Probing the State of a Mechanical Oscillator with an Ultrastrongly Coupled Quantum Emitter

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Performing accurate position measurements of a mechanical resonator by coupling it to some optically driven quantum emitter is an important challenge for quantum sensing and metrology. We fully characterize the quantum noise associated with this measurement process, by deriving master equations for the coupled emitter and the resonator valid in the ultrastrong coupling regime. At short timescales, we show that this noise sets a fundamental limit to the readout sensitivity and that the standard quantum limit can be recovered for realistic experimental conditions. At long timescales, the scattering of the mechanical quadratures leads to the decoupling of the emitter from the driving light, switching off the noise source. This method can be used to describe the interaction of any quantum system strongly coupled to a finite size reservoir.

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Coupling optical and mechanical degrees of freedom (d.o.f.) was first investigated in the pioneering field of cavity optomechanics, involving an electromagnetic resonator with a moving end-mirror coupled to a mechanical oscillator [1]. Optical drive of such systems has lead to efficient cooling of the mechanics [2,3] down to the ground state [4,5], opening the path to manipulation of the quantum states of a macroscopic oscillator [6,7]. With the recent developments of nanomechanics, a new class of systems has emerged where the optomechanical coupling is not mediated by a cavity, but by a single quantum emitter [8]. These hybrid systems are now implemented in a wide range of platforms coupling e.g., single spins [9], nitrogen-vacancy (NV) centers in diamonds [10,11], or semiconductor quantum dots [12,13] to vibrating nanowires, or else involving superconducting qubits embedded in oscillating membranes [14,15]. Interesting test beds to investigate the quantum-classical boundary or information thermodynamics [16], these devices are also especially appealing for quantum sensing and metrology [17]. Indeed, tiny variations in the position of the mechanics allow measuring ultralow forces, as the one created by a single spin [9]. Reciprocally, it is possible to extract information on the position of the mechanics from the properties of the light radiated by an embedded quantum emitter [18,19]. The intrinsic sensitivity of these devices can be enhanced by reducing the size of the resonator, or alternatively, by increasing the coupling between the mechanics and the quantum emitter: recently some experiments have reached the ultrastrong coupling regime where the emitter-mechanical coupling is comparable to the mechanical frequency [12,14].

These achievements have opened the way to the experimental study of the "single-photon regime" of optomechanics ruled by the nonlinearized optomechanical interaction [20], and characterized by new noise sources whose proper modeling is still to come. So far, indeed, most theoretical investigations of hybrid optomechanical systems have focused on the weak coupling regime [21–25] where the coupling to the quantum emitter is treated using perturbative techniques. A better understanding of state-of-the-art experimental devices now requires to model their evolution in the ultrastrong coupling regime.

In this Letter, we model the dynamics of a mechanical resonator interacting with an optically driven quantum emitter in the ultrastrong coupling regime. We demonstrate that it is possible to extract information on the mechanical position through the light radiated by the quantum emitter. This measurement process induces a backaction noise on the mechanical state, associated to the emitter's population fluctuations. To fully characterize this quantum noise, we go beyond the semiclassical approximation and derive master equations ruling the evolution of the hybrid system. At short timescales, the quantum noise translates into a nonsymmetric scattering of the mechanical quadratures, setting a fundamental limit to sensing precision. We show that the standard quantum limit can be recovered for realistic experimental conditions. At long timescales, this scattering leads to the decoupling of the quantum emitter from the driving light, switching off the noise source. In this picture, the driven quantum emitter behaves both as a measurement channel and as a source of dissipation for the mechanics. Our general method allows describing an

