by boundary spectrum under any boundary condition. The practical value of the finding is that the quasiperiodic Hamiltonians display a large number of topological gaps; hence one can generate localized wave modes in both space and energy by simply halving the system.

In this work, we put these general principles to the test in a completely different regime and we implement them for the first time using sound waves. Acoustic setups have been successfully used in the past to generate topological edge modes [30–32] and even to map the Hofstadter butterfly [33]. In particular, Refs. [32,33] introduced reconfigurable acoustic resonant structures where the building blocks are sealed acoustic chambers connected via thin bridges. They have isolated resonant modes; hence these structures fall under the umbrella of patterned resonators introduced in

Ref. [27] and they can be analyzed by similar methods. However, these types of acoustic structures are not breathable, which is a key requirement for many practical applications. As such, here we ask the question: Can one generate topological edge and interface modes by patterning the walls of an acoustic waveguide without impeding the air flow?

As we shall see, the answer is yes, but the methods of analysis are very different from those introduced in Ref. [27]. Indeed, the picture of coupled discrete resonators is no longer applicable and a full continuum medium treatment must be employed for the theoretical analysis. Furthermore, the topological character of the spectral gaps cannot be taken for granted because the waveguide supports many overlapping modes. As such, a new assessment of the topological character is introduced based on the continuum version of the lattice noncommutative Chern number proposed in Ref. [25] and achieved in Ref. [34]. This invariant is here evaluated numerically using the methods developed in Refs. [35,36]. Let us recall from Ref. [27] that the role of aperiodicity in this type of application is to generate virtual dimensions and, as we shall see [37], the Chern number mentioned above is defined on a three-dimensional noncommutative manifold, while for discrete patterns it is defined on a two-dimensional manifold.

At the experimental level, challenges exist because some of the spectral bands are very narrow and this, together with the aperiodicity, can lead to irregular mode profiles, although the bulk states are extended. As such, the only way to accurately map the bulk spectrum is to collect data from a large number of points along the waveguide. Following this protocol, we map not only the frequency but also the spatial profile of the bulk modes. Furthermore, inside the topological bulk gaps, we were able to detect sharp edge modes, which flow with the phason degree of

