Nondegenerate Solitons in Manakov System

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It is known that the Manakov equation which describes wave propagation in two mode optical fibers, photorefractive materials, etc., can admit solitons which allow energy redistribution between the modes on collision that also leads to logical computing. In this Letter, we point out that the Manakov system can admit a more general type of nondegenerate fundamental solitons corresponding to different wave numbers, which undergo collisions without any energy redistribution. The previously known class of solitons which allows energy redistribution among the modes turns out to be a special case corresponding to solitary waves with identical wave numbers in both the modes and traveling with the same velocity. We trace out the reason behind such a possibility and analyze the physical consequences.

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The discovery of solitons has created a new pathway to understand the wave propagation in many physical systems with nonlinearity [1]. In particular, the existence of optical solitons in nonlinear Kerr media [2] provoked the investigation on solitons from different perspectives, particularly from the applications point of view. By generalizing the waves propagating in an isotropic medium [3] to an anisotropic medium, a pair of coupled equations for orthogonally polarized waves has been obtained by Manakov [4,5] as

$$iq_{jz} + q_{jtt} + 2\sum_{p=1}^{2}|q_p|^2 q_j = 0, \qquad j = 1, 2, \tag{1}$$

where q_j, $j = 1, 2$, describe orthogonally polarized complex waves. Here the subscripts z and t represent the normalized distance and retarded time, respectively. Equation (1) also appears in many physical situations such as single optical field propagation in birefringent fibers [6], self-trapped incoherent light beam propagation in a photorefractive medium [7–9], and so on. The generalization of Eq. (1) to arbitrary N waves is useful to model optical pulse propagation in multimode fibers [10]. It has been identified [4] that the polarization vectors of the solitons change when orthogonally polarized waves nonlinearly interact with each other, leading to an energy exchange interaction between the modes [11]. The experimental observation of the latter has been demonstrated in Refs. [12–14]. The shape-changing collision property of such waves, which we designate here as a degenerate polarized soliton propagating with identical velocity and wave number in the two modes, gave rise to the possibility of constructing logic gates leading to all-optical computing at least in a theoretical sense [15–17]. Energy-sharing collisions among the optical vector solitons have been explored [16] by constructing multisoliton solutions explicitly to the multicomponent nonlinear Schrödinger equations. Furthermore, it has been shown that the multisoliton interaction process satisfies the Yang-Baxter relation [18]. It is clear from these studies that the shape-changing collision that occurs among the solitons with identical wave numbers in all the modes has been well understood. However, to our knowledge, studies on solitons with nonidentical wave numbers in all the modes have not been considered so far. Consequently, one would like to explore the role of such an additional wave number(s) on the soliton structures and collision scenario as well.

In the contemporary studies, a new class of multihump solitons has been identified in different physical situations. In birefringent dispersive nonlinear media, asymmetric double-hump–single-hump frozen states have been obtained [19]. Double-hump structure has been observed for the Manakov equation by considering two soliton solutions [20,21]. The first experimental observation of multihump solitons was demonstrated when the self-trapped incoherent wave packets propagate in a dispersive nonlinear medium [22]. These unusual solitons have been found in various nonlinear coupled field models [23]. The stability of multihump optical solitons has also been investigated in the case of a saturable nonlinear medium [24]. It is reported that in such a medium both two- and three-hump solitons do not survive after collision. N-self-trapped multihumped partially coherent solitons have also been explored in a photorefractive medium [25]. The coherent coupling between copropagating fields also gives rise to double-hump solitons in the coherently coupled nonlinear Schrödinger system [26]. In addition to the above, the dynamics of multihump structured solitons have also been studied in certain dissipative systems [27–30]. A double-hump phase-locked higher-order vector soliton has been observed, and its dynamics has been investigated in mode-locked fiber lasers [27,28]. Similarly in deployed fiber systems and fiber laser cavities, double-hump solitons have been observed during the buildup process of soliton molecules [31,32].