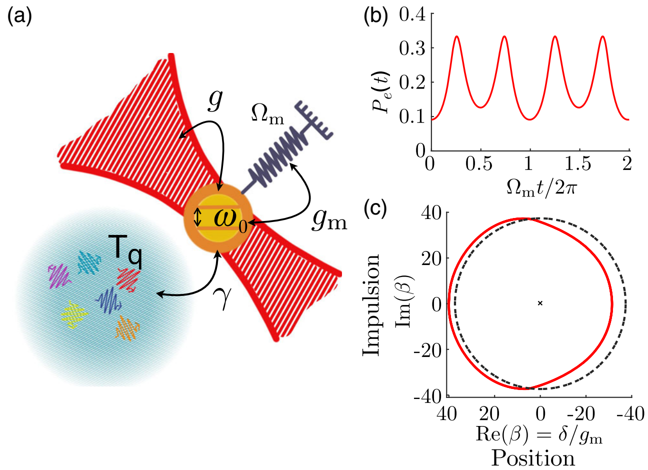

FIG. 1. (a) Hybrid system under study. (b) and (c) Semiclassical evolution of the hybrid system [cf. Eqs. (4) and (5)]. (b) Modulation of the TLS population $P_e(t) = \langle e|\rho_{\rm q}(t)|e\rangle$. (c) Shift of the mean position of the MO in the phase plane (mechanical complex amplitude $\beta(t) = \mathrm{Tr}\{b\rho_{\rm m}^0(t)\}$). Simulation parameters: $\Omega_{\rm m}/\gamma = 5\times10^{-3}$, $g_{\rm m}/\gamma = 0.1$, $g/\gamma = 1$, $T_{\rm q} = 0$. The initial MO state is a coherent state of amplitude $\beta(0) = -20$.

unusual situation of quantum optics involving an effective finite size reservoir (the quantum emitter), whose dynamics is sensitive to the evolution of the quantum system (the mechanical resonator).

System.—The hybrid mechanical system under study is depicted in Fig. 1(a): a two-level system (TLS) of frequency ω_0, of ground and excited states $|g\rangle$ and $|e\rangle$, respectively, is parametrically coupled to a mechanical oscillator (MO) of frequency $\Omega_{\rm m} \ll \omega_0$ and driven quasiresonantly by a classical monochromatic light source of frequency $\omega_L = \omega_0 - \delta_0$, where δ_0 stands for the drive-TLS detuning. The Hamiltonian of the driven TLS in the rotating wave approximation is $H_{\rm q}(t) = \hbar\omega_0\Pi_e + \hbar g(e^{i\omega_L t}\sigma_- + e^{-i\omega_L t}\sigma_+)$ with $\Pi_e = |e\rangle\langle e|$ the projector on the TLS excited state, g the classical Rabi frequency, and $\sigma_\pm$ the rising or lowering TLS operators. The MO dynamics is governed by the Hamiltonian $H_{\rm m} = \hbar\Omega_{\rm m}(b^\dagger b + \frac{1}{2})$ and the TLS-MO parametric coupling is $H_{\rm c} = \hbar g_{\rm m}\Pi_e(b^\dagger + b)$ where $\Omega_{\rm m}$ and b are, respectively, the mechanical frequency and the annihilation operator in the mechanical mode and $g_{\rm m}$ is the TLS-mechanical coupling strength. The total Hamiltonian of the hybrid system reads $H(t) = H_{\rm q}(t) + H_{\rm m} + H_{\rm c}$. In what follows, we focus on the ultrastrong coupling regime defined by $g_{\rm m} \geq \Omega_{\rm m}$. In addition, the TLS is coupled to an electromagnetic heat bath at thermal equilibrium of temperature $T_{\rm q}$ and correlation time $\tau_{\rm q}$, with γ being the spontaneous emission rate of the bare TLS (reached for $g_{\rm m} = 0$). We consider here the adiabatic limit $\gamma \gg g_{\rm m}$, which is fully compatible with the ultrastrong coupling condition as long as $\Omega_{\rm m} \ll \gamma$.

Semiclassical description.—We assume that, at time t_0, the hybrid system is in the factorized state characterized by the density matrix $\rho(t_0) = \rho_{\rm q}(t_0) \otimes |\phi_{\rm m}(t_0)\rangle\langle\phi_{\rm m}(t_0)|$ where $\rho_{\rm q}(t)$ is the reduced density matrix of the TLS and $|\phi_{\rm m}(t)\rangle$ the (pure) state of the MO. We first focus on the dynamics of the hybrid system over a few emission and absorption events after time t_0.

