Editors' Suggestion

Universal Hall Response in Interacting Quantum Systems

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We theoretically study the Hall effect on interacting M-leg ladder systems, comparing different measures and properties of the zero temperature Hall response in the limit of weak magnetic fields. Focusing on $SU(M)$ symmetric interacting bosons and fermions, as relevant for, e.g., typical synthetic dimensional quantum gas experiments, we identify an extensive regime in which the Hall imbalance Δ_H is universal and corresponds to a classical Hall resistivity $R_H = -1/n$ for a large class of quantum phases. Away from this high symmetry point we observe interaction driven phenomena such as sign reversal and divergence of the Hall response.

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In its semiclassical approximation [1] the Hall response of a conductor threaded by a weak perpendicular magnetic field B is independent of the bar geometry. Because of Galilean invariance, the ratio of the electric field E_y and the longitudinal current density j_x, the Hall coefficient $R_H = E_y/j_x B$, uniquely depends on the effective charge-q carrier density n, $R_H = -1/nq$, providing an extraordinary tool for the characterization of solid state systems [2–4]. Nevertheless, deviations from parabolic bands in realistic condensed matter systems lead dependence on the curvature of the Fermi surface [5–7] and large deviations of the Hall coefficient from its classical expressions are expected in strongly correlated phases, in particular when constrained to low dimensions, e.g., ladderlike systems. Several theoretical approaches addressed the Hall effect in strongly correlated quantum phases [8–16], but unbiased calculations of the Hall coefficient remain challenging in interacting systems.

Tremendous experimental progress with ultracold lattice gases [17–21] in artificial magnetic fields paves the way to the exciting study of the Hall effect in highly controllable clean many-body systems. Several experiments have so far observed the Hall [17,22,23] and quantum Hall [24,25] effect and a systematic measurement of the Hall coefficient was shown recently by Genkina *et al.* [26]. Several of these experiments were performed with synthetic-lattice dimensions [22,23,27–30], realizing Harper-Hofstadter (HH) like models on a ladder [31,32]. Promising ongoing efforts towards the realization of strong correlations in such systems [20,33] motivate the detailed theoretical analysis of the Hall effect in interacting many body systems.

In this Letter, we study the Hall response of strongly interacting fermions and bosons on quantum ladders. We introduce the Hall imbalance Δ_H, which can be directly observed in ultracold atom experiments. The key result is the observation of an extensive *universal* regime of parameters, where Δ_H is constant, corresponding to $R_H = -1/n$ behavior, independent of particle statistics and interaction strength. By means of numerical matrix product state DMRG simulations [34,35], supported by analytical arguments, we analyze the robustness of this effect and show its relevance for quench dynamics experiments with state-of-the-art quantum gases with synthetic dimensions. We discuss the breaking of this universality out of $SU(M)$ symmetry, in which divergencies or sign reversals of Δ_H signal phase transitions. We also provide an approximate formula to calculate Δ_H at equilibrium with open boundary conditions.

We consider HH ribbons on M legs, see Fig. 1(a). The Hamiltonian is $H = H^x_{\rm kin} + H^y_{\rm kin} + H_{\rm int}$, with hopping along the ladder $H^x_{\rm kin} = -t_x \sum_{j,m} e^{i\chi(m-m_0)/M} a^\dagger_{j,m} a_{j+1,m} +$ H.c. $\{m_0 = (M-1)/2$ and $m \in [0, M-1]\}$ and in the transverse direction $H^y_{\rm kin} = -t_y \sum_{j,m} a^\dagger_{j,m} a_{j,m+1} +$ H.c. Here, $a^{(\dagger)}_{j,m}$ is a fermionic or bosonic annihilation(creation) operator on the ladder rung j and leg m. In this work, we consider on-rung interactions as relevant for synthetic dimension experiments $H_{\rm int} = \sum_{j,m,m'} (U_{mm'}/2) n_{j,m} n_{j,m'}$, with $n_{j,m} = a^\dagger_{j,m} a_{j,m}$. Typical ultracold atom experiments realize an approximate $SU(M)$ symmetry $U_{m,m'} = U$ [36].

