Exciting a Bound State in the Continuum through Multiphoton Scattering Plus Delayed Quantum Feedback

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Excitation of a bound state in the continuum (BIC) through scattering is problematic since it is by definition uncoupled. Here, we consider a type of dressed BIC and show that it can be excited in a nonlinear system through multiphoton scattering and delayed quantum feedback. The system is a semi-infinite waveguide with linear dispersion coupled to a qubit, in which a single-photon, dressed BIC is known to exist. We show that this BIC can be populated via multiphoton scattering in the non-Markovian regime, where the photon delay time (due to the qubit-mirror distance) is comparable with the qubit's decay. A similar process excites the BIC existing in an infinite waveguide coupled to two distant qubits, thus yielding stationary entanglement between the qubits. This shows, in particular, that single-photon trapping via multiphoton scattering can occur without band edge effects or cavities, the essential resource being instead the *delayed* quantum feedback provided by a single mirror or the emitters themselves.

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Introduction.—Waveguide Quantum ElectroDynamics (QED) is a growing area of quantum optics investigating the coherent interaction between quantum emitters and the one-dimensional (1D) field of a waveguide [1–3]. In such systems, a growing number of unique nonlinear and interference phenomena are being unveiled, the occurrence of which typically relies on the 1D nature of such setups. Among these is the formation of a class of bound states in the continuum (BIC), which are bound stationary states that arise *within* a continuum of unbound states [4]. Topical questions are how to form and prepare such states so as to enable potential applications such as quantum memory, which requires light trapping at the few-photon level, of interest for quantum information processing [5–7]. We show that addressing these questions involves studying delayed quantum dynamics in the presence of nonlinearity.

An interesting class of BICs occurs in waveguide QED in the form of dressed states featuring one or more emitters, usually qubits, dressed with a *single* photon that is strictly confined within a finite region [8–15]. The existence of such BICs relies on the *quantum feedback* provided by a mirror or the qubits themselves (since a qubit behaves as a perfect mirror under 1D single-photon resonant scattering [16,17]). A natural way to populate these states is to excite the emitters and then let them decay: the system evolves towards the BIC with an amplitude equal to the overlap between the BIC and the initial state. This results in incomplete decay of the emitter(s) and, in the case of two or more qubits, stationary entanglement [14,15,18–20]. As a hallmark, this approach for exciting BICs is most effective in the Markovian regime where the characteristic photonic *time delays*, denoted by τ, are very short (e.g., the photon round-trip time between a qubit and mirror or between two qubits). Indeed, as the time delay grows, the qubit component of the BIC decreases in favor of the photonic component [11–13], making such decay-based schemes ineffective for large mirror-emitter or interemitter distances. This is a major limitation when entanglement creation is the goal [12].

In order to generate such dressed BICs in the non-Markovian regime of significant time delays, one needs initial states that overlap the BIC's photonic component, which in practice calls for *photon scattering*. A *single* photon scattered off the emitters cannot excite a BIC since the entire dynamics occurs in a sector of the Hilbert space orthogonal to the BIC. For *multiphoton* scattering, however, this argument does not hold because of the intrinsic qubit nonlinearity. Indeed, the role of two-photon scattering has been recognized previously [21,22] in the context of exciting normal bound states (i.e., outside the continuum) that occur in cavity arrays coupled to qubits [23–26].

We show that dressed BICs in waveguide-QED setups can be excited via multiphoton scattering in two paradigmatic setups: a qubit coupled to a semi-infinite waveguide (see Fig. 1) and a pair of distant qubits coupled to an infinite waveguide [see Fig. 5(a)]. A perfectly sinusoidal photon wave function and stationary excitation of the emitters

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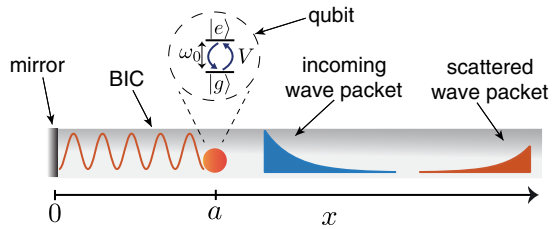

