Device-Independent Detection of Genuine Multipartite Entanglement for All Pure States

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We show that genuine multipartite entanglement of all multipartite pure states in arbitrary finite dimension can be detected in a device-independent way by employing bipartite Bell inequalities on states that are deterministically generated from the initial state via local operations. This leads to an efficient scheme for large classes of multipartite states that are relevant in quantum computation or condensed-matter physics, including cluster states and the ground state of the Affleck-Kennedy-Lieb-Tasaki (AKLT) model. For cluster states the detection of genuine multipartite entanglement involves only measurements on a constant number of systems with an overhead that scales linearly with the system size, while for the AKLT model the overhead is polynomial. In all cases our approach shows some robustness against experimental imperfections.

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Introduction.—Entanglement is an exclusive feature of quantum physics. As such it is believed to be the key ingredient in various quantum information processing tasks, like, e.g., quantum computation, quantum metrology and, to some extent, quantum key distribution. Entanglement is a direct consequence of the fact that quantum states are modeled as operators on the tensor product of the Hilbert spaces for each system. From this mathematical perspective the question of entanglement detection has been solved, most notably by the approach based on entanglement witnesses [1].

However, from a more physical perspective such a view is not fully satisfactory, as in order to be applied to an experiment it requires to assume a given dimension for the Hilbert space of each system and an exact quantum description of the measurement devices. Yet, it is hard to characterize a measurement device exactly, and, moreover, a physical system typically has access to more levels and degrees of freedom than one uses to describe its state. Hence neither of the assumptions can be fully verified in practice.

The most radical way to overcome these problems is offered by device-independent methods, which allow one to detect entanglement solely based on the Bell-like correlations of measurement outcomes collected in the experiment. In the bipartite case many results of fundamental interest have been obtained [2–6]. In particular, it is known that all pure entangled states violate some Bell inequality, meaning that the entanglement of any pure state can be detected in a device-independent way, a result sometimes referred to as the Gisin theorem [2,7]. Less is known, however, for the multipartite case, in particular when it comes to genuine multipartite device-independent entanglement. On one hand, extensions of Gisin's theorem to the multipartite case exist, but focus on the simplest form of entanglement instead of genuine multipartite entanglement [8–11]. More specific results, on the other hand, are only known for a few states [12–23]. This has to do with the difficulty to obtain multipartite Bell inequalities suited to specific states: typically, the starting point of the analysis is a fixed set of known Bell inequalities rather than the states themselves. In addition, multipartite Bell inequalities such as the Svetlichny inequality are inefficient to test experimentally, as they require an exponentially increasing number of measurement settings.

Here we circumvent these difficulties by introducing a scheme that allows one to detect genuine multipartite entanglement by testing bipartite Bell inequalities on states that are generated deterministically from the initial state via local operations and classical communication (LOCC). A similar approach was used in Ref. [24] to detect genuine multipartite nonlocality. This result, however, required testing bipartite Bell inequalities between all possible pairs of parties, hence being impractical for a large number of parties. More recently, the combined maximal violation of several bipartite Bell inequalities was used to self-test some families of multipartite states in Ref. [25]. Here, we derive a multipartite Bell inequality as the combination of several bipartite inequalities obtained for a covering set of pairs of parties, and show that a sufficiently large violation of this inequality allows one to certify genuine multipartite entanglement in a device-independent way.

Our approach is not limited to a specific family of states but works for all pure states. It thus answers an important open question raised in, e.g., Ref. [20] and generalizes Gisin's theorem to genuine multipartite entanglement: the genuine multipartite entanglement of a pure state can always be detected in a device-independent way.

Interestingly, the result obtained with this method comes with some built-in robustness against noise and

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imperfections: genuine multipartite entanglement is detected for mixed states that are sufficiently close to a pure genuinely multipartite entangled state. What is more, since our method does not involve all possible pairs of parties, we obtain a scheme that is experimentally efficient for large classes of interesting states, including all (weighted) graph states with constant degree [26–29] and ground states of 1D spin models such as the AKLT model [30]. That is, only a constant (logarithmically growing) number of parties needs to be measured, and the overhead in terms of measurement settings is only linear (polynomial) in the system size N, respectively, as opposed to previous schemes that scale exponentially with N.