Recent experiments, such as Ref. [26], accelerate the lattice gas by a linear potential $\Delta\mu \sum_{jm} j n_{jm}$. Subsequent monitoring of the evolution of the spin resolved momentum distribution allows for the measurement of the density polarization $P_y = 2\sum_{m=0}^{M} (m - m_0) n_{j,m}$ and the current $J_x = -it_x \sum_{j,m} e^{i\chi(m-m_0)} a^\dagger_{j,m} a_{j+1,m} +$ H.c. as function of time τ. Figures 1(a)–1(c) sketch this procedure for a $M = 3$ leg ladder system, initially prepared in the ground state. After the quench for $\tau > 0$ a total current develops $\langle J_x \rangle \neq 0$ as well as a finite density imbalance $\langle P_y \rangle \neq 0$. Both quantities essentially grow linearly with τ and, hence, the resulting Hall imbalance

$$\Delta_H = \left.\frac{\langle P_y \rangle}{\chi \langle J_x \rangle}\right|_{\chi\to 0} \tag{1}$$

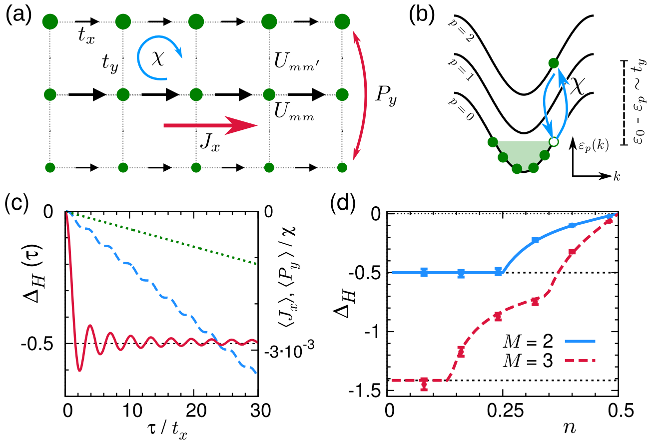

FIG. 1. (a) Particle (dots) and current (arrows) density on an M-leg ladder threaded by a flux χ, after applying a tilt ($M = 3$, subtracting $\tau = 0$ values for clarity). (b) Band structure of the M-leg ladder as function of the wave-vector k along the x direction. (c) Quench dynamics of the Hall imbalance Δ_H, polarization $\langle P_y \rangle / \chi$, and current $\langle J_x \rangle$ (solid, dashed, and dotted lines, respectively) for tilted free fermions ($t_x = t_y$, $n = 1/5$, $\chi = 0.01$, $\Delta\mu = 0.01 t_x / L$, see text). (d) Δ_H for free fermions as function of n. Solid lines correspond to Eq. (2), and points with error bars to time averages of the dynamics in (c). Δ_H is constant when only the lower band is occupied and kinks correspond to the occupation of upper bands in (b).

oscillates around a finite constant value for small enough times $\tau \lesssim L/2t$ as long as the finite size of the system can be neglected, see Fig. 1(c).

In the case of adiabatic dynamics, the Hall imbalance (1) can be equivalently derived *at equilibrium* for the same system on a ring with L rungs threaded by a Aharonov-Bohm flux ϕ, i.e., with periodic boundary conditions (PBC) along x and the substitution $a^\dagger_{j,m} a_{j,m+1} \to e^{i\phi/L} a^\dagger_{j,m} a_{j,m+1}$ [39]. The flux ϕ induces a persistent current $\langle J_x \rangle = L \partial_\phi \langle H \rangle |_{\phi \to 0}$, leading to a finite imbalance (1) for $\chi \neq 0$. This reactive Hall response has been discussed in detail by Prelovšek *et al.* [11]: the Hall coefficient R_H is found by adding an extra term $E_y P_y$ to the Hamiltonian, adjusted such that $\langle P_y \rangle = 0$ [11,40]. Nevertheless, the Hall imbalance Δ_H is a more direct and simpler observable in recent quantum gas experiments and we will consider both quantities in the following. Remarkably, both Δ_H and R_H can be expressed as derivatives of the ground state energy $\mathcal{E}_0(\phi, \chi, E_y)$ [40], $L\Delta_H = \partial_{\phi\chi E_y} \mathcal{E}_0 / \partial_{\phi\phi} \mathcal{E}_0$ and $R_H = -L\Delta_H / \partial_{E_y E_y} \mathcal{E}_0$ for $\phi, \chi, E_y = 0$.

