Dark Matter Interactions, Helium, and the Cosmic Microwave Background

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The cosmic microwave background (CMB) places a variety of model-independent constraints on the strength interactions of the dominant component of dark matter with the standard model. Percent-level subcomponents of the dark matter can evade the most stringent CMB bounds by mimicking the behavior of baryons, allowing for larger couplings and novel experimental signatures. However, in this Letter, we will show that such tightly coupled subcomponents leave a measurable imprint on the CMB that is well approximated by a change to the helium fraction, Y_{He}. Using the existing CMB constraint on Y_{He}, we derive a new upper limit on the fraction of tightly coupled dark matter, f_{TCDM}, of $f_{\mathrm{TCDM}} < 0.006$ (95% C.I.). We show that future CMB experiments can reach $f_{\mathrm{TCDM}} < 0.001$ (95% C.I.) and confirm that the bounds derived in this way agree with the results of a complete analysis. These bounds provide an example of how CMB constraints on Y_{He} have applications beyond studying big bang nucleosynthesis, since tightly coupled dark matter plays no direct role in the formation of light nuclei. We briefly comment on the implications for model building, including millicharged dark matter.

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Introduction.—The nature of dark matter is one of the central questions in physics, intersecting the fields of astrophysics, cosmology, and particle physics. While it is common (and simpler) to assume dark matter is made entirely from a single type of weakly interacting particles, the possibility that the dark sector involves a richer set of particles and interactions is compatible with (or even motivated by) the state of experimental and observational dark matter searches (see, e.g., Refs. [1,2] for recent reviews). A variety of new searches have been proposed that target this type of exotic physics in the dark sector [3].

Cosmology provides a particularly useful window into the nature of dark matter. It is the domain where the influence of dark matter can be inferred most directly and unambiguously from observations. In the linear regime, predictions for the evolution of the density fluctuations in the presence of cold dark matter (CDM) have been verified at high precision with cosmological observations [4]. These observations put strong limits on potential interactions between the dominant component of dark matter and the standard model [5–8] and/or other particles in the dark sector (e.g., Refs. [9–11]).

The cosmological bounds on these interactions are drastically reduced for percent-level subcomponents of the dark matter as they can mimic the signatures of baryons [12,13]. Baryons make up roughly 5% of the energy density in the Universe today and have substantial self-interactions and interactions with the photons. Since it is these interactions that distinguish baryons from the dark matter in many cosmological observables, a subcomponent of dark matter strongly interacting with the standard model acts effectively as baryonic matter from the cosmological perspective. These limitations on cosmological constraining power are particularly relevant to light dark matter subcomponents that interact with baryons either through electromagnetism (e.g., millicharge) or some new direct force. These examples are not well constrained by current experiments but can have observable signatures in a variety of proposed searches [3]. These models have also been proposed to explain unusual astrophysical signals like the EDGES measurement of the global 21 cm absorption feature [14–16].

While the above discussion suggests that there is a degeneracy between baryons and tightly coupled dark matter (TCDM), we will show that a more precise description is that the phenomenology of TCDM is effectively the same as a change to the primordial helium abundance, Y_{He}, as far as the cosmic microwave background (CMB) is concerned: neutral helium does not efficiently interact with CMB photons, yet it is tightly coupled to protons and electrons and contributes to the physical baryon density, ω_b. At fixed total effective baryon density, increasing the mass fraction in the form of helium or TCDM therefore increases the mean free path of photons by decreasing the density of scatterers which, in turn, increases diffusion (Silk) damping of the power spectrum. From this observation, the CMB limits on the fraction of TCDM can be derived directly from the existing CMB measurement of Y_{He} and does not require a separate analysis of the data. Furthermore, this bound does not depend on the detailed form of the interactions between dark matter and the baryons. Additionally, the constraints on TCDM derived from the CMB demonstrate that CMB measurements of Y_{He} have value beyond probing big bang nucleosynthesis (BBN), since TCDM plays no direct role in BBN. With current limits on Y_{He} from Planck [4], we will show that TCDM can be at most 0.6% of the dark matter, and upcoming CMB observations should improve these limits by a factor of 5. This new bound excludes the most of the viable parameter space for a millicharged dark matter interpretation of the EDGES measurement [16,17] and is similarly powerful in other settings as a broad model-independent constraint.