We choose a coarse graining time $\Delta t_{\rm q}$, verifying $\gamma^{-1} \gg \Delta t_{\rm q} \gg \tau_{\rm q}$ such that the bath d.o.f. can be traced out [26]. As $\Delta t_{\rm q} \ll \Omega_{\rm m}^{-1}$, $g_{\rm m}^{-1}$, the MO remains in the pure state $|\phi_{\rm m}(t)\rangle$ and acts as a classical external operator on the TLS, shifting its frequency by $\delta_{\rm m}(t) = g_{\rm m}\langle\phi_{\rm m}(t)|b + b^\dagger|\phi_{\rm m}(t)\rangle$ [16,25,27] (see Supplemental Material [28] Sec. I). We consider here a mechanical state $|\phi_{\rm m}(t)\rangle$ with a well-defined position; i.e., the quantum variance $V_X = \langle\phi_{\rm m}(t)|X^2|\phi_{\rm m}(t)\rangle - \langle\phi_{\rm m}(t)|X|\phi_{\rm m}(t)\rangle^2$ of the MO position operator $X = x_0(b + b^\dagger)$, x_0 being the zero point motion, fulfills:

$$V_X \ll \frac{\gamma}{g_{\rm m}} x_0^2. \tag{1}$$

This condition ensures that the shift δ_m of the TLS frequency takes a well-defined value. The master equation for the total hybrid system reads:

$$\dot{\rho}(t) = -\frac{i}{\hbar}[H(t), \rho(t)] + (\mathcal{L}_{\rm q}^{\delta(t)} \otimes \mathbb{1}_{\rm m})[\rho(t)], \tag{2}$$

$\delta(t) = \delta_0 + \delta_{\rm m}(t)$ is the total detuning between the drive and the TLS, $\mathbb{1}_{\rm m}$ the identity superoperator in the MO Hilbert space. We have introduced the Lindbladian due to the coupling to the heat bath:

$$\mathcal{L}_{\rm q}^{\delta} = \gamma(n_{\rm q}^{\delta} + 1)D[\sigma_-] + \gamma n_{\rm q}^{\delta} D[\sigma_+], \tag{3}$$

where $D[\hat{A}]\rho = \hat{A}\rho\hat{A}^\dagger - \frac{1}{2}\hat{A}^\dagger\hat{A}\rho - \frac{1}{2}\rho\hat{A}^\dagger\hat{A}$ for any operator $\hat{A}$ and $n_{\rm q}^{\delta} = [e^{\hbar(\omega_L+\delta)/k_B T_{\rm q}} - 1]^{-1}$ is the mean number of thermal photons. Note that we have neglected the modifications of Lindbladian [Eq. (3)] due to the drive, which are solely noticeable for very strong driving $\sqrt{g^2 + \delta^2} \gtrsim \omega_0$, very high temperature $k_B T_{\rm q} \sim \hbar\omega_0$, or structured vacuum [36] (see Ref. [28] Sec. II). The reduced master equation for the TLS is straightforwardly derived:

$$\dot{\rho}_{\rm q} = -\frac{i}{\hbar}[H_{\rm q}(t) + V_{\rm q}(t), \rho_{\rm q}(t)] + \mathcal{L}_{\rm q}^{\delta(t)}\rho_{\rm q}(t), \tag{4}$$

where we have introduced the effective Hamiltonian $V_{\rm q}(t) = \hbar\delta_{\rm m}(t)\Pi_e$.

Equation (2) is our starting point to investigate the dynamics of the MO over many fluorescence cycles of typical duration γ^{-1}. The essence of our approach can be grasped by noticing that the MO-TLS coupling $g_{\rm m}$ is much lower than the TLS spectral width γ; hence the TLS behaves like an effective Markovian reservoir of typical correlation time γ^{-1}, such that a master equation for the reduced mechanical density matrix $\rho_{\rm m}(t)$ can be derived. We thus choose a coarse graining time $\Delta t_{\rm m}$ verifying that

$\gamma^{-1} \ll \Delta t_{\mathrm{m}} \ll \gamma g_{\mathrm{m}}^{-2}$, Ω_{m}^{-1}. On this timescale, quantum correlations between the TLS and the MO build up and vanish, such that the density matrix of the hybrid system is always factorized $\rho(t) = \rho_{\mathrm{m}}(t) \otimes \rho_{\mathrm{q}}(t)$, even in the ultrastrong coupling case. At first order in g_{m}, the mechanical evolution is governed by the Von Neumann equation:

$$\dot{\rho}_{\mathrm{m}}^{0} = -\frac{i}{\hbar}[H_{\mathrm{m}} + V_{\mathrm{m}}(t), \rho_{\mathrm{m}}^{0}(t)]. \tag{5}$$

$V_{\mathrm{m}}(t) = \hbar g_{\mathrm{m}} P_e(t)(b + b^{\dagger})$ describes the action of the TLS on the MO and $P_e(t) = \mathrm{Tr}[\rho_{\mathrm{q}}(t)\Pi_e]$ is the mean excitation of the TLS.