Motivated by the above, in this Letter, we present a new class of generalized soliton solutions for the Manakov model, exhibiting various interesting structures under general parametric conditions. A fundamental double-hump soliton (as well as other structures described below) sustains its shape even after a collision with another similar soliton. This behavior is in contrast to the one which exists in saturable nonlinear media, where two and three humps do not survive after a collision. The soliton solutions presented in this Letter also have both symmetric and asymmetric natures analogous to the partially coherent solitons in a photorefractive medium. Under a specific parametric restriction on wave numbers, they degenerate into the standard Manakov solitons exhibiting shape-changing collisions [11,16].

To explore the new family of soliton solutions for Eq. (1), we consider the bilinear forms of Eq. (1) as $(iD_z+D_t^2)g^{(j)}f=0$, $j=1, 2$, and $D_t^2 ff=2\sum_{n=1}^{2} g^{(n)}g^{(n)*}$, which are obtained through the dependent variable transformations $q_j = g^{(j)}/f$, $j = 1, 2$. Here D_z and D_t are the well-known Hirota bilinear operators [33], and $g^{(j)}(z,t)$ are complex functions, whereas f is a real function, and $*$ denotes complex conjugation. In principle, multisoliton solutions of Eq. (1) can be constructed by solving recursively the system of linear partial differential equations which results by substituting the series expansions $g^{(j)} = \epsilon g_1^{(j)} + \epsilon^3 g_3^{(j)} + \cdots$ and $f = 1 + \epsilon^2 f_2 + \epsilon^4 f_4 + \cdots$ for the unknown functions $g^{(j)}$ and f in the bilinear forms. Here ϵ is a formal expansion parameter.

Considering two different seed solutions for $g_1^{(1)}$ and $g_1^{(2)}$ as $\alpha_1^{(1)}e^{\eta_1}$ and $\alpha_1^{(2)}e^{\xi_1}$, respectively, where $\eta_1 = k_1 t + ik_1^2 z$, $\xi_1 = l_1 t + il_1^2 z$, and $\alpha_1^{(j)}$, $j = 1, 2$, k_1 and l_1 are, in general, independent complex wave numbers, to the resultant linear partial differential equations $(iD_z+D_t^2)g_1^{(j)}1=0$, $j = 1, 2$, which arise in the lowest order of ϵ, the series expansion gets terminated as $g^{(j)}=\epsilon g_1^{(j)}+\epsilon^3 g_3^{(j)}$ and $f=1+\epsilon^2 f_2+\epsilon^4 f_4$. The explicit forms of the unknown functions present in the truncated series expansions constitute a new fundamental one soliton solution to Eq. (1) in the form

$$q_1 = (\alpha_1^{(1)}e^{\eta_1} + e^{\eta_1+\xi_1+\xi_1^*+\Delta_1^{(1)}})/D_1,$$
$$q_2 = (\alpha_1^{(2)}e^{\xi_1} + e^{\eta_1+\eta_1^*+\xi_1+\Delta_1^{(2)}})/D_1, \qquad (2)$$

where $D_1 = 1 + e^{\eta_1+\eta_1^*+\delta_1} + e^{\xi_1+\xi_1^*+\delta_2} + e^{\eta_1+\eta_1^*+\xi_1+\xi_1^*+\delta_{11}}$, $e^{\delta_1} = [|\alpha_1^{(1)}|^2/(k_1+k_1^*)^2]$, $e^{\delta_2} = [|\alpha_1^{(2)}|^2/(l_1+l_1^*)^2]$, $e^{\delta_{11}} = \{[|k_1-l_1|^2|\alpha_1^{(1)}|^2|\alpha_1^{(2)}|^2]/[(k_1+k_1^*)^2(k_1^*+l_1)(k_1+l_1^*)(l_1+l_1^*)^2]\}$, $e^{\Delta_1^{(1)}} = \{[(k_1-l_1)\alpha_1^{(1)}|\alpha_1^{(2)}|^2]/[(k_1+l_1^*)(l_1+l_1^*)^2]\}$, and $e^{\Delta_1^{(2)}} = -\{[(k_1-l_1)|\alpha_1^{(1)}|^2\alpha_1^{(2)}]/[(k_1+k_1^*)^2(k_1^*+l_1)]\}$. From the above, it is evident that the fundamental solitons propagating in the two modes are characterized by four arbitrary complex parameters k_1, l_1, and $\alpha_1^{(j)}$, $j = 1, 2$. These nontrivial parameters determine the shape, amplitude, width, and velocity of the solitons which propagate in the Kerr media or photorefractive media. The amplitudes of the solitons that are present in the two modes q_1 and q_2 are governed by the real parts of the wave numbers k_1 and l_1, whereas velocities are described by the imaginary parts of them. Note that $\alpha_1^{(j)}$, $j = 1, 2$, are related to the unit polarization vectors of the solitons in the two modes.