We first identify the universal regime of interest shown in Fig. 1, in which Δ_H is a constant function of n, corresponding to $R_H = -1/n$, which can be well understood in free particle systems. For χ, $E_y = 0$, the generic spectrum of M coupled wires on the lattice coupled by hopping to each other is made of M bands $\varepsilon_p(k) = \varepsilon_x(k) + \varepsilon_y(p)$, that are labeled with the index $p \in [0, M-1]$, in k space (wave vector along the x direction). As shown in Fig. 1(b) the bands are split by the transverse hopping strength t_y. In the specific case of the HH model we have $\varepsilon_x(k) = -2t_x \cos(k)$ and $\varepsilon_y(p) = -2t_y \cos[\pi(p+1)/(M+1)]$. Analytical expressions of Δ_H are readily found in perturbation theory in χ, E_y, ϕ [40]. For noninteracting fermions one finds

$$\Delta_H = \frac{\sum_{p<P} v_{F,p} \mathcal{I}_p}{\sum_{p<P} v_{F,p}}, \qquad R_H = \frac{-\Delta_H}{\sum_{p<P} n_p \mathcal{I}_p}. \tag{2}$$

in which $v_{F,p}$ and n_p are the Fermi velocity and density of fermions in band p and $P \leq M$ is the number of occupied bands, see Fig. 1(b). The coefficients $\mathcal{I}_p$ depend on the details of the Hamiltonian [40]. Figure 1(d) shows examples of Δ_H as a function of the total density $n = \sum_p n_p$, from Eq. (2). The Hall imbalance Δ_H exhibits a series of kinks, corresponding to the change in the number of occupied bands P. It is remarkable, as shown in panels (c)–(d) of Fig. 1, that sudden quench behavior (even though $\Delta\mu$ is a "weak" perturbation) is perfectly reproduced by equilibrium calculations. Because of particle-hole symmetry the Hall response vanishes identically at half filling [13] and for the case of hole conductance $n > 0.5$ the sign of the Hall imbalance is inverted [11]. The Hall response (2) is finite when only the lowest band $p = 0$ is occupied:

$$\Delta_H = q\mathcal{I}_0(M, t_\perp) \quad \text{and} \quad R_H = -1/n. \tag{3}$$

This is an interesting result as it recovers *exactly* the usual $1/n$ behavior for R_H of free particles in the continuum, generally violated on the lattice [5–7] and *always* applies for noninteracting bosons as well [40]. Note, that Eq. (3) shows the possibility to observe a finite Hall response in systems in which only two Fermi points are present, for which one would naively have expected a single chain behavior and thus the absence of Hall effect. This Letter focuses on the regime corresponding to Eq. (3). We will now provide numerical calculations supported by analytical arguments that this observation carries over to the correlated regime for generic $SU(M)$ symmetric bosons or fermions in a large family of ground states.

Figures 2(a)–2(b) show DMRG results for Δ_H for interacting fermions on $M = 2$ leg ladder as function of n and U (see the Supplemental Material [40] for further examples). The solid lines of Fig. 2(b) indicate the different phase transitions for $\chi = 0$ known for the integrable Fermi-Hubbard model for finite t_y. In the fully polarized (FP) state, for which all N particles occupy the lowest band $p = 0$, we find $\Delta_H = -1/2$ is independent of interaction strength and density in perfect accordance with Eq. (3).

Because of particle-hole symmetry in the Mott (MI) and band insulator (BI) phases at half filling Δ_H vanishes identically. As the fully paired phase (PSF), superfluid of composite bosonic pairs, for attractive interactions can be related to the insulating MI phase by means of a Shiba transformation [48], we also find here identically vanishing