Equations (4) and (5) are the semiclassical equations of the hybrid optomechanical system. The corresponding evolution is pictured in Figs. 1(b) and 1(c) in the case where the MO is initially in a coherent state. On the one hand, the TLS exerts a force on the MO, which translates into a shift of the mechanical rest position. In particular, in the ultrastrong coupling regime, a single excitation in the quantum emitter creates a measurable displacement (i.e., larger than x_0 the zero point fluctuations of the MO). This hybrid force is a fully analogous to the radiation pressure force in cavity mediated optomechanical setups.

Reciprocally, the MO modulates the frequency of the TLS by $\delta_{\mathrm{m}}^{0}(t) = g_{\mathrm{m}}\mathrm{Tr}[\rho_{m}^{0}(t)(b + b^{\dagger})]$, varying the value of the total detuning $\delta(t) = \delta_{\mathrm{m}}^{0}(t) + \delta_0$ between the TLS and the driving laser. In the adiabatic regime considered here, this creates a modulation of the TLS population, which follows the steady-state solution of optical Bloch equations $P_e^{\infty}[\delta(t)] = \{2 + [2\delta(t)/g]^2 + (\gamma/g)^2\}^{-1}$. Therefore, measures of the TLS population, e.g., by recording the intensity of the radiated light can be used to measure the mechanical position. This measurement process is associated with a noise of quantum origin that we shall now characterize.

Backaction noise induced by the TLS.—We now focus on the evolution of the hybrid system on timescales larger than $\gamma g_{\mathrm{m}}^{-2}$, for which the semiclassical description presented above is not valid anymore as the noise induced by the TLS on the MO (i.e., terms of second order in g_{m}) starts to play a noticeable role. Our strategy is inspired by the method of Zwanzig [37], which was initially developed to describe standard reservoirs involving a large number of d.o.f. We have adapted this method to the present situation where the spectrally broad TLS plays the role of a "finite-size reservoir," sensitive to the evolution of the MO. By coarse graining the evolution of the hybrid system over a timescale $\Delta t_{\mathrm{m}} \gg \gamma^{-1}$, we derived the reduced mechanical equation of motion (see Ref. [28] Sec. III):

$$\dot{\rho}_{\mathrm{m}}(t) = -\frac{i}{\hbar}[H_{\mathrm{m}} + V_{\mathrm{m}}(t), \rho_{\mathrm{m}}(t)] + \mathcal{L}_{\mathrm{h}}[\rho_{\mathrm{m}}(t)], \tag{6}$$

where

$$\mathcal{L}_{\mathrm{h}}[\rho_m] = -\frac{\Gamma_{\mathrm{h}}(t)}{2x_0^2}[X, [X, \rho_{\mathrm{m}}]]. \tag{7}$$

The rate $\Gamma_{\mathrm{h}}(t)$ is related to the fluctuation spectrum of the TLS population at zero frequency, and is defined as:

$$\Gamma_{\mathrm{h}}(t) = 2g_{\mathrm{m}}^2 \mathrm{Re}\int_0^{\infty} d\tau \langle \delta\tilde{\Pi}_e(t)\delta\tilde{\Pi}_e(t-\tau)\rangle. \tag{8}$$

We have introduced $\delta\Pi_e(t) = \Pi_e - P_e(t)$, the tilde standing for the interaction representation with respect to the semiclassical evolution (see Supplemental Material [28] Sec. III. 1). Such a Linbladian has no effect on the average position X or momentum $P = (\hbar/2x_0)i(b^{\dagger} - b)$ of the MO, but increases the momentum variance $V_P(t) = \mathrm{Tr}_{\mathrm{m}}[\rho_{\mathrm{m}}(t)P^2] - \mathrm{Tr}_{\mathrm{m}}[\rho_{\mathrm{m}}(t)P]^2$, leading to a nonsymmetric scattering of the mechanical quadratures. Equation (7) describes the backaction noise of the continuous weak measurement on the mechanical position encoded in the fluorescence light [38]. In the adiabatic regime, considered here that $\Gamma_{\mathrm{h}}(t)$ verifies (see Ref. [28] Sec. IV):

$$\Gamma_{\mathrm{h}}(t) = \frac{g_{\mathrm{m}}^2}{\gamma}\frac{2g^2[4\delta(t)^2 + \gamma^2](g^2 + 2\gamma^2)}{[4\delta(t)^2 + 2g^2 + \gamma^2]^3}. \tag{9}$$