FIG. 1. One-qubit setup: a *semi-infinite* waveguide, whose end lies at $x = 0$ and acts as a perfect mirror, is coupled to a qubit at $x = a$. When a resonant standing wave can fit between the qubit and the mirror ($k_0 a = m\pi$), an incoming two-photon wave packet is *not* necessarily fully scattered off the qubit: a fraction remains trapped in the form of a dressed single-photon BIC.

represents a clear signature of single-photon trapping. This provides a solvable example of non-Markovian quantum dynamics in a nonlinear system, a scenario of interest in many areas of contemporary physics [27–33].

Model and BIC.—Consider first a qubit coupled to the 1D field of a semi-infinite waveguide [Fig. 1(a)] having a linear dispersion $\omega = v|k|$ (with v the photon group velocity and k the wave vector). The qubit's ground and excited states $|g\rangle$ and $|e\rangle$, respectively, are separated in energy by $\omega_0 = vk_0$ (we set $\hbar = 1$ throughout). The end of the waveguide at $x = 0$ is effectively a perfect mirror, while the qubit is placed at a distance a from the mirror. The Hamiltonian under the rotating wave approximation (RWA) reads [16,34–37]

$$\hat{H} = \omega_0 \hat{\sigma}^\dagger \hat{\sigma} - iv \int_0^\infty dx \left(\hat{a}_R^\dagger(x) \frac{d}{dx} \hat{a}_R(x) - \hat{a}_L^\dagger(x) \frac{d}{dx} \hat{a}_L(x) \right) + V \int_0^\infty dx [(\hat{a}_L^\dagger(x) + \hat{a}_R^\dagger(x))\hat{\sigma} + \text{H.c.}] \delta(x - a), \tag{1}$$

with $\hat{\sigma} = |g\rangle\langle e|$, $\hat{a}_{R(L)}(x)$ the bosonic field operator annihilating a right-going (left-going) photon at position x, and V the atom-photon coupling. Due to the RWA, the total number of excitations $\hat{N} = \hat{\sigma}^\dagger \hat{\sigma} + \sum_{\eta=R,L} \int dx\, \hat{a}_\eta^\dagger(x) \hat{a}_\eta(x)$ is conserved.

In the single-excitation subspace ($N = 1$), the spectrum of Eq. (1) comprises an infinite continuum of unbound dressed states $\{|\phi_k\rangle\}$ with energy $\omega_k = v|k|$ [12,16,34–37], each a scattering eigenstate in which an incoming photon is completely reflected. Notably, a further stationary state $|\phi_b\rangle$ exists when the condition $k_0 a = m\pi$ (with $m = 0, 1, \ldots$) is met. This BIC has the same energy $\omega_b = \omega_0$ as the qubit and is given by [11,38]

$$|\phi_b\rangle = \varepsilon_b \left[\hat{\sigma}^\dagger \pm i \sqrt{\frac{\Gamma}{2v}} \int_0^a dx \left(e^{ik_0 x} \hat{a}_R^\dagger(x) - e^{-ik_0 x} \hat{a}_L^\dagger(x) \right) \right] |g\rangle|0\rangle, \tag{2}$$

with $\Gamma = 2V^2/v$ the qubit's decay rate (without mirror). The qubit's excited-state population (referred to simply as "population" henceforth) is given by

$$|\varepsilon_b|^2 = \frac{1}{1 + \frac{1}{2}\Gamma\tau}, \tag{3}$$

where $\tau = 2a/v$ is the *delay time*. Equations (2) and (3) fully specify the BIC. The photonic wave function has shape [we set $|x\rangle = \hat{a}^\dagger(x)|0\rangle$ with $\hat{a}^\dagger(x) = \hat{a}_R^\dagger(x) + \hat{a}_L^\dagger(x)$]

$$\langle x|\phi_b\rangle \propto \sin(k_0 x) \quad \text{for } 0 \le x \le a, \tag{4}$$

while it vanishes at $x \notin [0, a]$: the BIC is formed strictly between the qubit and the mirror, where the field profile is a pure sinusoid. When the BIC exists (i.e., for $k_0 a = m\pi$) the qubit does not fully decay in vacuum [11,39,40]: since the overlap of the initial state $|e, 0\rangle$ with the BIC is ε_b, $|\varepsilon_b|^2$ is also the probability of generating the BIC via vacuum decay. This probability decreases monotonically with delay time [Eq. (3)], showing that vacuum decay is most effective when $\Gamma\tau$ is small.