To identify certain special features of the obtained four complex parameter family of soliton solution (2), we first consider (for simplicity of analysis) the special case where the imaginary parts of the wave numbers $k_{1I} = l_{1I}$ but with $k_{1R} \neq l_{1R}$. The latter case yields at least the following four different symmetric wave profiles, apart from similar asymmetric wave profiles, from solution (2) by incorporating the condition $k_{1R} < l_{1R}$ with further conditions and with suitable choices of parameters (examples given in Ref. [34]): (i) single-hump–single-hump soliton, $\alpha_{1R}^{(1)} > \alpha_{1R}^{(2)}$ and $\alpha_{1I}^{(1)} = \alpha_{1I}^{(2)}$; (ii) double-hump–single-hump soliton, $\alpha_{1R}^{(1)} = \alpha_{1R}^{(2)}$ and $\alpha_{1I}^{(1)} < \alpha_{1I}^{(2)}$; (iii) double-hump-flattop soliton, $\alpha_{1R}^{(1)} = \alpha_{1R}^{(2)}$ and $\alpha_{1I}^{(1)} \approx \alpha_{1I}^{(2)}$; (iv) double-hump–double-hump soliton, $\alpha_{1R}^{(1)} > \alpha_{1R}^{(2)}$ and $\alpha_{1I}^{(1)} = \alpha_{1I}^{(2)}$. Similar conditions can be given for $k_{1R} > l_{1R}$ also. We have not listed the asymmetric wave profiles here for brevity, which also exhibit the properties discussed below. A similar classification can be made for the case $k_{1I} \neq l_{1I}$, so that the solitons propagate in the two modes with different velocities and exhibit similar interaction properties. These will be discussed separately.

To illustrate the symmetric case, we display only the intensity profile of the double-hump soliton in Fig. 1. We call the solitons that have two distinct wave numbers in both the modes as in Eq. (2) nondegenerate solitons (which can exist as different profiles as described above), while the solitons which have identical wave numbers in all the modes (which exist only in single-hump form) are designated as degenerate solitons. In particular, in the special case when $k_1 = l_1$, the forms of q_j given in Eq. (2) degenerate into the standard bright soliton form [4,11]

$$q_j = \frac{\alpha_1^{(j)}e^{\eta_1}}{1+e^{\eta_1+\eta_1^*+R}}, \qquad j = 1, 2, \qquad (3)$$

which can be rewritten as

$$q_j = k_{1R}\hat{A}_j e^{i\eta_{1I}}\operatorname{sech}\left(\eta_{1R}+\frac{R}{2}\right), \qquad (4)$$

where $\eta_{1R} = k_{1R}(t-2k_{1I}z)$, $\eta_{1I} = k_{1I}t + (k_{1R}^2 - k_{1I}^2)z$, $\hat{A}_j = \{[\alpha_1^{(j)}]/\sqrt{(|\alpha_1^{(1)}|^2 + |\alpha_1^{(2)}|^2)}\}$,