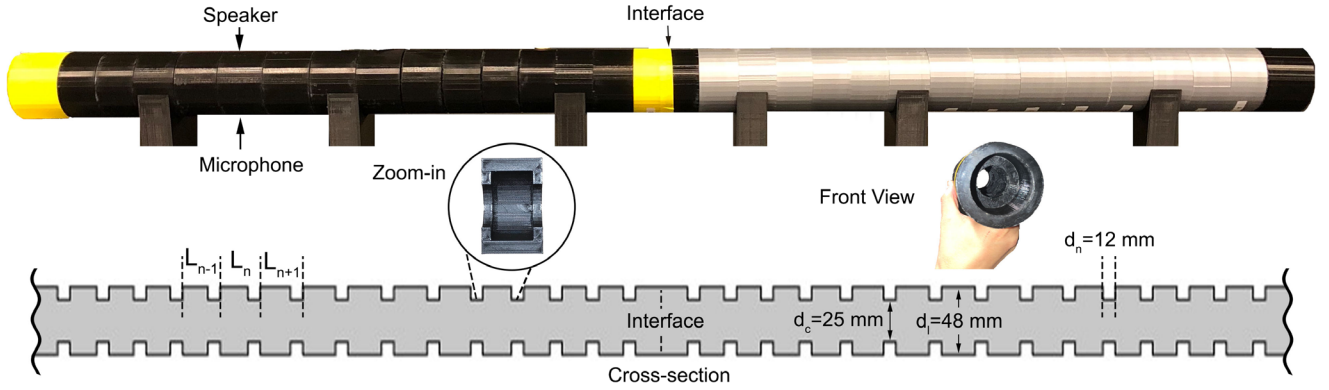


FIG. 1. Top: Photograph of the waveguide configuration used to measure topological interface modes. Bottom: Cross section and geometrical parameters. The waveguide consists of interlocking 3D-printed PLA parts as shown in the inset and it is mirrored relative to the domain wall indicated by the dashed line. For experimentation, a speaker is placed at portholes accessible in each chamber and a piezoelectric microphone is inserted into an opposite porthole. The portholes that are not in use are sealed. The lengths L_n were generated with Eq. (1) and their average was fixed at $L_{av} = 40$ mm. The parameters in Eq. (1) were fixed at $\Delta L = 0.2L_{av}$ and $\theta = (2\pi/\sqrt{117})$. This particular irrational fraction of 2π accepts a good rational approximation $\theta = (9\pi/48) + \mathcal{O}(10^{-3})$, which was used in some of the numerical calculations. The system was also run without a domain wall, for bulk and edge measurements.

freedom in a manner consistent with the computed Chern numbers.

The quasiperiodic acoustic waveguide consists of a uniform cylindrical tube decorated with walls. The parts were 3D printed out of polylactic acid (PLA) using an Ultimaker 3 and then assembled as in Fig. 1. The walls have identical thickness, but the spacings between adjacent walls are modulated according to the algorithm

$$L_n = L_{av} + \Delta L \sin(n\theta + \phi), \quad n \in \mathbb{Z}. \quad (1)$$

The geometric parameters used in the experiments are supplied in Fig. 1. To make the above labels meaningful, we assume that the waveguide is centered at a point inside L_0 . In Eq. (1), θ is an angle incommensurate with 2π , which will be kept fixed during the measurements, and ϕ is the phason, which should be let to vary. For example, a simple relabeling $n \rightarrow n + m$, which corresponds to recentering the waveguide, will change ϕ into $(\phi + m\theta) \bmod 2\pi$. Since θ is incommensurate, these relabelings alone will sample the phason densely in the $[0, 2\pi]$ interval. L_{av} in Eq. (1) is the average distance between the walls and ΔL sets the magnitude of the fluctuations in L_n .

In the inset of Fig. 1, we show a front view of the waveguide, confirming that air can flow freely through the structure. It is then somewhat striking that, with the proposed patterning, we can stop sound propagation over several intervals of frequencies and, furthermore, we can generate, very much on demand, topological sound modes localized at any desired location along the tube. As opposed to an ordinary resonant mode produced in a fully sealed acoustic chamber, the interface modes produced in the present work have less contact with the boundary; hence they are expected to have very high Q factors, a much desired characteristic for practical applications.

To understand the effect of the patterning, we report in Fig. 2 the dispersion of the acoustic modes for clean and periodically ($L_n = L_{av}$) patterned waveguides, as well as the resonant spectrum of the aperiodically patterned waveguide [L_n set by Eq. (1)]. As expected for quasi-one-dimensional wave propagation, the periodic pattern opens spectral gaps in the gapless spectrum of the clean tube. These gaps, however, are not topological. The role of aperiodicity is to open additional gaps in the spectrum that, as one can see, resemble quite closely the Hofstadter butterfly [38], when mapped as a function of θ . As we shall see, these are the gaps that carry nontrivial bulk-topological invariants prompting the topological edge and interface modes. Let us mention that the spectra in Fig. 2 were produced with an in-house FORTRAN code, which diagonalizes the Laplace operator expressed in the

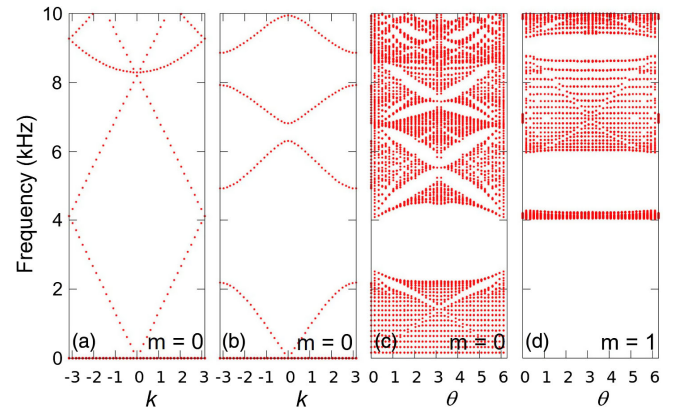


FIG. 2. (a) Dispersion of the acoustic modes for the unpatterned waveguide, for $m = 0$ sector. (b) The band structure of a periodically patterned waveguide (i.e., $\theta = 0$), for $m = 0$ sector. (c), (d) Resonant spectrum of a patterned waveguide as function of θ , for $m = 0, 1$ sectors, respectively.