Tight coupling and helium.—In this note, we will assume that the dark matter has sufficiently large interactions with the baryons to be tightly coupled with the photon-baryon fluid during the era of recombination. We will also assume its direct scattering cross section with the photons to be negligible as to not alter the visibility function. These assumptions are relevant to direct forces between baryons and the dark matter, for example, due to the exchange of a new particle or a millicharge [18].

Our definition of tight coupling is that the TCDM component and baryons behave effectively as a single fluid. Assuming the dark matter is nonrelativistic during recombination (mass greater than 10 eV), its contribution to this fluid is only to increase the energy density. Prior to recombination, the baryon-TCDM fluid couples to the photons to form a single relativistic fluid with a sound speed,

$$c_s = \frac{1}{\sqrt{3(1+R_b)}}, \qquad R_b \equiv \frac{3\rho_b}{4\rho_\gamma}, \tag{1}$$

leading to characteristic temperature fluctuations $\Delta = \Delta T/T$ of the schematic form,

$$(\Delta+\psi)(k,\eta) \propto \cos[kr_s(\eta)], \qquad r_s(\eta) = \int^\eta d\eta' c_s(\eta'), \tag{2}$$

where ψ is the Newtonian gauge (time-time) metric perturbation.

From this description, one might imagine that there is no discernible difference between TCDM and true baryon density as measured by the CMB power spectrum. Indeed, the first cosmological bounds on the fraction of TCDM [12] came from comparing the measurement of ω_b inferred from the CMB (with Y_{He} fixed) to that from primordial light element abundances produced during BBN. TCDM does not contribute to ω_b measured from BBN and therefore the difference is a measure of the TCDM fraction.

Fortunately, we can get a better constraint from the CMB alone. While baryons and TCDM are indistinguishable both well before recombination when baryons and photons are tightly coupled as well as after decoupling (but before reionization), the degeneracy is broken (or modified) by the finite mean free path of photons which allows photon diffusion out of the gravitational potential wells.

$$(\Delta+\psi)(\vec{k},\eta) = A_{\vec{k}} \cos[kr_s(\eta)] e^{-k^2/k_d^2}, \tag{3}$$

where $A_{\vec{k}}$ is the primordial amplitude. The diffusion (Silk) damping scale k_d is given by [19–21]

$$\frac{1}{k_d^2(\eta)} = \int^\eta d\eta' \frac{1}{an_{\mathrm{H}}x_e\sigma_T}\frac{c_s^2}{2}\left(\frac{R_b^2}{(1+R_b)}+\frac{16}{15}\right), \tag{4}$$

where σ_T is the Thomson scattering cross section, n_{H} is the number density of hydrogen, and x_e is the ionization fraction. In this notation $n_{\mathrm{H}}x_e$ is the free electron number density, and x_e can exceed unity when helium is ionized.

One noteworthy feature of the damping scale k_d (and the mean free path in general) is that it is determined not by n_b but by n_{H}, which depends both on the effective baryon density and the form of the baryons. This distinction is important even in ΛCDM, due to the presence of primordial helium. At the time of recombination, helium is tightly coupled to the protons, but is electrically neutral (not ionized) and does not efficiently couple to CMB photons. As a result, at fixed $\omega_b \equiv \Omega_b h^2$, the damping scale is sensitive to the helium mass fraction Y_{He} through $n_{\mathrm{H}} = n_b(1-Y_{\mathrm{He}})$. It is predominantly through this effect that the CMB can constrain Y_{He}, leading to the current measurement [4],

$$Y_{\mathrm{He}} = 0.247^{+0.026}_{-0.027} \quad (95\%\ \mathrm{C.I.}), \tag{5}$$

assuming a $\Lambda\mathrm{CDM} + Y_{\mathrm{He}}$ cosmology (we use the $\mathrm{TT,TE,EE} + \mathrm{lowP} + \mathrm{lensing}$ Planck likelihood throughout this work).