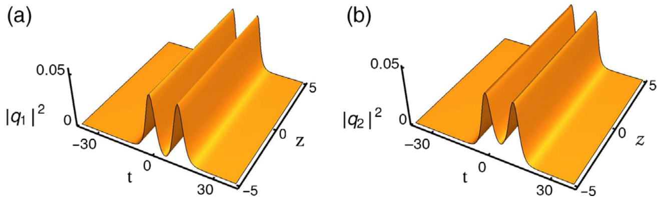

FIG. 1. Nondegenerate symmetric double-hump one soliton in the two modes: (a) and (b) denote the intensities of the components q_1 and q_2, respectively. The parameters are chosen as $k_1 = 0.316 + 0.5i$, $l_1 = 0.333 + 0.5i$, $\alpha_1^{(1)} = 0.49 + 0.45i$, and $\alpha_1^{(2)} = 0.45 + 0.45i$.

$e^R = [(|\alpha_1^{(1)}|^2 + |\alpha_1^{(2)}|^2)/(k_1 + k_1^*)^2]$, and $j = 1, 2$. Note that the above fundamental bright soliton always propagates in both the modes q_1 and q_2 with the same velocity $2k_{1I}$. The polarization vectors $(\hat{A}_1, \hat{A}_2)^\dagger$ have different amplitudes and phases, unlike the nondegenerate case where they have only different phases $[A_1 = (\alpha_1^{(1)}/\alpha_1^{(1)*})^{1/2}$, $A_2 = (\alpha_1^{(2)}/\alpha_1^{(2)*})^{1/2}]^\dagger$ [vide Eq. (2)] but the same unit amplitude. We call the above type of soliton (3) or (4) a degenerate soliton [35].

In order to understand the collision dynamics of the soliton solution of the kind (2), it is essential to construct the corresponding two-soliton solution. In the latter case, the series expansion for q_j, $j = 1, 2$, gets terminated as $g^{(j)} = \epsilon g_1^{(j)} + \epsilon^3 g_3^{(j)} + \epsilon^5 g_5^{(j)} + \epsilon^7 g_7^{(j)}$ and $f = 1 + \epsilon^2 f_2 + \epsilon^4 f_4 + \epsilon^6 f_6 + \epsilon^8 f_8$. The obtained explicit forms of $g^{(j)}$ and f, $j = 1, 2$, in the above truncated expansions constitute the nondegenerate two-soliton solution of Eq. (1), which reduces to the known form given in Ref. [11] for $k_i = l_i$, $i = 1, 2$. The complicated profiles of the present nondegenerate two-soliton solution are governed by eight arbitrary complex parameters k_j, l_j, $\alpha_1^{(j)}$, and $\alpha_2^{(j)}$, $j = 1, 2$ (see Supplemental Material [34]).

To study the collision dynamics between the nondegenerate two solitons, as an example, we again confine ourselves to the case of symmetric double-hump solitons by fixing the imaginary parts of the wave numbers as $k_{iI} = l_{iI}$, $i = 1, 2$. For other types also, a similar analysis has been carried out. By carefully examining the behavior of the obtained nondegenerate two-soliton solution in the asymptotic regimes, $z \to \pm\infty$, we find that the phases of the fundamental nondegenerate double-hump solitons in both the modes change during the collision process, while the intensities remain unchanged. This can be verified by defining the transition amplitudes as $T_j^l = (A_j^{l+}/A_j^{l-})$, $j = 1, 2$ and $l = 1, 2$, where subscript j represents the mode and superscript $l\pm$ denote the nondegenerate soliton numbers 1 and 2 designated as S_1 and S_2, respectively, in the asymptotic regimes $z \to \pm\infty$.