One direct consequence of our result is that pure states with arbitrary entanglement depth M can be detected device independently. Indeed, in any such state it is possible to trace out $N-M$ parties such that the remaining M parties are genuinely multipartite entangled. Our result then guarantees that it is possible to detect the genuine M-partite entanglement within these parties in a device-independent way.

Statement of the main results.—Let $N \in \mathbb{N}$ denote the number of parties, $V = \{1, \ldots, N\}$, $E_k = \{i_k, j_k\}$ with i_k, $j_k \in V$ be pairs of parties and $E = \{E_1, \ldots, E_K\}$ their union. We define a graph $G = (V, E)$ by associating the parties with vertices and the pairs E_k with edges. We say that E is a covering set of pairs for the N-partite system if the corresponding graph G is connected.

The main result of this Letter is the following:

Theorem 1—Let $|\psi\rangle$ be a state in the Hilbert space $\bigotimes_{i=1}^{N} \mathbb{C}^{d_i}$, where $d_i \in \mathbb{N}$ for all $i \in \{1, \ldots, N\}$. Assume that there exists a covering set such that for each pair in it there exist local operations and classical communication such that one can produce an entangled pure state between the two parties in all branches of the LOCC protocol. Then one can show genuine multipartite entanglement between all N parties in a device-independent way.

Sketch of the proof: The idea behind the proof is the following. One constructs a Bell expression by considering the sum of all bipartite Bell expressions for the pairs of parties appearing in the covering set E and all branches of the LOCC protocol (that all result in a pure entangled state of the chosen pair E_k). The bipartite Bell inequalities are chosen to reach the quantum bound β_* for each of the states, which is in fact possible due to a recent result [4]. One then shows that the quantum state achieves a value for the multipartite Bell expression which is incompatible with any biseparable quantum state

$$\varrho_{\mathrm{BS}} = \sum_{\lambda} \sum_{\substack{g_1 \cap g_2 = \emptyset \\ g_1 \cup g_2 = \{1, \ldots, N\}}} p(\lambda) \rho_{g_1}(\lambda) \otimes \rho_{g_2}(\lambda) \qquad (1)$$

of arbitrary local dimensions. Here the sum over g_1 and g_2 covers all possible splittings of the N parties in two groups, ρ_{g_1} are ρ_{g_2} are arbitrary joint quantum states of the parties belonging to the corresponding group, and λ is the hidden variable.

For each pair of parties and each component of the biseparable state expansion appearing in Eq. (1) there are two possibilities regarding the expectation value of the bipartite Bell expressions in the multipartite Bell inequality. Either the two parties appearing in the bipartite inequality belong to same group g_1 or g_2, in which case they may always achieve the maximal value of the Bell expression β_*. Or they belong to different groups g_1 and g_2, in which case they end up in a separable state in each branch of the LOCC protocol and can at most contribute a value corresponding the local bound $\beta_L < \beta_*$. Since E is a covering set of pairs, for each grouping $g_1|g_2$ in the biseparable state expansion of Eq. (1) there will be at least one term in the Bell expression, corresponding to some pair E_k, whose value is limited to $\beta_L < \beta^*$. Hence, the expected value of the overall Bell expression on a biseparable state ρ_{BS} is also strictly smaller than β^*. For a detailed proof see Ref. [31].

In fact, in the ideal case, where the observed violation of the constructed Bell expression is maximal β^*, we show that it is incompatible with any quantum state $(1-\varepsilon_{\mathrm{BS}})\varrho_Q + \varepsilon_{\mathrm{BS}}\varrho_{\mathrm{BS}}$ that has a nonzero biseparable weight $\varepsilon_{\mathrm{BS}}$, i.e., with any mixture of a genuinely multipartite entangled state and a biseparable state with arbitrarily small nonzero weight. We will also use this property to show the robustness our result later in the Letter.

Theorem 2.—All genuine multipartite entangled pure states fulfill the assumption (namely, that there exists a covering set such that for each pair in it there exist local operations and classical communication such that one can produce an entangled pure state between the two parties in all branches of the LOCC protocol) from Theorem 1.

Sketch of the proof: One can show that almost any measurement brings a genuinely multipartite entangled N-partite state $|\Psi\rangle$ to d_1 postmeasurement $(N-1)$-partite genuinely multipartite entangled states $|\Psi_k\rangle$, where d_1 is the local Hilbert space dimension. This can be iterated until one arrives at a bipartite state. The set of measurements for which it does not work is of measure zero in each step. This guarantees that there are measurements on $N-2$ parties such that one obtains an entangled bipartite state in all branches. For a detailed proof see Ref. [31].