BIC generation scheme.—Bound state [Eq. (2)] cannot be generated, however, via single-photon scattering, which involves only the unbound states $\{|\phi_k\rangle\}$ that are all orthogonal to $|\phi_b\rangle$: during a transient time the photon may be absorbed by the qubit, but it is eventually fully released. We thus send a *two-photon* wave packet such that the initial joint state is $|\Psi(0)\rangle = A \iint_0^\infty dx\, dy [\varphi_1^L(x)\varphi_2^L(y) + 1 \leftrightarrow 2]\, \hat{a}_L^\dagger(x)\hat{a}_L^\dagger(y)|g\rangle|0\rangle$, where A is for normalization, $\varphi_i^L(x)$ is the wave function of a single left-propagating photon, and the qubit is not excited. The ensuing dynamics in the two-excitation sector ($N = 2$) is given by

$$|\Psi(t)\rangle = \left[\sum_{\eta=R,L} \int_0^\infty dx\, \psi_\eta(x,t) \hat{a}_\eta^\dagger(x) \hat{\sigma}^\dagger + \sum_{\eta,\eta'=R,L} \frac{1}{\sqrt{2}} \iint_0^\infty dx\, dy\, \chi_{\eta\eta'}(x,y,t) \times \hat{a}_\eta^\dagger(x) \hat{a}_{\eta'}^\dagger(y) \right] |g\rangle|0\rangle, \tag{5}$$

where $\chi_{\eta\eta'}(x,y,t)$ is the wave function of the two-photon component, while $\psi_\eta(x,t)$ is the amplitude that the qubit is excited and a right-(left-)propagating photon is found at position x. We define

$$P_e(t) \equiv \sum_{\eta=R,L} \int_0^\infty dx |\psi_\eta(x,t)|^2,$$
$$P_{ph}(t) \equiv 2 \sum_{\eta,\eta'=R,L} \int_0^a dx \int_a^\infty dy |\chi_{\eta\eta'}(x,y,t)|^2 \tag{6}$$

as, respectively, the qubit population and the probability that one photon lies in region $[0, a]$ and one in (a, ∞).