cylindrical coordinates (ρ, z) and resolved over the azimuthal symmetry sectors. In appropriate units, the operator reads

$$\Delta_m = -\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + \frac{m^2}{\rho^2} - \frac{\partial^2}{\partial z^2}, \quad m = 0, \pm 1, \dots, \quad (2)$$

and the von Neumann condition is considered at the boundary. Recall that the latter is set by θ and ϕ ; hence Δ_m depends in a fundamental way on these parameters. The Laplace operator was discretized using finite differences.

The protocol for acoustic data acquisition was as follows. Sinusoidal signals of duration 1 s and amplitude of 0.5 V were produced by a Rigol DG1022 function generator, amplified by a Crown XLS 2502 power amplifier with the gain set to 6, and then applied on a CUI Inc. GF0501 speaker, placed at one of the portholes. A PCB Piezotronics Model-378C10 microphone and a PCB Piezotronics Model-485B12 power conditioner acquired the acoustic signals at a porthole opposite the speaker (see Fig. 1). To account for the frequency-dependent response of the components, a separate measurement is performed with the waveguide removed but speaker and microphone kept in the same positions. All readings are normalized by the output of these measurements. The outputs were read by a custom LABVIEW code via a National Instruments USB-6112 data acquisition box and the ratio of the two measurements is stored on a computer for graphic renderings.

For the bulk measurements, the protocol was repeated for all 48 chambers of a patterned waveguide, with frequency scans from 500 to 6000 Hz in 25 Hz steps. The results are reported in Fig. 3. When the data are rendered as a function of frequency and chamber index, clear extended acoustic modes can be identified. Furthermore, when the data are collapsed on the frequency axis, clear spectral gaps can be identified, two of which are predicted to be topological. Unfortunately, the $m = 0, 1$ spectra overlap above the nontopological gap (see Fig. 2) and the higher frequency topological gaps could not be experimentally resolved. Let us note that the agreement between experiment and theory in Fig. 3 is better than 95%.

To assess the topological character of the gaps, we employ the bulk-boundary correspondence for continuum models established in Ref. [34]. The bulk-topological invariant is supplied by the noncommutative Chern number of the gap projection $P_G = \chi_{(-\infty, G]}[\Delta_m(\phi) - G]$:

$$\text{Ch}(P_G) = \text{Tr}_L \{ P_G [\partial_\phi P_G, (Z, P_G)] \}, \quad (3)$$

where Z is the position operator parallel to the tube and Tr_L is the trace per length. The invariant can be computed at any arbitrary but fixed phason value, which is a consequence of the Birkhoff ergodic theorem [39]. With the Laplacian