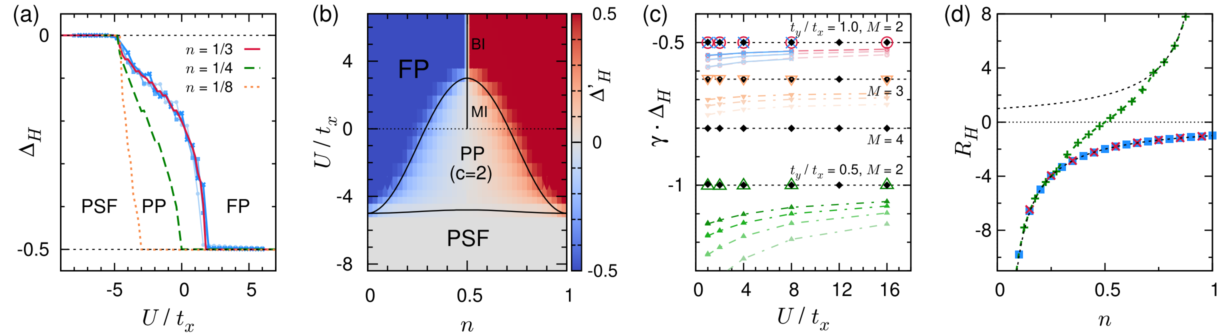

FIG. 2. Hall effect in $SU(M)$ symmetric ladders. (a) and (b) Δ_H for $M = 2$ fermions with on-rung interactions U (DMRG data, $t_x = t_y$). (a) Data for different fillings $n = 1/3$, $1/4$, and $1/8$—symbols depict PBC results for $L = 24$, 36, 48 (empty circle, empty square, cross) rungs for $n = 1/3$. The other lines correspond to Δ'_H (OBC, $L = 96$ rungs). The solid lines in (b) correspond to the phase transitions for the one-dimensional Fermi-Hubbard model at $\chi = 0$ with Zeeman field of strength t_y [48]. The phases include fully polarized, partially paired, and paired superfluid, and Mott insulator and band insulator phases (see text for details). (c) Δ_H of the $SU(M)$-symmetric Bose Hubbard model for $M = 2$ and $t_x = t_y$, $n = 1/2$, and $n = 1/4$ as well as $t_x = t_y/2$, $n = 1/4$ (small symbols: cross, empty circle, empty triangle) and $M = 3$, $t_x = t_y$, $n = 1/8$ (inverted triangle) with (bottom to top) $L = 24$, 28, 32, and 36 rungs—larger symbols depict the extrapolated value in the thermodynamic limit $L \to \infty$ [40]. The filled diamond symbols show Δ'_H data for $M = 2$ and 3 and 4 leg ladders ($L = 60$, $n = 1/2M$). The horizontal dashed lines depict Eq. (3). Scaled by $\gamma = 4/M^2$ for clarity. (d) $1/n$-behavior (dotted lines) of the Hall resistivity R_H for $M = 2$ bosons with $U = 16t_x$, $t_x = t_y$ (cross) and $U = 8t$, $t_x = t_y/2$ (empty square) as well as fermions $U = t_x = t_y$ (plus).

Δ_H. In the partially paired (PP) two component phase (with a central charge $c = 2$), as in the noninteracting case, Δ_H strongly deviates from the universal form interpolating smoothly between results of the FP and PSF phases.

Remarkably, this universal behavior carries over to interacting bosons. Figure 2(c) shows the universal values for softcore $SU(M)$ symmetric bosons for $M = 2$, 3, and 4, different densities and interactions strengths. As anticipated, Δ_H is just given by Eq. (3), that is a function of t_y/t_x, independent of the interaction strength. Note, that J_x and P_y themselves exhibit a complicated dependence on the parameters n and U. Interestingly, for the bosonic model, as it is not particle-hole symmetric, we observe approaching of the same finite constant even for half filling, where for sufficiently strong interactions the system enters an insulating state. In order to verify the universality of the corresponding Hall coefficient, in Fig. 2(d) we show R_H for some of the previous examples. As conjectured we find $R_H = -1/n$ for bosons and fermions independent of M, density, and interaction strength as long as only the lower band $p = 0$ is occupied.

The universality of Eq. (3) is understood by inspecting the expectation values of P_y and J_x in general many-body perturbation theory. Upon introduction of the eigenstates $\{|\alpha\rangle\}$ of $H(\chi = 0)$, of energy $\mathcal{E}_\alpha$, with $|0\rangle$ the ground state, the leading contribution to the polarization reads [40]

$$\langle 0|P_y|0\rangle = \chi \sum_{\alpha \neq 0} \frac{\langle 0|P_y|\alpha\rangle\langle\alpha|\tilde{J}_x|0\rangle + \text{c.c.}}{\mathcal{E}_0 - \mathcal{E}_\alpha}, \quad (4)$$