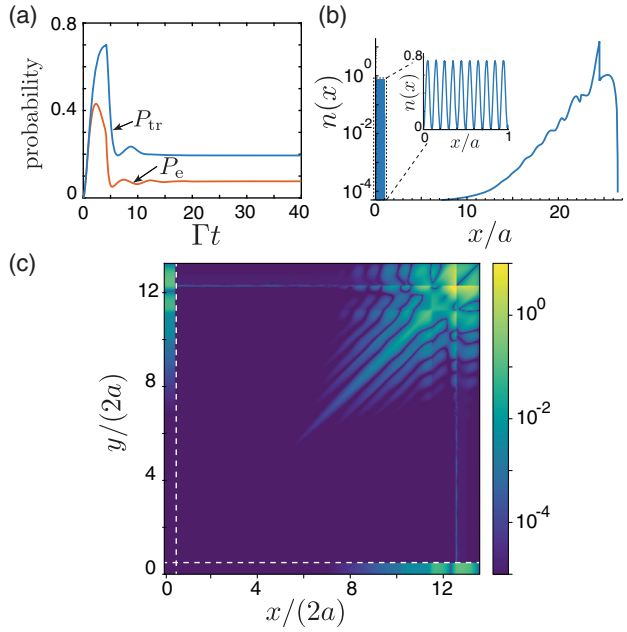

FIG. 2. BIC generation via two-photon scattering. (a) Qubit population P_e and trapping probability P_{tr} as a function of time in units of Γ^{-1}. (b) Spatial profile of the field intensity $n(x)$ at the end of scattering. The inset highlights the sinusoidal wave function in the range $0 \leq x \leq a$. (c) Two-photon probability density function $\sum_{\eta,\eta'=R,L}|\chi_{\eta\eta'}(x,y)|^2$ after scattering is complete ($t = t_f$). The white dashed lines $x = a$ and $y = a$ mark the qubit position. [Panels (b) and (c) are plotted on a log scale and with $t_f = 80/\Gamma$. We considered a two-photon exponential wave packet with $\Gamma\tau = \pi$, $k_0 a = 10\pi$ and $\Delta k = \Gamma/2v$.]

We first consider for simplicity a two-photon exponential wave packet (sketched in Fig. 1): $\varphi^L_{1,2}(x) = e^{-\Delta k|x-a|-ik_0(x-a)}\theta(x-a)$ where $v\Delta k$ is the bandwidth, the carrier wave vector k_0 is resonant with the qubit, and the wave front reaches the qubit at $t = 0$. In Fig. 2, we plot results for the dynamics described by Eq. (1) obtained numerically (for details, see the Supplemental Material (SM) [41]). As the wave packet impinges on the qubit, its population P_e [Fig. 2(a)] exhibits a rise followed by a drop (photon absorption then reemission) eventually converging to a small—yet *finite*—steady value. This shows that part of the excitation absorbed from the wave packet is never released back.

The photon field in the same process is shown in Figs. 2(b) and 2(c) displaying, respectively, the field intensity $n(x) = \langle\Psi(t_f)|\hat{a}^\dagger(x)\hat{a}(x)|\Psi(t_f)\rangle$ and the total two-photon probability density $\sum_{\eta,\eta'=R,L}|\chi_{\eta\eta'}(x,y,t_f)|^2$ at a time t_f after the scattering process is complete. The wave packet is not entirely reflected back: a significant fraction remains trapped between the mirror and qubit, forming a perfectly sinusoidal wave with wave vector k_0 [Fig. 2(b)]. Remarkably, this stationary wave is of single-photon nature. Indeed, Fig. 2(c) shows that either both photons are reflected (top right corner) or one is scattered and the other remains trapped in the mirror-qubit interspace (top left and bottom right). Note that the probability that both photons are trapped (bottom left) is zero.

These outcomes, in light of the features of the BIC [Eq. (2)], suggest that, after scattering, the joint state has the form

$$|\Psi(t_f)\rangle = \int_a^\infty dx\,\xi_R(x,t_f)\hat{a}^\dagger_R(x)|\phi_b\rangle + \iint_a^\infty dx\,dy\,\beta_{RR}(x,y,t_f)\hat{a}^\dagger_R(x)\hat{a}^\dagger_R(y)|g,0\rangle, \quad (7)$$

where in the first line a single photon has left the BIC region, while the last line describes two outgoing photons. Let $P_{tr} = P_e + P_{ph}$ be the probability that either the qubit is excited or a photon is trapped between the mirror and qubit. It then follows from Eq. (7) [41] that the asymptotic values of P_{tr} and P_e fulfill

$$P_{tr}(\infty) = \int_a^\infty dx\,|\xi_R(x,\infty)|^2 = \left(1 + \frac{1}{2}\Gamma\tau\right)P_e(\infty), \quad (8)$$

which is naturally interpreted as the probability of generating the BIC, $P_{BIC} \equiv P_{tr}(\infty)$. The time dependence of P_{tr} shown in Fig. 2(a) demonstrates that it reaches a finite steady value satisfying Eq. (8), confirming Eq. (7) and, thus, the generation of the BIC. The identity [Eq. (8)] was checked in all of the numerical results presented.