While this works for all pure states, in general the protocol is not efficient. From the proof of Theorem 2 one sees that up to $N-2$ parties need to carry out measurements with at least two outcomes each. This gives rise to exponentially many branches in which one needs to test a bipartite Bell inequality. In the general case one can choose a covering set with $N-1$ pairs. In the worst case for each pair the other $N-2$ parties perform measurements with d outcomes (where for simplicity we assume here that all parties have the same local Hilbert space dimension d). For each of the d^{N-2} possible outcomes the pair has to perform 4 measurements in the case of qubits ($d=2$) [32], and 8 in the case of qudits [4]. Thus in the worst case, the overall effort scales as $4(N-1)2^{N-2}$ for qubits and $8(N-1)d^{N-2}$

for qudits. The (worst case) number of settings for each party is given by $2^N + 1$ for qubits and $7d^{N-2} + 1$ for qudits.

However, there are families of states for which only a limited number of parties are involved in the LOCC protocol for each pair, and thus the protocol is efficient. This is described in more detail in the next section.

The results are illustrated for GHZ states in Ref. [31] but they are very general and widely applicable. We now illustrate their usefulness with some examples of classes of states for which one can (efficiently) show genuine multipartite entanglement in a device-independent way.

Connected, generalized and weighted graph states.—This family of states plays an important role in quantum information, in particular in the context of quantum error correction, measurement-based quantum computation, and quantum networks. It has first been defined for qubits [26–28] and later been generalized to higher local dimensions [33,34]. The toric code and its generalizations [35] also belong to this family. For qubits, weighted graph states [29] can be defined by $|G\rangle = \prod_{\{i,j\}\in E} U_{ij}|+\rangle^{\otimes N}$ Here, $U_{ij} = \mathrm{diag}(1,1,1,e^{i\phi_{ij}})$ in the computational basis, $|+\rangle$ is the $+1$ eigenstate of the Pauli X operator and V and E are sets of vertices and edges as above. If $G = (V,E)$ is connected, then the (weighted) graph state $|G\rangle$ is said to be connected. For a discussion of the case of local dimension $d > 2$ see Refs. [33,34].

It is easy to see that these states fulfill the requirements from Theorem 1. The set E itself is a covering set of pairs and any pair in it can be isolated via measurements of all qubits in the neighborhood in the computational basis [26–28], since the gate U_{ij} commutes with the measurement in the Z basis. The efficiency of the protocol depends on the number of outcomes for the measurements of the qubits in the neighborhood of each pair. The number of neighbors is specified by the degree $\deg(G)$ of the graph, and is at most $2\cdot\deg(G)$. For all measurement outcomes, one obtains a state that is equivalent up to local Pauli corrections to $U_{ij}|+\rangle^{\otimes 2}$. The covering set can always be chosen to contain at most N pairs. This can be achieved by first choosing one vertex and adding all edges connecting this vertex to the set E'. One then continues this step for all neighbors of this vertex, but adds only those edges that connect vertices which were not already connected in the previous round. The size of the neighborhood, i.e., the number of vertices adjacent to a pair, enters exponentially in the total number of operators which need to be measured. This is because one has to take all measurement outcomes into account. For qubit graph states one has to optimize over all possible covering sets and over all local unitary (LU) equivalent states for each pair individually. The concept of local complementation can substantially change the degree of a graph; e.g., for a binary tree graph a sequence of local complementations [27,28] can change the degree from 3 to $N-1$ and vice versa. In particular, as long as the maximal degree of the graph grows at most logarithmically with the number of vertices N the protocol is efficient. For constant degree one indeed obtains a linear scaling, as there are only linearly many terms in the Bell inequality, and each has support on a constant number of parties only. This holds, e.g., for prominent graphs states defined on square and triangular lattices, which are also universal resources for measurement-based quantum computation [28,36].

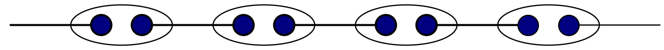

FIG. 1. Pictorial representation of the AKLT state. There are two virtual qubits (blue dots) at each site. Dots connected by an edge represent singlet states $|\psi^-\rangle$ and ellipses refer to projections onto the three-dimensional triplet subspace, where one makes the following identification: $|\tilde{0}\rangle = |00\rangle$, $|\tilde{1}\rangle = |11\rangle$, $|\tilde{2}\rangle = |\psi^+\rangle = (1/\sqrt{2})(|01\rangle + |10\rangle)$.