In the nondegenerate double-hump soliton case, the amplitudes of the solitons S_1 and S_2 in the first mode $(2k_{1R}A_1^{1-}, \{[(k_1-k_2)(k_2-l_1)^{1/2}(k_1+k_2^*)(k_2^*+l_1)^{1/2}]/[(k_1^*-k_2^*)(k_2^*-l_1^*)^{1/2}(k_1^*+k_2)(k_2+l_1^*)^{1/2}]\}2k_{2R}A_2^{1-})$ before a collision change to $(\{[(k_1-k_2)(k_1-l_2)^{1/2}(k_1^*+k_2)\times(k_1^*+l_2)^{1/2}]/[(k_1^*-k_2^*)(k_1^*-l_2^*)^{1/2}(k_1+k_2^*)(k_1+l_2^*)^{1/2}]\}\times 2k_{1R}A_1^{1+}, 2k_{2R}A_2^{1+})$ after a collision, where $A_1^{1\pm} = \sqrt{[\alpha_1^{(1)}/\alpha_1^{(1)^*}]}$ and $A_2^{1\pm} = \sqrt{[\alpha_2^{(1)}/\alpha_2^{(1)^*}]}$. Similarly in the second component, the amplitudes of the solitons S_1 and S_2 are $(2l_{1R}A_1^{2-}, \{[(l_1-l_2)(k_1-l_2)^{1/2}(l_1+l_2^*)(k_1+l_2^*)^{1/2}]/(l_1^*-l_2^*)(k_1^*-l_2^*)^{1/2}(k_1^*+l_2)^{1/2}(l_1^*+l_2)\}2l_{2R}A_2^{2-})$ before a collision which change to $(\{[(l_1-l_2)(l_1-k_2)^{1/2}\times(l_1^*+l_2)(k_2+l_1^*)^{1/2}]/[(l_1^*-l_2^*)(k_2^*-l_1^*)^{1/2}(l_1+l_2^*)\times(k_2^*+l_2)^{1/2}]\}2l_{1R}A_1^{2+}, 2l_{2R}A_2^{2+})$ after a collision, where $A_1^{2\pm} = \sqrt{[\alpha_1^{(2)}/\alpha_1^{(2)^*}]}$ and $A_2^{2\pm} = \sqrt{[\alpha_2^{(2)}/\alpha_2^{(2)^*}]}$. However, the intensity redistribution does not occur among the modes of the solitons, which can be confirmed by taking the absolute squares of the transition amplitudes which turn out to be unity, that is, $|T_j^l|^2 = 1$. This shows that, in the nondegenerate case, $k_i \neq l_i$, $i = 1, 2$, the polarization vectors do not contribute to intensity redistribution among the modes. Consequently, the double-hump solitons in each mode exhibit a shape-preserving collision corresponding to an elastic nature. This is illustrated in Fig. 2 for the parameter values given there by actually plotting the two-soliton solution (given in Supplemental Material [34]). From this figure, it is easy to identify that the intensity or energy of the double-hump solitons in the two modes propagates without change after a collision with another double-hump soliton except for a phase shift. A similar scenario exists generally for all other cases of $k_i \neq l_i$, $i = 1, 2$, the details of which will be published elsewhere. We also find that the phases of the soliton S_1 in the two modes change from $(\{[\Delta_{11} - \rho_1]/2\}$, $\{[\gamma_{11} - \rho_2]/2\})$ to $(\{[\Delta_{51} - \Phi_{22}]/2\}, \{[\gamma_{51} - \chi_{22}]/2\})$ during the collision process, while the phases of soliton S_2 change from $(\{[\Delta_{15} - \Theta_{11}]/2\}$, $\{[\gamma_{15} - \nu_{11}]/2\})$ to $(\{[\Lambda_{22} - \hat{\rho}_1]/2\}, \{[\mu_{22} - \hat{\rho}_2]/2\})$ after a collision. Here $\rho_j = \log \alpha_1^{(j)}$, $\hat{\rho}_j = \log \alpha_2^{(j)}$, $j = 1, 2$, Δ_{11}, γ_{11}, Δ_{51}, γ_{51}, Δ_{15}, Θ_{11}, γ_{15}, ν_{11}, Λ_{22} and μ_{22} are constants (see [34]).