Dependence on time delay.—A substantial delay time is essential for exciting the BIC. The parameter set in Fig. 2, for instance, corresponds to $\Gamma\tau \simeq 3.14$. To highlight this dependence, we report in Fig. 3(a) that the steady state values of P_e, P_{ph}, and P_{tr}, optimized with respect to Δk, as functions of $\Gamma\tau$. Both photon trapping and stationary qubit excitation are negligible in the Markovian regime $\Gamma\tau \ll 1$, in sharp contrast to vacuum-decay schemes for which this is instead the optimal regime. A delay time $\Gamma\tau \gtrsim 1$ is required to make our BIC generation scheme effective; indeed, each of the three probabilities reaches a maximum at a delay of order $\Gamma\tau \sim 1$. Remarkably, P_e becomes negligible compared to P_{ph} for $\Gamma\tau \gtrsim 10$, showing that the photon component is dominant at large delays as expected from Eqs. (2) and (3): in this regime, we thus get almost *pure* single-photon trapping.

Dependence on bandwidth and detuning.—The efficiency of BIC generation depends on the width, Δk, of the injected wave packet. In Fig. 3(b) the optimal value is close to $\Gamma/2v$. Thus, photon absorption is maximum when the wave packet width is of order the qubit decay rate, in agreement with general expectations [43–45]. The optimal Δk as a function of delay time is given in the SM [41]; for large $\Gamma\tau$, the optimal value saturates near $0.2\Gamma/v$.

Nonresonant photons can also be used to generate the BIC: results for a wave packet of two photons detuned oppositely in energy are shown in Fig. 3(b). The optimal

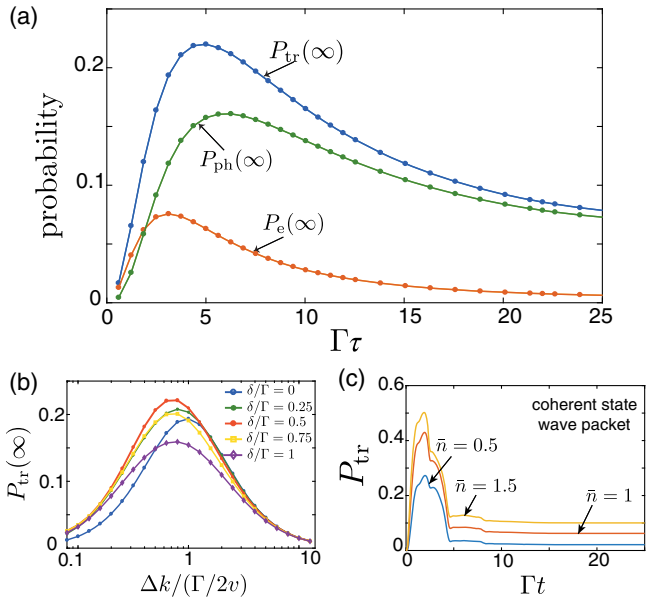

FIG. 3. (a) Asymptotic values of $P_{\rm e}$, $P_{\rm ph}$, and $P_{\rm tr}$ as a function of the rescaled time delay $\Gamma\tau$ for $\delta = 0$. At each point, Δk is set so as to maximize $P_{\rm tr}$. Here we used the optimized Δk shown in the SM [41]. (b) Asymptotic value of $P_{\rm tr}(\infty) = P_{\rm BIC}$ against the wave packet bandwidth for $\Gamma\tau = \pi$, $k_0 a = 10\pi$ and different values of detuning δ, where we assumed that one photon has carrier wave vector $k_1 = k_0 + \delta/v$ and the other has $k_2 = k_0 - \delta/v$. (c) $P_{\rm tr}$ versus time for coherent-state wave packets with the same shape. For computational reasons, only contributions up to three-photon Fock states are retained. The parameters are the same as in Fig. 2.

wave packet width changes but remains of order Γ. As the detuning increases, the maximum $P_{\rm BIC}$ initially rises and then decays; note that the optimal detuning is $\delta \approx \Gamma/2$. At this value the nonlinear scattering flux was shown to peak [46,47], confirming that the intrinsic *nonlinearity* of the emitters is key to generating the BIC [41].