Affleck-Kennedy-Lieb-Tasaki model.—The AKLT model [30] is a generalization of the one-dimensional (quantum) Heisenberg spin model, with Hamiltonian $H = \sum_j \mathbf{S}_{(j)}\cdot\mathbf{S}_{(j+1)} + \frac{1}{3}(\mathbf{S}_{(j)}\cdot\mathbf{S}_{(j+1)})^2$ where $\mathbf{S}_j$ is the spin-1 operator acting on system j. The model is exactly solvable and can be viewed as a prototype of a matrix product state (MPS) [37] (for reviews see Refs. [38,39]). One can certify genuine multipartite entanglement in the AKLT model in a device-independent way efficiently. The preparation of entangled states of pairs $(j, j+1)$, as required in Theorem 1, can be achieved by measuring only a small neighborhood of each pair, and for all measurement outcomes one is left with an entangled pair. Using the notation introduced in Fig. 1, it suffices to measure the neighboring spins of the pair in the basis $\{|\tilde{0}\rangle, |\tilde{1}\rangle, |\tilde{2}\rangle\}$, where each outcome occurs with probability one-third. For outcomes corresponding to $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$ the chain is decoupled, and for the outcome corresponding to $|\tilde{2}\rangle$, which corresponds to entanglement swapping at the level of the virtual links, one has shifted the problem of cutting to the next site. One can then repeat the probabilistic cutting. Measuring n sites on each side of the pair results in a success probability of cutting out the pair of $p_{\mathrm{cut}} \geq 1–2(\frac{1}{3})^n$. The state of the resulting pair depends on the outcome of the measurements, and is always entangled and pure [40]. The success probability goes to one exponentially fast with n. From the results presented below, it follows that $n = O(\log N)$, as such a reduced success probability yields a smaller, nonmaximal violation of the bipartite inequality. This corresponds to a polynomial number of measurement settings and hence an efficient scheme to detect genuine multipartite entanglement in the AKLT model. See also Ref. [31].

Dicke states.—Dicke states [41] are an important class of multipartite entangled states. An N-qubit Dicke state with k excitations is given by $|D_k^N\rangle = \binom{N}{k}^{-1/2}\sum_{\mathrm{permutations}}|1\rangle^{\otimes k}|0\rangle^{\otimes N-k}$, where the sum refers to all permutations of the parties. Entangled states for any pair of parties can be produced deterministically via Pauli Z and X measurements (see Ref. [31]) and thus it follows from Theorem 1 that one can show genuine multipartite entanglement in a device-independent way. In contrast to the cases discussed above our method is not efficient for Dicke states, as there are exponentially many branches in the LOCC protocol.

Robustness and experimental feasibility.—The robustness to imperfections is crucial for the experimental feasibility of any protocol. We show that our method to reveal genuine multipartite entanglement is robust to noise and study two different situations. First we demonstrate the robustness to noise in the general case. Then we study the case of cluster states in more detail, where we also investigate different error models.

From the analysis presented in Ref. [31] it follows that in the presence of a nonmaximal violation of the bipartite inequality one can show genuine multipartite entanglement only for up to

$$M = \left\lfloor \frac{\beta_* - \beta_L}{\beta_* - \beta} \right\rfloor \tag{2}$$