in which we introduced the asymmetric current $\tilde{J}_x = -t_x \sum_{j,m} i e^{i\phi/L}(m - m_0) a^\dagger_{j,m} a_{j+1,m} + \text{H.c.}$ Consider the commutator $[P_y, H(\chi=0)] = \sum_{j,p,p'} C_{p,p'}(\epsilon_p - \epsilon_{p'}) \tilde{a}^\dagger_{j,p} \tilde{a}_{j,p'}$ in which we switched to operators $a_{j,p}$ diagonalizing H^y_{kin} and thus annihilating particles on band p (the factor $C_{p,p'}$ is given in the Supplemental Material [40]). We consider the FP ground state, stabilized, e.g., by repulsion, large t_y, or bosonic enhancement. For excited states $|\alpha_p\rangle$, with 1 particle in band $p > 0$, the commutator leads to the fact that $[\mathcal{E}_{\alpha_p} - \mathcal{E}_0]\langle\alpha_p|P_y|0\rangle = [\epsilon_y(p) - \epsilon_y(0)]\langle\alpha_k|P_y|0\rangle$. For the cases in which $\langle\alpha_p|P_y|0\rangle \neq 0$, the energy difference of the interacting many-body states becomes trivial $\mathcal{E}_{\alpha_p} - \mathcal{E}_0 = \epsilon_y(p) - \epsilon_y(0)$. As an important consequence, the ground state polarization reads $\langle P_y\rangle = \chi \mathcal{I}_0 \langle 0|J_x|0\rangle$ [40], leading to the remarkably simple expression Eq. (3) for the Hall imbalance for any single component $SU(M)$ symmetric quantum state on a M-leg ladder. The behavior $R_H = -1/n$ follows by a similar argument to calculate $\partial^2\mathcal{E}_0/\partial E_y^2$ in perturbation theory [40]. Note that for generic ladder models these results are generally true in the large coupling limit $t_y/t_x \gg 1$.

We consider now breaking the $SU(M)$ symmetry. Note, that the Hall response (3) for fermions in the lowest band is robust, since after diagonalization of H^y_{kin}, H_{int} takes the general form $H_{\text{int}} = \sum_{p,p',q,q'} \mathcal{U}_{p,p',q,q'} a^\dagger_{j,p} a_{j,p'} a^\dagger_{j,q} a_{j,q'}$, which projects to zero if only the lowest band $p = 0$ is occupied. This is not the case for bosons. Important insight into the deviations of the bosonic Hall effect from Eq. (3) may be obtained by a simple mean-field description, justified for typical experiments with large particle numbers per site [26]. For small fields χ and ϕ, each site can be described by a coherent state with fixed density $n_{j,m} = n + (m - m_0)\delta n$, leading to a classical description of the system [49,50]. The density variation δn is found by minimization of the energy $\langle H\rangle$ and the total current is given by the twist between subsequent sites $\langle J_x\rangle = 4t_x n \sin(\phi/L) \approx 4t_x n\phi/L$. The Hall imbalance Δ_H is then derived and for $M = 2$ [40]

$$\Delta_H = -\frac{t_x}{2t_y + 2n(U_{00} - U_{01})}. \quad (5)$$

A rich phenomenology of the Hall effect is thus suggested. At the $SU(2)$ symmetric point, the interaction part of the ground-state energy simplifies to Un^4 and, hence, becomes independent of rung density variations δn, which allows for an intuitive explanation of the universal Hall response in this regime: Δ_H remains independent of U and n obeying Eq. (3).

Examples for the generally more complex dependence of Δ_H on filling and interaction strengths are shown in Fig. 3 for a strongly interacting $M = 2$ Bose-Hubbard model for several ratios of on-rung and on-site interactions $\delta = U_{01}/U_{00}$. In the low filling regime $n \to 0$ we observe a good qualitative agreement with the mean field result of Eq. (5) (inset of Fig. 3). Generally the Hall imbalance may unveil the presence of phase transitions and gaped phases; e.g., we observe a finite jump in Δ_H close to the commensurate-incommensurate phase transition at half filling (compare Fig. 3, $\delta = 0.5$). The more remarkable is the vanishing of these features at the exact $SU(2)$ point ($\delta = 1$ curve in Fig. 3).

While the mean-field description leading to Eq. (5) indicates that for $U_{00} < U_{01}$ the Hall imbalance vanishes with increasing density or interaction strength, in the strong coupling regime for $\delta \to 0$ (and $n > 1/2$) we find an interesting sign change of the Hall imbalance. This property can be attributed to the restored particle hole symmetry in the hardcore boson limit which leads to the same sign change as discussed above for free fermions. We may also understand this as a precursor of the topological phase transition previously reported previously by Huber *et al.* [15,16].