Returning to TCDM, the above discussion shows that ω_{TCDM} has a much tighter degeneracy with Y_{He} than with ω_b (with Y_{He} fixed by BBN consistency). We explain below how we can therefore use the CMB constraint on Y_{He} of Eq. (5), in combination with the Y_{He} value predicted by BBN to derive constraint on the TCDM fraction.

In the presence of TCDM, we will simply define

$$n_b \equiv \frac{\rho_b}{m_{\mathrm{H}}} \equiv n_{\mathrm{H}} + \frac{m_{\mathrm{He}}}{m_{\mathrm{H}}}n_{\mathrm{He}} + \frac{\rho_{\mathrm{TCDM}}}{m_{\mathrm{H}}}, \tag{6}$$

such that ρ_b and n_b include the TCDM. Even in the presence of TCDM, we will still define Y_{He} in terms of the true baryons as they are the relevant quantity during BBN,

$$Y_{\rm He} \equiv \frac{m_{\rm He} n_{\rm He}}{m_{\rm H} n_{\rm H} + m_{\rm He} n_{\rm He}} = \left(1 - \frac{\omega_{\rm TCDM}}{\omega_b}\right)^{-1} \frac{m_{\rm He}}{m_{\rm H}} \frac{n_{\rm He}}{n_b}. \quad (7)$$

At fixed ω_b, the number density of hydrogen is therefore

$$n_{\rm H} = n_b \left[1 - Y_{\rm He} - (1 - Y_{\rm He}) \frac{\omega_{\rm TCDM}}{\omega_b}\right] \equiv n_b [1 - Y_{\rm He} - F_{\rm TCDM}]. \quad (8)$$

This formula defines $F_{\rm TCDM}$ as it is only through this parameter that $\omega_{\rm TCDM}$ enters the Boltzmann equations.

It should be clear that a change to $Y_{\rm He}$ or $F_{\rm TCDM}$ (while holding the other fixed) has the same effect on Eq. (8). We can therefore interpret the measurement of $Y_{\rm He}$ in Eq. (5) as the measurement of $Y_{\rm He}^{\rm BBN} + F_{\rm TCDM}$, using the BBN prediction of $Y_{\rm He}^{\rm BBN} = 0.24534 \pm 0.00061$. Combined with the measurement $\omega_b/\omega_{\rm cdm} = 0.187 \pm 0.004$ [4], we can derive a constraint

$$f_{\rm TCDM} \equiv \frac{\omega_{\rm TCDM}}{\omega_{\rm cdm}} < 0.0067 \quad (95\% \text{ C.I.}). \quad (9)$$

This upper limit of approximately 0.7% tightly coupled dark matter is almost 10% weaker than the true bound because it includes a change to $x_e(z)$ in Eq. (4) due to helium ionization that is not associated with TCDM. We will demonstrate this small additional effect in the next section and confirm that extrapolation of the limits on $Y_{\rm He}$ is consistent with the treatment of TCDM in a full Boltzmann code. In addition, we will see that significant improvements in these bounds are expected in the next generation of experiments. The bound derived here is already an improvement over the best previously published constraint [13] (where the TCDM was assumed to be millicharged dark matter) from the inclusion of Planck polarization data.

This bound is particularly relevant to dark matter-baryon interactions invoked to explain the EDGES measurement [14,15]. Given existing experimental and observational constraints, the most viable such model is a tightly coupled millicharged subcomponent of the dark matter with $f_{\rm TCDM} \sim 0.003$–0.01 [16,17]. Our new bound significantly reduces the available parameter space for these models and should be covered entirely by the current generation of ground-based CMB observations.

The bounds presented here also have implications to experimental searches for light dark matter. While direct detection searches have placed strong limits on the dark matter-baryon cross sections for masses above a GeV, few experiments directly probe smaller masses (see, e.g., Refs. [3,22] for reviews). A variety of indirect measurements limit the allowed parameter space for the dominant component of dark matter [4,8,23,24]. These indirect constraints can be weakened significantly for subcomponents of the dark matter and leave open the opportunity of direct detection in the lab. The limits presented here further restrict the viable parameter space for such models.

Validation and forecasts.—The above physical argument suggests that the identification between $F_{\rm TCDM}$ and $Y_{\rm He}$ should be valid for a CMB power spectrum analysis. However, the constraints on $Y_{\rm He}$ are derived using a Boltzmann code like CAMB [25] or CLASS [26] to compute the CMB power spectrum, and it is reasonable to ask if this identification is valid at the level of precision needed both in current and future data.