Note that $\Gamma_{\mathrm{h}} > 0$, such that the momentum variance can only increase: therefore no cooling can be realized by driving the TLS. The rate Γ_{h} characterizing the backaction noise is plotted in Fig. 2(a). Taking into account the intrinsic coupling of the MO to its own thermal bath with temperature T_{m}, the full master equation governing the mechanical evolution reads:

$$\begin{aligned}\dot{\rho}_{\mathrm{m}} = &-\frac{i}{\hbar}[H_{\mathrm{m}} + V_{\mathrm{m}}(t), \rho_{\mathrm{m}}] + \mathcal{L}_{\mathrm{h}}[\rho_m] \\ &+ \Gamma_{\mathrm{m}}(n_m + 1)D[b]\rho_{\mathrm{m}} + \Gamma_{\mathrm{m}} n_m D[b^{\dagger}]\rho_{\mathrm{m}}. \end{aligned}\tag{10}$$

We have introduced Γ_{m} the MO damping rate in the bath, and $n_{\mathrm{m}} = (e^{\hbar\Omega_{\mathrm{m}}/k_B T_{\mathrm{m}}} - 1)^{-1}$ the mean number of thermal phonons. The weak coupling regime of small g_{m} corresponds to $\Gamma_{\mathrm{m}} \gg \Gamma_{\mathrm{h}}$: the TLS slightly modifies the effective bath parameters, but the nature of the relaxation remains thermal [23] (see Ref. [28] Sec. V). More interestingly, Eq. (10) allows describing a less explored situation characterized by $\Gamma_{\mathrm{h}} \sim g_{\mathrm{m}}^2/\gamma \gg (n_{\mathrm{m}} + 1)\Gamma_{\mathrm{m}}$ where the TLS-induced mechanical fluctuations largely overcome the Brownian motion. Such scattering stems from the emission and absorption of photons by the driven dissipative TLS [26]. An experimental characterization of this hybrid noise is within reach [11,12,19].

Eventually, this quantum noise sets a fundamental limit to the sensitivity of measurements based on the optical detection of the MO position [40,41], just like radiation pressure in cavity optomechanics [39]. To illustrate this point, we focus on the readout of a static MO deflection δ_1

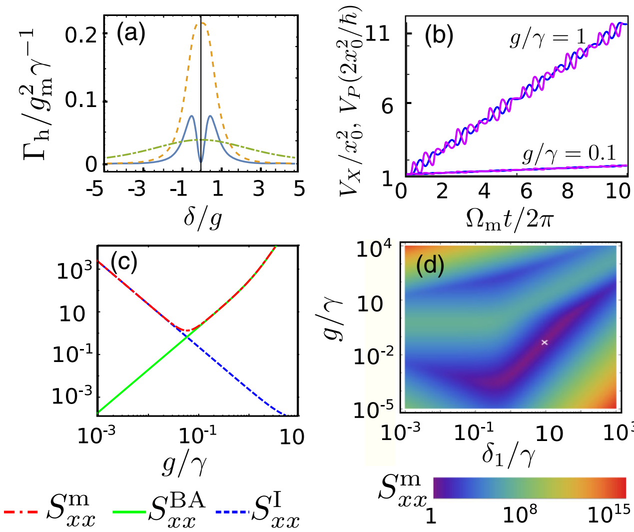

FIG. 2. MO position measurement through TLS fluorescence. (a) Hybrid heating rate $\Gamma_h(t)$ induced by the TLS on the MO [Eq. (9)] as a function of the total detuning $\delta(t)$ between the TLS and the driving field for three different Rabi frequencies $g = 10\gamma$ (blue solid), $g = \gamma$ (orange dashed), and $g = 0.1\gamma$ (green dash dotted). (b) Evolution of the variances of the MO's position V_X and momentum V_P along 10 mechanical periods for $g/\gamma = 1$ (top) and $g/\gamma = 0.1$ (bottom). The other parameters for (a) and (b) are the same as Fig. 1. (c) Total measurement noise spectrum S_{xx}^{m} and the two contributions: the backaction and imprecision noise spectra S_{xx}^{BA} and S_{xx}^{I} at the mechanical frequency, for the optimum value of the detuning $\delta_1 \simeq 9.3\gamma$ allowing us to reach the global minimum of the total noise, as a function of the Rabi frequency. (d) Sensitivity of the MO position measurement as a function of the detuning δ_1 and the Rabi frequency g in units of γ. The spectra in (c) and (d) are normalized to the minimum value of S_{xx}^{m}, i.e., 4.01×10^{-29} m^2/Hz, reached for $\delta_1 \simeq 8.8\gamma$ and $g \simeq 0.06\gamma$ (white cross), which is just above the standard quantum limit for the measurement of the position of an oscillator $2x_0^2/\Gamma_m = 4\times 10^{-29}$ m^2/Hz [39]. Parameters: $\Omega_m = 5$ MHz, $g_m = 0.1$ GHz, $\gamma = 1$ GHz, $\Gamma_m = \Omega_m/10^6$, $T_q = 0$, and $x_0 = 10^{-2}$ pm.