Coherent-state wave packet.—It is natural to wonder whether, instead of a two-photon pulse, the BIC can be excited using a coherent-state wave packet, which is easier to generate experimentally. In Fig. 3(c) we consider the same setup, parameters, and wave packet shape $\varphi(x)$ as in Fig. 2 but for a low-power coherent-state pulse [36] $|\alpha\rangle = e^{-|\alpha|^2}\sum_{n=0}^{\infty}(\alpha^n/n!)(\int dx\,\varphi(x)\hat{a}_L^\dagger(x))^n|g,0\rangle$ with the average photon number given by $\bar{n} = |\alpha|^2$. For $\bar{n} = 1.5$, $P_{\rm tr}(\infty)$ is comparable to the one obtained with the two-photon pulse, demonstrating the effectiveness of using coherent states.

Increasing the BIC generation probability.—We find that the trapping probability depends sensitively on the shape of the incoming wave packet. While we have mostly used (Figs. 2, 3, and 5) the exponential pulse that is standard in the literature [45,48], Fig. 4 shows how engineering the wave packet shape strongly enhances $P_{\rm BIC}$ [49]. We set here $\Gamma\tau = 5$, which roughly corresponds to the maximum of

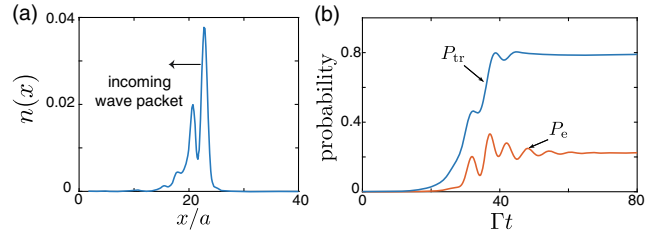

FIG. 4. BIC generation scheme for the one-qubit setup using a structured-shape two-photon wave packet (see SM [41]). (a) Photon density profile of the incoming wave packet. (b) $P_{\rm BIC}$ and $P_{\rm e}$ versus time. For this plot we fixed the distance to $k_0 a = 20\pi$ and the time delay to $\Gamma\tau = 5$ to maximize the photon trapping probability [see Fig. 3(b)].

$P_{\rm tr}(\infty) = P_{\rm BIC}$ in Fig. 3(a). The engineered incoming two-photon wave packet in Fig. 4(a) (for methods see SM [41]) yields $P_{\rm BIC} \simeq 80\%$, a value about four times larger.

Two-qubit BIC.—A BIC very similar to the one addressed above occurs in an infinite waveguide (no mirror) coupled to a pair of identical qubits [8,10,12–15]. With the qubits placed at $x_1 = -a/2$ and $x_2 = a/2$ and for $k_0 a = m\pi$ [Fig. 5(a)], there exists a BIC given by

$$|\varphi_b\rangle = \varepsilon_b\left[\hat{\sigma}_\pm^\dagger - i\sqrt{\frac{\Gamma}{4v}}\int_{-a/2}^{a/2} dx\left(e^{ik_0(x+a/2)}\hat{a}_R^\dagger(x) - e^{-ik_0(x+a/2)}\hat{a}_L^\dagger(x)\right)\right]|g_1, g_2\rangle|0\rangle, \tag{9}$$

where now $|\varepsilon_b|^2 = 1/(1+\Gamma\tau/4)$, $\hat{\sigma}_\pm = (\hat{\sigma}_1 \pm \hat{\sigma}_2)/\sqrt{2}$, and plus (minus) is used if m is odd (even). By tracing