The identification between $F_{\rm TCDM}$ and $Y_{\rm He}$ would seem to be exact with respect to the Boltzmann equations for the evolution of the photon and baryon density perturbations. Helium is included only as a subcomponent of the baryon density and is not given its own evolution equations. For fixed ω_b and ionization history $x_e(z)$, the only impact of $Y_{\rm He}$ is to change n_e as described above, so the evolution of the perturbations treats helium and TCDM in the same way.

However, there is more to the cosmology of helium than simply altering n_e at recombination. In particular, the ionization history, as defined by $x_e(z)$, must include helium ionization both in the early and late universe. The atomic physics of helium is included when computing the ionization or recombination history [27–29]. From the output of such a calculation, the distinction between helium and TCDM can be easily seen from the fact that one finds redshifts where $x_e(z) > 1$ in the presence of (ionized) helium. As shown in Fig. 1, the response of $x_e(z)$ to changes in $Y_{\rm He}$ and $F_{\rm TCDM}$ is nearly identical around recombination where x_e is responding only to the change in n_e. In contrast, the results are significantly different at

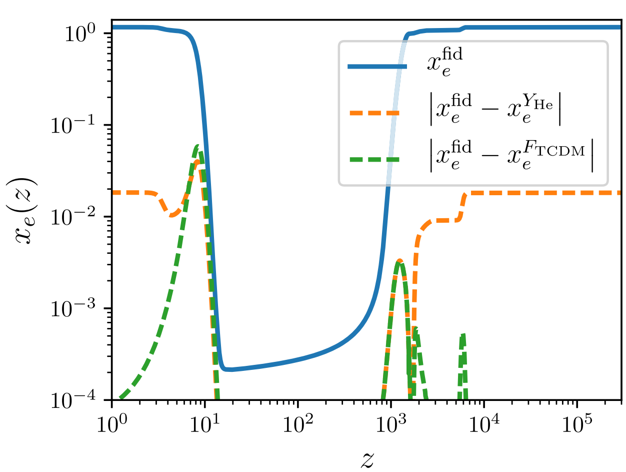

FIG. 1. Changes to the ionization fraction resulting from an increase of 0.02 to $Y_{\rm He}$ and to $F_{\rm TCDM}$. The dominant effects around hydrogen recombination are almost identical for both parameters. First and second helium recombination are visible in the orange curve as rapid changes in $x_e(z)$ around $z \simeq 6000$ and 1800 and likewise for helium reionization around $z \simeq 3$. These features are easily visible when we change $Y_{\rm He}$ but are not affected by changes to $F_{\rm TCDM}$.

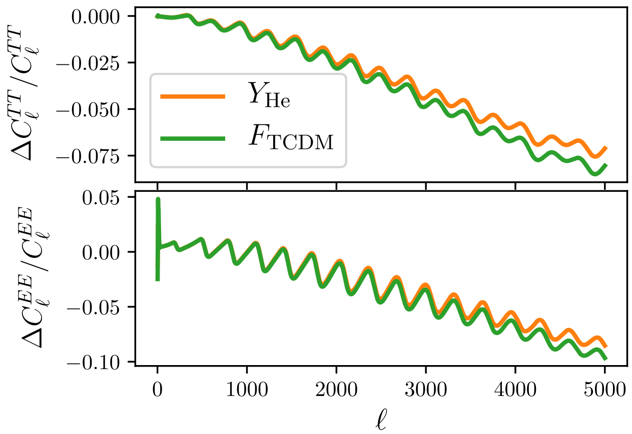

FIG. 2. Relative changes to the temperature (top) and E-mode polarization (bottom) power spectra resulting from an increase of 0.02 to $Y_{\rm He}$ and $F_{\rm TCDM}$.

high and low redshifts where helium ionization is important.