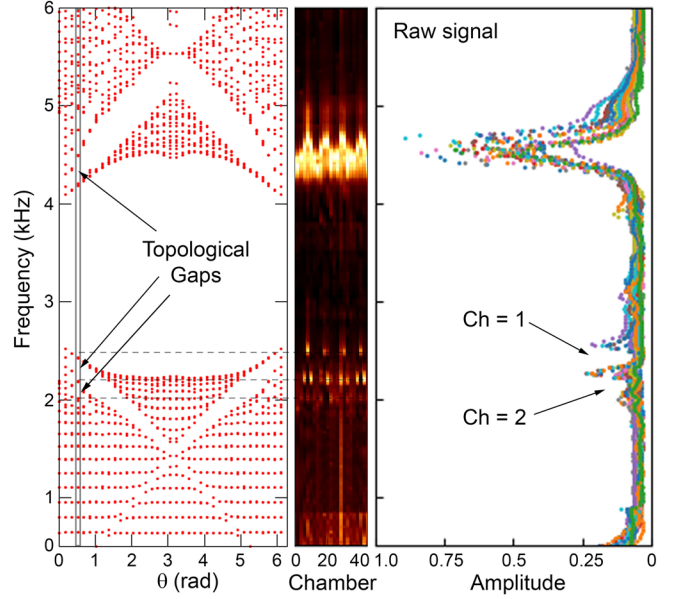


FIG. 3. Bulk resonant spectrum for the geometry described in Fig. 1. Left: Theoretical resonant spectrum reproduced from Fig. 2(c), with arrows indicating the topological gaps. The vertical marking identifies $\theta = (2\pi/\sqrt{117})$, used in experiments. Center: Normalized microphone readings from the center of 48 chambers, recorded over a wide frequency interval. Right: Collapse on the frequency axis of the intensity plot reported in the mid panel. Three spectral gaps can be clearly identified in the experimental data and seen to be well aligned with the theoretical calculations. The values of the Chern numbers for the two topological gaps are also indicated.

discretized on a lattice via finite differences, Eq. (3) was evaluated using methods which are by now standard [35,36]. The results are reported in Fig. 3, confirming that the smaller gaps are topological. Furthermore, Ref. [34] established the existence of a boundary topological invariant which counts the number of chiral boundary bands, as well as the equality between the bulk and boundary invariants.

The presence of chiral modes, in accordance with the above bulk-boundary principle, is confirmed by our numerical simulations reported in Fig. 4(a). To map the boundary modes experimentally, the acquisition protocol was applied on the second chamber from the left physical edge, which was plugged. The frequency was swept from 2.0 to 2.6 kHz in steps of 25 Hz and the value of the phason was modified by moving the physical edge sequentially to the right, hence, from L_0 to L_n , $n = 1, 2, \dots$. The results are presented in Fig. 4(b) and they indeed confirm the existence of one chiral band in the upper topological gap and two such bands in the lower topological gap. For reference, we reproduced in Fig. 4(c) the experimental data from Fig. 3, from where the exact position of the bulk edges can be inferred. As one can see, the boundary resonances occur inside the bulk gaps and the dispersion with ϕ is consistent with the theoretical prediction.

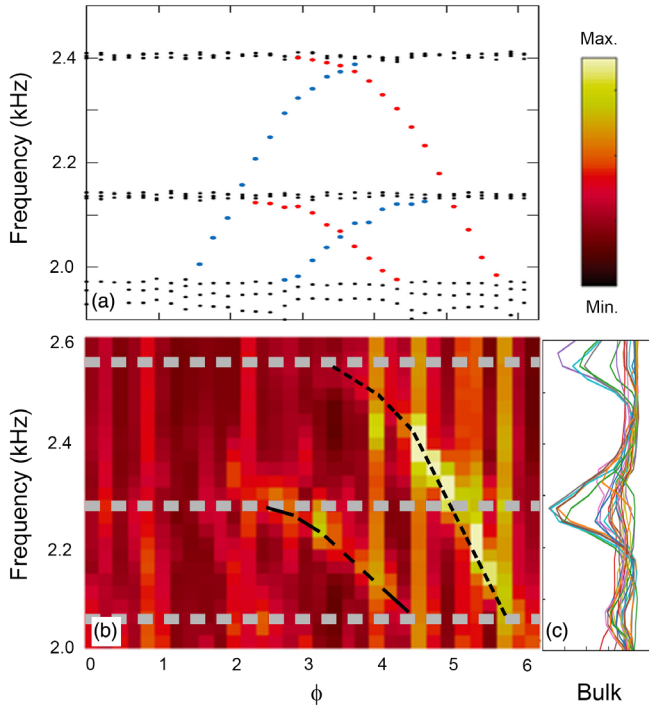


FIG. 4. Topological edge spectrum. (a) Theoretical prediction of the spectral flow against the phason parameter ϕ , demonstrating the existence of chiral bands. The red (blue) marks relate to the left (right) edge of the waveguide. (b) Experimental mapping of the spectral flow, confirming the existence of chiral bands. (c) The measurements for bulk spectrum, reproduced from Fig. 3, indicating the position of the bulk gap edges.