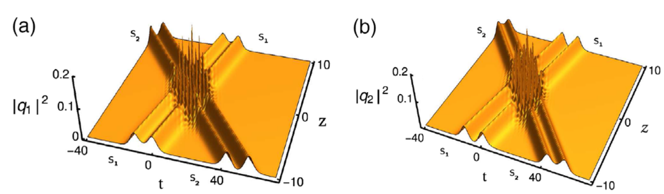

FIG. 2. Nondegenerate solitons exhibiting shape-preserving collisions: (a) and (b) denote the elastic collision of two symmetric double-hump solitons for the parametric values $k_1 = 0.333 + 0.5i$, $k_2 = 0.3–2.2i$, $l_1 = 0.3 + 0.5i$, $l_2 = 0.333–2.2i$, $\alpha_1^{(1)} = 0.45 + 0.45i$, $\alpha_2^{(1)} = 0.49 + 0.45i$, $\alpha_1^{(2)} = 0.49 + 0.45i$, and $\alpha_2^{(2)} = 0.45 + 0.45i$.

In addition to the above, we have also observed a similar shape-preserving collision in the case of a symmetric single-hump soliton when it collides with another identical soliton. The flattop soliton also preserves its structure when it collides with a symmetric double-hump soliton. However, while testing the stability property of a double-hump soliton interacting with a single-hump soliton, we come across a slightly different collision scenario. During this interaction process, the symmetric double-hump soliton experiences a strong perturbation due to the collision with the symmetric single-hump soliton. The result of their collision is reflected only in a change in the shape of the symmetric double-hump soliton into a slightly asymmetric form but without a change in energy. However, the symmetric single-hump soliton does not undergo any change (see [34]).

In contrast to the nondegenerate case, the nonlinear superposition of degenerate fundamental solitons ($k_i = l_i$, $i = 1, 2$) in the Manakov system exhibits an interesting shape-changing collision due to intensity redistribution among the modes as shown in Ref. [11]. The intensity redistribution occurs in the degenerate case due to the arbitrary polarization vectors in the two modes getting mixed up, which is illustrated in Fig. 3, where the intensity redistribution occurs because of the enhancement or suppression of intensity in any one of the modes in either one of the degenerate solitons with a corresponding suppression or enhancement of intensity of the same soliton; see Eq. (4) [11]. To hold the energy conservation between the two modes, the intensities of the two solitons S_1 and S_2 change appropriately. It is well known that the degenerate soliton or Manakov soliton [Eq. (3)] reported in Refs. [10,11], in general, exhibits a shape-changing collision through energy redistribution among the modes (except for the very special case $[\alpha_1^{(1)}/\alpha_2^{(1)}] = [\alpha_1^{(2)}/\alpha_2^{(2)}]$ [10,16], where elastic collision occurs). We have also verified the elastic nature of a double-hump soliton collision using the Crank-Nicolson method [36].

We also further wish to point out that, considering the notion dissipative solitons, they also exhibit an elastic collision property. However, this collision scenario, for example, in a fiber laser cavity, is entirely different from the one that occurs in our present case. In the fiber laser cavity, during the collision between the soliton pair (bound state or doublet) and single-soliton state (singlet), the single soliton destroys the bound state, but another pair is formed that moves away with the same velocity, leaving one of the solitons of the previously moving pair in rest [37,38]. During this collision scenario, the energy or momentum is not conserved in the dissipative system (fiber laser cavity). To bring the above elastic collision, it is essential to set up the binding energy of solitons to be nonzero, and the difference in velocities of the pair and the singlet is fixed and must be the same before and after collision [37,38]. Also, no explicit analytical form of such a dissipative soliton is available for a direct analysis.