CMB observations do not directly constrain $x_e(z)$ and we must therefore address whether this difference between TCDM and helium impacts the constraints on $f_{\rm TCDM}$ derived from constraints on $Y_{\rm He}$. In order to validate the assumption made in the previous section, we modified CAMB [25] to include a TCDM component. This was achieved by reducing the number density of protons (and thereby electrons) compared to the density of effective baryons as shown in Eq. (8). The density of baryons used in the recombination and reionization parts of the code was adjusted to match the true baryon density (rather than the effective density which includes the TCDM). With these modifications an equal change to either $Y_{\rm He}$ or $F_{\rm TCDM}$ leads to an ionization history which is the same when helium is neutral, but differs before helium recombination and after helium reionization.

The effects on the CMB power spectra for equal changes to $Y_{\rm He}$ and $F_{\rm TCDM}$ are very similar, as seen in Fig. 2. There is a slight difference in the amount of damping, which can be straightforwardly understood as being due to the presence of ionized helium at high redshift. Models with increased $Y_{\rm He}$ have higher $x_e(z)$ at very early times before helium recombined. Helium ionization adds additional electrons, which reduces the mean free path of photons at higher redshifts and thus increases k_d, as can be seen in Eq. (4) and in Fig. 3. Directly calculating k_d gives an 8% difference in the change to k_d^2 between $Y_{\rm He}$ and $F_{\rm TCDM}$ relative to the fiducial model.

There is also a difference in the way reionization proceeds when either $Y_{\rm He}$ or $F_{\rm TCDM}$ is adjusted. Our modifications held fixed the optical depth to reionization τ, which requires a slightly different redshift of reionization in the two cases, since helium undergoes reionization while TCDM does not. This difference is reflected in the impact on the largest scales of the E-mode polarization power

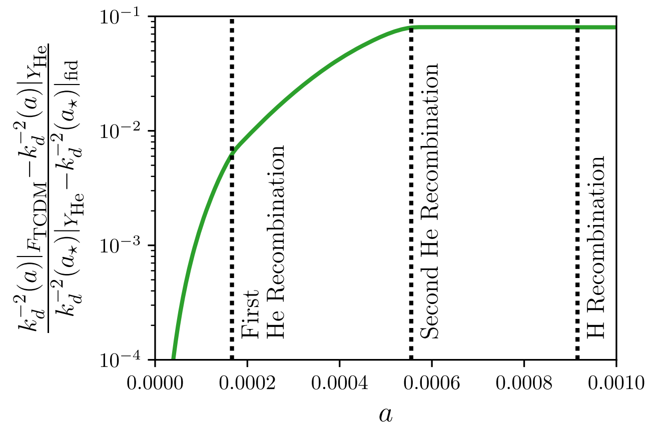

FIG. 3. Difference in $k_d^{-2}(a)$ [see Eq. (4)] between models where $Y_{\rm He}$ and $F_{\rm TCDM}$ are increased by 0.02 relative to the difference in $k_d^{-2}(a_\star)$ between a model with $Y_{\rm He}$ increased by 0.02 and the fiducial model, where $a_\star$ is the scale factor at last scattering. It is clear that the difference in the damping scale accumulates during the very early times when helium has not yet recombined and the value of the ionization fraction $x_e(z)$ differs between the two models. The difference reaches a constant 8% offset after helium recombination.

spectrum, shown in the bottom panel of Figure 2, though these modes are subject to large cosmic variance and do not drive the constraints shown below. The response of $x_e(z)$ to changes in $Y_{\rm He}$ and $F_{\rm TCDM}$ also alters the visibility function during recombination but we find this effect is much smaller than the change to k_d.

The similarities seen in the ionization history and the power spectra translate directly to the measurements of $Y_{\rm He}$ and $f_{\rm TCDM}$. We show the forecasts for $\Lambda{\rm CDM} + Y_{\rm He}$ and $\Lambda{\rm CDM} + f_{\rm TCDM}$ in Fig. 4. Given our analytic argument, rescaling $Y_{\rm He}$ forecasts by a factor of $(\omega_b/\omega_{\rm cdm})(1-Y_{\rm He})^{-1}$ should produce nearly the same results. The forecasts show that the two agree very well, but in detail

$$\sigma(f_{\rm TCDM}) \approx 0.90(\omega_b/\omega_{\rm cdm})(1-Y_{\rm He})^{-1}\sigma(Y_{\rm He}). \quad (10)$$