parties, where β is the observed value of the bipartite inequality [42]. On the one hand M puts a bound on the system size, such that one can show genuine multipartite entanglement. On the other hand it can be viewed as the number of parties of a subset of the whole system of $N > M$ parties, for which genuine M-partite entanglement can be shown. A genuine M-partite entanglement is also sometimes referred to as a state with entanglement depth M, i.e., a state that is not $(M-1)$ producible. One sees that the robustness is determined by how much the noise affects the violation of each bipartite inequality, as well as the gap of the local bound β_L and the quantum bound β_* of the inequality. Nevertheless, one obtains a certain robustness for *any* entangled pure state $|\psi\rangle$. That is, there exists an ϵ-ball around each entangled pure state where we can confirm genuine multipartite entanglement with our method. To see this consider an arbitrary state ρ which is close in fidelity to the target state $|\psi\rangle$, that is $|\langle\psi|\rho|\psi\rangle| \geq 1-\epsilon$. The observed violation β of each bipartite inequality for ρ is then bounded by $\beta \geq (1-\epsilon)\beta_* + \epsilon\beta_{\min}$, where $\beta_{\min}$ is the algebraic minimum of the inequality, a fixed number. This is a consequence of the fact that $|\psi\rangle$ is the eigenstate of the operator corresponding to the Bell inequality with eigenvalue β_*, while all its other eigenvalues are lower bounded by $\beta_{\min}$. It follows from Eq. (2), that genuine multipartite entanglement can be shown for up to $M = \lfloor (1/\epsilon)(\beta_* - \beta_L/\beta_* - \beta_{\min}) \rfloor$. In other words, the tolerated noise ϵ is inversely proportional to M times a constant depending on the bipartite inequalities (e.g., it is ≈ 0.15 for the CHSH inequality). Note that in many cases imperfect measurements can be described as ideal ones preceded by noisy channels, in all such cases the argument presented above also promises some robustness with respect to measurement imperfections.

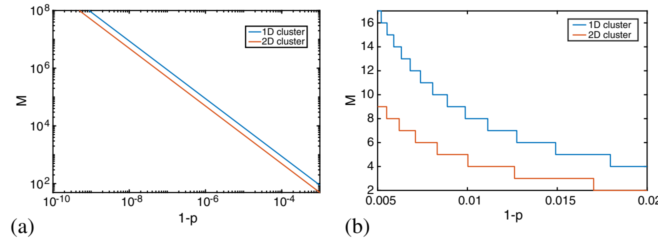

FIG. 2. (a) Plot of the maximal number of parties for which one can certify entanglement as a function of the noise $1-p$. (b) Similar plot for larger, experimentally better accessible values of $1-p$.

We now turn to an explicit example and consider the impact of local depolarizing noise (LDN) acting on each qubit of a cluster state. LDN can be viewed as a worst case local noise model [43]. It is parametrized by $p \in [0,1]$, where $p = 1$ corresponds to no noise and $p = 0$ to complete depolarization and is described by a map $\mathcal{E}(p)\rho = p\rho + (1-p)/4\sum_j \sigma_j \rho \sigma_j$. We choose 1D and 2D cluster states for testing the robustness and assume an infinite system size (or periodic boundary conditions). In Fig. 2 we plot the number of parties M for which one can show genuine multipartite entanglement as a function of the noise $1-p$. We choose a covering set which only contains nearest neighbor pairs. For cluster states it is possible to establish maximally entangled states between any pairs and hence we employ the CHSH inequality [44] as the bipartite Bell inequality, which has $\beta_* = 2\sqrt{2}$ and $\beta_L = 2$.

The plots suggest a polynomial relation between p and M.

For a discussion of a setup where 1D cluster states are generated via imperfect gates see Ref. [31].

Conclusion and outlook.—In this work we have introduced a scheme to detect genuine multipartite entanglement in a device-independent way based on bipartite Bell tests of entangled pairs that are deterministically generated from the initial state via LOCC. Our scheme detects all genuinely multipartite entangled pure states, therefore showing that neither knowledge of the Hilbert space dimension nor of the calibration of measurement devices are necessary in order to certify the genuine multipartite entangled nature of pure quantum states.

Our result is also applicable to mixed states with a sufficiently small amount of noise. The robustness of the scheme is directly related to the ratios of the local bound and the quantum violation of bipartite Bell inequalities, and any improvement on such inequalities directly leads to a larger set of states whose genuine multipartite entanglement can be certified device independently using our approach. Despite its generality our scheme is surprisingly efficient for important classes of states including the AKLT model and all (weighted) graph state with a bounded connectivity. In the latter case, in order to show genuine multipartite device-independent entanglement, one only needs to measure linearly many correlators with a fixed number of settings. Together with its relative robustness this makes our scheme very promising for future experiments.

We remark that a similar approach can be employed to reveal genuine multipartite nonlocality for large classes of states [45], where in this case the criterion is more stringent as maximally entangled qubit pairs on a covering set need to be generated deterministically. Still, many multipartite states including the toric code, the ground state of the AKLT model or all connected graph states, can be shown to be genuine multipartite nonlocal.

Finally, it would be interesting to explore how this approach can be used to detect more detailed entanglement structures [46].

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