We now demonstrate that a localized topological edge mode can be created without the assistance of any plug. For this, we consider a domain wall configuration,

$$\dots|L_{31}|L_{30}|L_{29}|L_{29}|L_{30}|L_{31}| \dots,$$

where the waveguide is mirror reflected relative to the left edge of the L_{29} chamber. This particular index was chosen because moving the origin to that chamber generates a phason, $\phi = (29\theta) \bmod 2\pi$, which coincides with the value where strong midgap edge modes were observed in the first topological gap. Since Eq. (3) is odd under reflection, with this patterning, an interface between topological systems with opposite Chern numbers is created. As such, the bulk-boundary principle predicts the emergence of 2Ch acoustic modes localized at the interface.

The experimental measurements are reported in Fig. 5(a). The frequencies were swept as in Fig. 4 and, in order to probe the localization of the acoustic modes, the speaker and microphone were placed at several port-holes at and away from the interface. A strong and sharp resonance was detected in the first topological gap (Ch = 1), when the measurements were performed one and two chambers away from the interface. The resonance

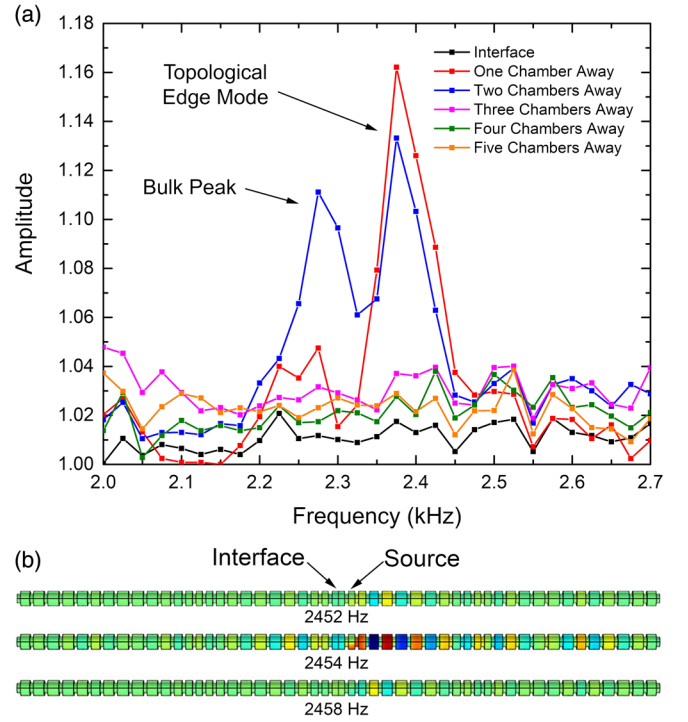


FIG. 5. (a) Topological interface mode, measured for a waveguide configuration similar to that in Fig. 1. The spatial localization of the interface mode was mapped by moving the speaker and microphone incrementally away from the domain wall. (b) The topological interface mode is also observed in COMSOL simulations. Red, blue, and green colors represent high, low, and zero pressure variations, respectively.

was not detectable farther away from the interface or at the interface itself. A similar resonance can be detected at the other side of the interface, leading to a full confirmation of the topological bulk-boundary prediction. The interface mode is also observed in a standard COMSOL simulation, as shown in Fig. 5(b).

In conclusion, we have demonstrated that topological edge and interface modes can be created by a simple quasiperiodic patterning of an acoustic waveguide. The topological gaps can be easily identified when the resonant spectrum is mapped as a function of modulation parameter θ . Furthermore, a topological invariant was computed and shown to be in agreement with the number of observed topological chiral edge modes.

As we have seen, quasiperiodicity opens topological gaps inside the bands of the periodic structure, which resemble the Hofstadter butterfly when mapped as a function of θ . Optimization over ΔL in Eq. (1) and the geometric parameters of the tube, as well as improvements in materials (e.g., by replacing the polymer with metal), can highly enhance these topological gaps and the Q factors of the topological boundary and interface modes. Other than that, the procedure requires no further fine-tuning and, due to its simplicity, we believe it can be easily incorporated in practical applications. The present analysis can also serve

as a model for acoustic implementations of many other promising aperiodic structures [40].

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*david.j.apigo@njit.edu

†prodan@yu.edu

*cprodan@njit.edu

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