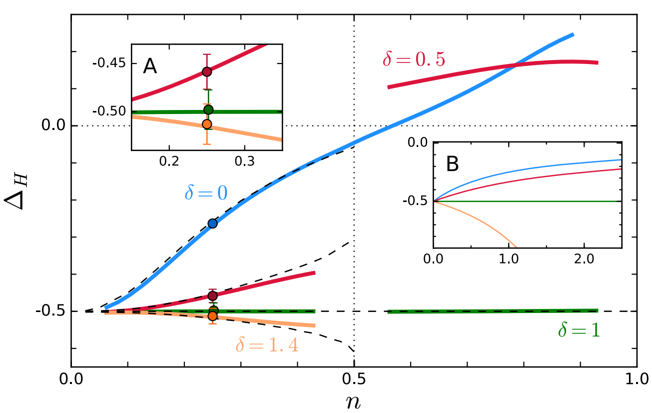

FIG. 3. Hall effect out of $SU(2)$-symmetry for the Bose-Hubbard model. Δ_H as function of the density n for different values of interactions $\delta = U_{01}/U_{00} = 0$, 0.5, 1.0 and 1.4 (extrapolated DMRG data for $L = 24$, 28 and 32 rungs, $n_\mathrm{max} = 2$, $U_{00} = 24t$, $t_y = t_x$). Dashed lines depict Δ'_H. Symbols with error bars are time averages of the quench simulations for $10 < \tau/t_x < 20$ ($L = 48$ rungs, $\Delta\mu = 0.05t_x/L, \chi = 0.01$). Inset *A* shows a zoomed view of the plot. Inset *B* depicts the coherent state approximation Eq. (5) for $U_{00} = t_x = t_y$ and the same values of $\delta = 0$, 0.5, 1, and 1.4 (top to bottom).

For large fillings and $\delta > \delta_c$, for some critical $\delta_c = \delta_c(n, t_y/t_x) > 1$, we observe a quantum phase transition to a biased ladder phase (BLP), where a majority of the particles accumulates on one leg of the ladder [40], corresponding to a ferromagnetic state with fully polarized spin [51–53]. Because of the spontaneous breaking of $\mathbb{Z}_2$ symmetry in the thermodynamic limit this state exhibits $\langle P_y\rangle \neq 0$ at $\chi = 0$ and, hence, we expect a diverging Hall imbalance [40]. This can be also seen in the classical model Eq. (5) in which for $n > t_y/(U_{01} - U_{00})$ the system becomes unstable and develops a spontaneous imbalance $\langle P_y\rangle > 0$ at vanishing field, resulting in a diverging Hall imbalance. Possibly such a giant Hall response can be observed within ferromagnetic alkali species such as ^{23}Na or with the help of tuning of scattering lengths by means of optical Feshbach resonances [54].

We conclude by discussing the particular case in which $[J_x, \mathcal{H}] = 0$, valid, e.g., for noninteracting particles, but also interacting fermions when projected on the lower bands. In this case, the off-diagonal matrix elements of J_x vanish, leading to a compact simplified expression for Δ_H, $\Delta'_H = \sum_{\alpha\neq 0}(\langle 0|P_y|\alpha\rangle/\mathcal{E}_0 - \mathcal{E}_\alpha)(\langle\alpha|\tilde{T}_x|0\rangle/\langle 0|T_x|0\rangle)$ [40]. For interacting systems, one generally finds $\Delta'_H \neq \Delta_H$ (except for the interesting cases mentioned above), but the former is a remarkably good approximation in many cases, see Figs. 2 and 3. It is also remarkable that Δ'_H can be efficiently evaluated with open-boundary conditions (OBC) as well by means of DMRG calculations [40].

Summarizing, we have studied the Hall response of an interacting ladder. While generally the Hall effect strongly depends on the precise type and form of interactions, for certain single component states of $SU(M)$ symmetric models we observed the Hall imbalance Δ_H to be independent from filling and interactions strength, corresponding to a universal $1/n$ behavior of the Hall constant R_H. Note, that this property remains true for strongly interacting particles where an interpretation of n as effective density of long-lived quasiparticles is not necessarily correct. In this work we have focused on the cases relevant or compatible with the experimental measurement procedures such as Refs. [22,23,25,26], and the results should be reproducible with current experimental setups (see Supplemental Material for examples on experimental parameters). In general, the reactive Hall coefficient may depend on the details of the measurement procedure [55]. Further interesting extensions could include the role of interactions at finite field strengths where already in two-ladder systems a wealth of quantum phases and phenomena has been reported [51–53,56–78], which might exhibit unconventional Hall responses observable in current quantum gas experiments.

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