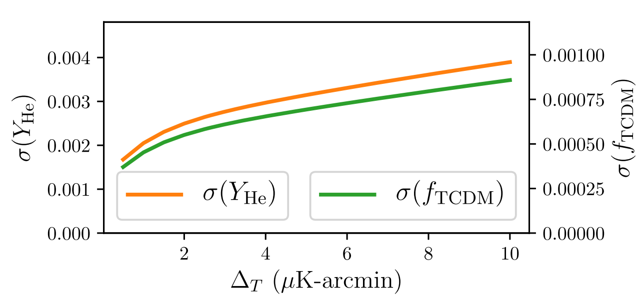

FIG. 4. Forecasts for 1σ constraints on $Y_{\rm He}$ in $\Lambda{\rm CDM} + Y_{\rm He}$ compared to constraints on $f_{\rm TCDM}$ in $\Lambda{\rm CDM} + f_{\rm TCDM}$ for a range of noise levels, assuming a 1.5 arc min beam. Our forecasts show that to a very good approximation the direct constraints on tightly coupled dark matter can be obtained from rescaling constraints using Eq. (10).

This additional factor of 0.90 is related to the 8% difference in k_d arising from the helium ionization at early times through the factor of $x_e(z)$ in Eq. (4), with the remaining 2% difference being due to the approximate hydrodynamical treatment of the fluctuations and the effects of helium reionization at late times. Since F_{TCDM} leads to slightly more damping than Y_{He}, the constraint on f_{TCDM} derived from Y_{He} is weaker than the true bound by 10%. For completeness, we computed the actual bound using the Planck 2015 (TT, TE, EE + lowP + lensing) likelihood [30] and a modified version of CosmoMC [31] to find

$$f_{\mathrm{TCDM}} < 0.0064 \quad (95\%\ \mathrm{C.I.}) \tag{11}$$

in a $\Lambda\mathrm{CDM} + f_{\mathrm{TCDM}}$ model with Y_{He} fixed by BBN consistency, which very nearly matches 0.90 times the constraint quoted in Eq. (9) obtained from rescaling the Y_{He} constraint. Since the ratio of the constraints is largely independent of noise level, one could further rescale the bounds of the previous section to give limits accurate to about a percent.

Finally, the forecasts shown in Fig. 4 illustrate the significant improvement in sensitivity to TCDM expected for the coming generations of CMB surveys. A survey like CMB-S4 [32] with map noise levels near 1 μK-arcmin can reach $f_{\mathrm{TCDM}} < 0.001$ (95% C.I.). This factor of 5–6 improvement is unsurprising given the relationship to Y_{He} we have shown here, but it further illustrates that the degeneracy with ω_b does not fundamentally limit our sensitivity to TCDM.

Summary.—Subcomponents of dark matter that interact with baryons can evade some of the stringent model-independent constraints placed by cosmology. These subcomponents are appealing as experimental targets and signatures of complex dark sectors. In this Letter, we showed that tightly coupled subcomponents of dark matter would effectively increase the helium fraction as measured by the CMB and can be excluded by current Planck data if they make up more than 0.6% of the dark matter. Upcoming CMB observations will improve on this bound by about a factor of 5.

These results are relevant to the broader question of how cosmological data more generally informs our understanding of dark matter. It has long been understood that the CMB measurement of Y_{He} will continue to improve with observational sensitivity [32]. However, the potential impact of these measurements was largely thought to be restricted to probing BBN, which is already well characterized by primordial abundance measurements. Nonetheless, we have shown that unlike primordial abundances, the CMB constraint is not particularly sensitive to the atomic or nuclear properties of helium and can broadly characterize new physics which does not directly impact BBN. This further suggests that combining primordial abundance measurements and CMB observations may yield further insights into the nature of dark matter.

More generally, the tools we use to characterize dark matter from cosmology are being significantly extended with improvements in CMB sensitivity. Although the signatures of tightly coupled dark matter are degenerate with ω_b to leading order, the subleading impact on the damping tail offers a powerful constraint. As intuition for cosmological data evolves to match the exponential improvements in survey sensitivity, new windows into the nature of dark matter are likely to emerge.

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