using the intensity of the TLS fluorescence. The sensitivity of the measurement scheme is quantified by the noise spectrum S_{xx}^{m}, which is the sum of two contributions: $S_{xx}^{m} = S_{xx}^{I} + S_{xx}^{BA}$ (see Ref. [28] Sec. VI for analytical expressions). The imprecision noise spectrum S_{xx}^{I} comes from intensity fluctuations of the fluorescence and dominates at low driving [see Fig. 2(c)]. At strong driving the quantum noise induced by the TLS S_{xx}^{BA} characterized above dominates. The total noise S_{xx}^{m} is minimized when both quantum and imprecision noises are equal. S_{xx}^{m} is plotted in Fig. 2(d) as a function of the detuning δ_1 and the Rabi frequency g. Its minimum value is reached for $(\delta_1/\gamma, g/\gamma) \simeq (8.8, 0.06)$ and fulfills $S_{xx}^{m} = 4.01 \times 10^{-29}$ m^2/Hz. This value corresponds to the standard quantum limit for the position measurement on an oscillator $2x_0^2/\Gamma_m = 4 \times 10^{-29}$ m^2/Hz [39]: thus probing this bound within hybrid optomechanical devices is within experimental reach.

Quantum trajectory of the mechanical oscillator.—At first sight, it may seem that the momentum variance of the MO and thus the mechanical energy continuously increases under the action the noise, apparently violating the second law of thermodynamics. Actually, over long timescales $t \gg \gamma g_m^{-2}$, the MO position fluctuations randomize the TLS frequency, reducing the coupling to the driving light and eventually switching off the noise source. In this limit the master Eq. (10) is no longer valid, as Eq. (1) breaks down because of the increase of V_X. To describe the decoupling, we adopt a quantum trajectory approach taking into account the partial information about the MO position encoded in the radiated light. In this description, at each time t the MO is in a pure state $|\phi_m^{st}(t)\rangle$ verifying a stochastic Schrödinger equation obtained by unravelling Eq. (10) [38]:

$$
\begin{aligned}
d|\phi_m^{st}(t)\rangle = &-\frac{idt}{\hbar}[H_m + V_m(t)]|\phi_m^{st}(t)\rangle \\
&-\frac{dt}{2}\Gamma_h(t)[X - \langle X(t)\rangle_{st}^2]|\phi_m^{st}(t)\rangle \\
&+\sqrt{\Gamma_h(t)}dW(t)[X - \langle X(t)\rangle_{st}]|\phi_m^{st}(t)\rangle. \qquad (11)
\end{aligned}
$$

$dW(t)$ is a real normalized Wiener increment of zero average and verifying $dW^2(t) = dt$. We have used Itō's convention for stochastic calculus and denoted $\langle\cdot\rangle_{st}$ the expectation value in state $|\phi_m^{st}(t)\rangle$. $\Gamma_h(t)$ is computed by supposing that the position variance verifies Eq. (1) at any time, such that the effective detuning $\delta(t)$ is defined. A trajectory $|\phi_m^{st}(t)\rangle$ represents the dynamics of the MO conditioned to a given measurement record $\{X_{st}(t)\}_{t_i\le t\le t_f}$, of the position measurement performed via the TLS. The measurement outcome at time t is $X_{st}(t) = \langle X(t)\rangle + dW(t)/2\sqrt{\Gamma_h}dt$ and is stochastic due to the intrinsic randomness of quantum measurement [38]. We have solved numerically Eq. (11) for a single realization of the process. At each time t, the scattering rate Γ_h is computed from the instantaneous mechanical state $|\phi_m^{st}(t)\rangle$, generating the trajectory. We find that, along each trajectory, the MO follows a random walk in phase space, leading first to the heating of the mechanics [see Figs. 3(a) and 3(b)]. Meanwhile, the detuning of the TLS is also scattered, inducing a spectral wandering of the TLS emission line. At longer timescales, the mechanical amplitude becomes large compared with g/g_m and the MO spends most of the time away from its rest position. The TLS is brought off resonance with the drive, leading to a vanishing heating rate Γ_h and a saturation of the mechanical energy $E_m(t)$.