In principle, one can construct the N-soliton solution of the nondegenerate type to the Manakov system by following the procedure given above. For the N-nondegenerate soliton, the power series expansion should be of the form $g^{(j)} = \sum_{n=1}^{2N-1} \epsilon^{2n-1} g_{2n-1}^{(j)}$ and $f = 1 + \sum_{n=1}^{2N} \epsilon^{2n} f_{2n}$. The shape of the profile will be determined by the $4N$ complex parameters which are present in the N-soliton solution. The degenerate soliton solutions can be recovered from the nondegenerate N-soliton solution by fixing the wave numbers as $k_i = l_i$, $i = 1, 2, \ldots, N$. The symmetric profile of the multinondegenerate soliton can be obtained by fixing the imaginary parts as $k_{iI} = l_{iI}$, $i = 1, 2, \ldots, N$. We also point out that the symmetric and asymmetric cases of the nondegenerate soliton solution given in Eq. (2) can be compared with a partially incoherent soliton in a photorefractive medium [25]. The profile of the partially coherent soliton is determined by only three real parameters for $N = 2$ as a special case of the degenerate soliton [10,16] (Manakov case), whereas in the present nondegenerate case, the profiles of the single soliton are governed by four complex parameters. In the incoherent limit (the number of modes is infinity), the shape of the partially coherent soliton can be arbitrary, since the number of parameters involved in the underlying analytical form is N-free real parameters. However, in the incoherent limit, the presence of $2N$ free complex parameters in the nondegenerate fundamental one soliton would bring in more complex shapes than the above-mentioned partially coherent soliton reported in the photorefractive medium.

To observe the existence of nondegenerate solitons (2) experimentally, one may consider the mutual-incoherence procedure given in Refs. [12,22] with two different laser sources of different characters (instead of a single laser source). Using polarizing beam splitters, the extraordinary beams coming out from the two laser sources can be further split into four individual incoherent fields. These four fields can act as two nondegenerate individual solitons in the two modes. Furthermore, the collision angle must be large enough to observe the elastic collision between these two nondegenerate solitons in both the modes [12,13]. The experimental procedure with a single laser can be used to

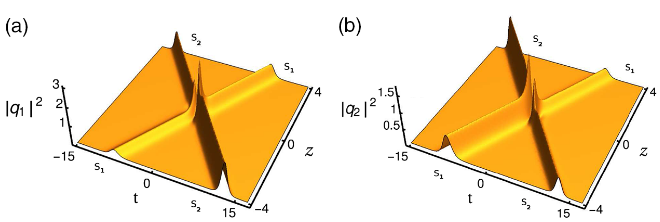

FIG. 3. Degenerate solitons exhibiting a shape-changing collision: (a) and (b) denote the energy-sharing collision in the two modes for the parametric values $k_1 = l_1 = 1 + i$, $k_2 = l_2 = 1.51 - 1.5i$, $\alpha_1^{(1)} = 0.5 + 0.5i$, and $\alpha_2^{(1)} = \alpha_1^{(2)} = \alpha_2^{(2)} = 1$.

observe the Manakov solitons and multimode multihump solitons that arise in a dispersive nonlinear medium [12,22].

Finally, it is essential to point out the application of our above-reported soliton solutions. Our results open up a new possibility to investigate nondegenerate solitons in both integrable and nonintegrable systems and will give rich coherent structures when the four-wave-mixing phenomenon is taken into account. Our studies can also be extended to fiber arrays and multimode fibers where the pulse propagation is described by Manakov-type equations. Experimental observations of Manakov solitons in AlGaAs planar wave guides [13] and multihump solitons in the multimode self-induced wave guides [22] give the impression that our results will be important to an interaction of the optical field in coupled field models. The shape-preserving collision which occurs among the nondegenerate solitons can be used for the optical communication process. The double-hump nature of the nondegenerate solitons can be useful for the information process as described in the concept of a soliton molecule [31]. As far as the degenerate soliton is concerned, it has already been shown that it is useful in the computation process [15,16]. We note that under the appropriate conditions, namely, $k_{1I} \approx k_{2I}$ and $l_{1I} \approx l_{2I}$, the nondegenerate solitons reported in the present conservative system can be seen as the soliton molecule observed in the deployed fiber systems and in fiber laser cavities [31,32,39–42].

In conclusion, we have shown that the Manakov model under a general physical situation admits interesting nondegenerate solitons exhibiting shape-preserving collisions, thereby leading to explain the interaction of the elastic nature of a light-light interaction under general initial conditions. The fascinating energy-sharing collisions exhibiting the nonlinear superposition of degenerate multisolitons can be extracted from the nondegenerate soliton solutions under the specific physical restrictions, which leads to the construction of optical logic gates [15].

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