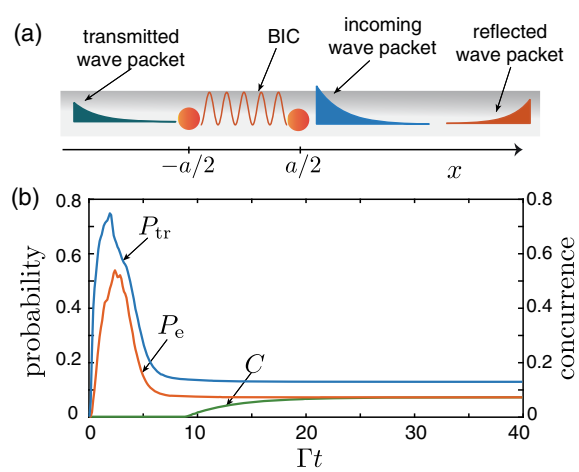

FIG. 5. (a) Two-qubit setup: an *infinite* waveguide (no mirror) is coupled to a pair of qubits. (b) Probability to excite at least one qubit $P_{\rm e}$, trapping probability $P_{\rm tr}$, and qubit-qubit concurrence C versus time in a two-photon scattering process (see SM [41] for definition of $P_{\rm e}$, $P_{\rm tr}$, and C). The wave packet and parameters are the same as in Fig. 2. The scheme generates a dressed BIC in a way analogous to the one-qubit setup in Fig. 1, yielding however stationary entanglement between the qubits.

out the photonic field, Eq. (9) clearly entails *entanglement* between the qubits. (In the familiar limit $\Gamma\tau \ll 1$, for instance, the entangled state is $\hat{\sigma}_{\pm}^{\dagger}|g_1, g_2\rangle|0\rangle$, namely the sub- or super-radiant, maximally entangled state [18–20, 37,50–52].) Thus, in the two-qubit setup of Fig. 5(a), our scattering-based approach to exciting the BIC can in particular generate entanglement.

This expectation is confirmed in Fig. 5(b), for which the same injected exponential wave packet was used as in Fig. 2. In addition to the probability to excite at least one qubit P_e and the probability to generate a BIC (9), we plot the amount of entanglement between the qubits, as measured by the concurrence C [53]. As for one qubit, the two-qubit BIC population reaches a steady value after scattering, resulting in an excitation stored in the qubits and hence stationary entanglement. Note the typical [54] "sudden birth" of the entanglement.

Conclusions.—We have shown that dressed BICs occurring in waveguide-QED setups can be generated via multiphoton scattering. This enables single-photon capture and, for multiple emitters, production of stationary entanglement. These BICs differ significantly from purely excitonic subradiant states, as well as from BICs located entirely within the side-coupled quantum system, in that they involve the field of the waveguide itself.

For our method, it is critical to have *nonlinear* emitters such as qubits; replacing them by bosonic modes, e.g., will invalidate the whole scheme [41].

While preparing this Letter, we became aware of a related scheme by Cotrufo and Alù [49]. There however the BIC arises from a single system, comprised of a qubit and two cavities to provide feedback, side-coupled to an infinite waveguide. Here, instead, no cavities are present and the necessary quantum feedback is provided by a mirror (cf. Fig. 1) or the emitters themselves [cf. Fig. 5(a)]. Remarkably, in order to generate the BIC, this feedback needs to be *delayed* [cf. Fig. 3(a)].

Investigating the non-Markovian effects of non-negligible delays is a new frontier of quantum optics [11,13,30–33,37,39,45,52,55–70]. Here we have taken advantage of such delays, demonstrating that their role can be constructive [32,64,69]. In particular, within the range considered, as shown in Fig. 3(a), long delays ($\Gamma\tau \gtrsim 20$) enable almost pure single-photon trapping (instead of hybrid atom-photon excitation). Remarkably, adding qubit losses, denoted by γ_a, makes the trapped photon decay slowly at the rate $\gamma_a|\varepsilon_b|^2$ [41], suggesting that our scheme is more robust against emitter loss for larger delay τ.

Targets of ongoing investigation include exploring the regime of very long delays {beyond $\Gamma\tau \simeq 25$ in Fig. 3(a) allowed by our current computational capabilities [41]} and deriving a systematic criterion to increase the generation probability by wave packet engineering (possibly by exploiting time reversal symmetry [43,49]). We expect this line of research will become important to, e.g., long-distance communication over quantum networks.

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