We have plotted the Wigner function $\mathcal{W}(x,p) = (1/\pi\hbar)\int_{-\infty}^{\infty} dy \langle\phi_m^{st}(t)|x+y\rangle\langle x-y|\phi_m^{st}(t)\rangle e^{2ipy/\hbar}$ of the MO at the initial time and after 3×10^3 mechanical oscillations [see Figs. 3(c) and 3(d)]. One clearly sees the deformation of the shape of the MO state in phase space due to the quadrature-dependent scattering. We also note that the

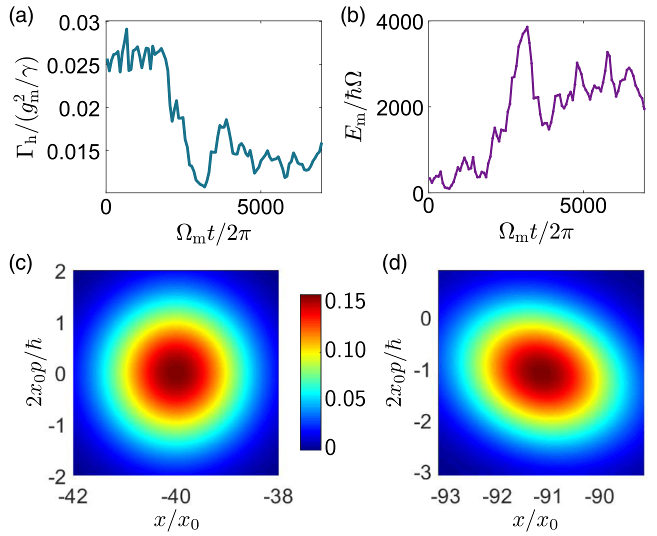

FIG. 3. (a) and (b) Evolution of the hybrid heating rate $\Gamma_h(t)$ (c) and of the mechanical energy $E_m(t) = \hbar\Omega_m\langle b^\dagger b\rangle_{st}$ (d) along 3×10^3 mechanical oscillations. Each point is averaged over 60 subsequent mechanical oscillations to remove the fast oscillations of the mechanical quantities (see Fig. 2). (c) and (d) Wigner function $\mathcal{W}(x,p)$ of the MO at the initial time when the drive of the TLS is switched on (c) and after 3×10^3 mechanical oscillations (d). The simulation parameters are the same as Fig. 1.

position variance remains of the order of unity along any trajectory (even on large timescales $t \gg \Omega_m^{-1}$): indeed, state $|\phi_m^{st}\rangle$ is continuously updated with the information extracted by the continuous position measurement performed by the TLS, which reduces the quantum uncertainty on X. Therefore, the effective detuning $\delta(t)$ between the drive and the TLS is defined at any time, validating the trajectory based approach.

Conclusion.—Our model shows that a hybrid optomechanical system in the ultrastrong coupling regime can be described by semiclassical equations at short times, provided that the TLS is strongly dissipative. Beyond the semiclassical regime, the TLS induced mechanical fluctuations either generate an effective thermal bath (small g_m), or the nonsymmetrical scattering of the MO quadratures (large g_m). This quantum noise is the equivalent of the radiation pressure noise in cavity optomechanics, and appears as a fundamental limit of hybrid optomechanical detection sensitivity. Noticeable deviations from the semiclassical description can be observed over longer timescales and our study allows us to simulate quantum trajectories of the system using a stochastic Schrödinger equation, which is a precious tool to describe the TLS and MO fluctuating observables. These quantities are especially relevant to investigate the quantum limit of sensing [42] in the context of hybrid optomechanics, and to perform further thermodynamic studies, e.g., probe fluctuation theorems [43], or design nanoheat engines based on optomechanical devices [44–46]. Finally, the method presented here is a quite general one allowing us to treat the case of any quantum system in strong interaction with a finite size reservoir [47–50].

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