Uniaxial Dynamical Decoupling for an Open Quantum System

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Dynamical decoupling (DD) is an active and effective method for suppressing decoherence of a quantum system from its environment. In contrast to the nominal biaxial DD, this work presents a uniaxial decoupling protocol that requires a significantly reduced number of pulses and a much lower bias field satisfying the "magic" condition. We show this uniaxial DD protocol works effectively in a number of model systems of practical interest, e.g., a spinor atomic Bose-Einstein condensate in stray magnetic fields (classical noise), or an electron spin coupled to nuclear spins (quantum noise) in a semiconductor quantum dot. It requires only half the number of control pulses and a 10–100 times lower bias field for decoupling as normally employed in the above mentioned illustrative examples, and the overall efficacy is robust against rotation errors of the control pulses. The uniaxial DD protocol we propose shines new light on coherent controls in quantum computing and quantum information processing, quantum metrology, and low field nuclear magnetic resonance.

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Introduction.—Decoherence, due to coupling of a system to its surrounding environment, is a key obstacle in practical applications of quantum technologies [1–3]. Reliable quantum operations cannot proceed effectively or coherently without the decoherence of a quantum state under control [4]. One may naively hope for the existence of a system perfectly isolated from its environment. However, this imposes a heavy resource requirement and extreme conditions, such as ultralow temperature, ultrahigh vacuum, ultraweak or ultrastrong magnetic fields [5–7], etc., some of which for all practical reasons cannot be achieved. Alternatively, one can search for strategies capable of slowing down or suppressing decoherence.

Dynamical decoupling (DD) is one such frequently employed decoherence-suppression method. It is capable of reducing effectively the coupling structure as well as the strength between a quantum system and its environment, thereby decoupling (or isolating) the system from its environment [8–11]. DD has been widely employed for more than half a century in nuclear magnetic resonance (NMR) to isolate subsystems of nuclear spins from nearby spins and more recently in demonstrating robust quantum memory and universal quantum gate operations [12–18]. Experiments in electron-nuclear spins and nitrogen vacancy centers have further established it as a powerful technique, e.g., capable of preserving arbitrary quantum states over extended times [18–23].

Most DD protocols employ biaxial resonant rotations, which in the presence of a large bias magnetic field along the z axis (the quantization axis), can be constructed in terms of two rotations along two orthogonal (e.g., x or y) axes, respectively. These resonant rotations can be either discrete or continuous, respectively, associated with bang-bang DD or continuous DD [9,12,23–35]. For bang-bang DD, many sophisticated protocols have been developed, e.g., periodic DD (PDD) or concatenated DD, which requires hard (delta function) pulses with a total number scaling as $4L$ for L cycles and progression to even 4^L for L-level concatenation [26,36]. Several advanced schemes were utilized to optimize DD protocols, respectively, given rise to the Uhrig DD [31], concatenated Uhrig DD [37], and quadratic DD [38], with variable pulse delays to suppress high order noise correlations [39]. A question of great importance is: can one reduce the number of pulses, e.g., to $2L$ for L-cycle DD while maintaining the same level of noise suppression as in $4L$ PDD?

The uniaxial DD (Uni-DD) protocol we present in this Letter achieves such a challenging goal, making it a more efficient replacement for the usual biaxial PDD protocol. We show that the number of pulses reduces to the order of $2L$ for L-cycle DD while the performance remains similar to or better than the usual $4L$ PDD protocol. In addition, the z-axis bias magnetic field is reduced to about 100 times the average noise fields in the examples we studied, which is much less than the $1000 \sim 10\,000$ times typically required in NMR experiments [10,40,41]. Numerical simulations reveal the superior performance of our Uni-DD in a spinor Bose-Einstein condensate (BEC) decohered by stray magnetic fields and in a semiconductor quantum dot (QD) electron spin qubit decohered by nuclear spins. Our result can be applied to research in low field DD in quantum information, NMR, magnetic resonance imaging, and quantum sensing beyond standard quantum limit [42–44].

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Uni-DD protocol.—We first briefly review the usual PDD protocol which forms the basis of the more advanced DD protocols. A qubit is decohered in general by stochastic interactions along three orthogonal directions, the longitudinal along the z axis causes dephasing while the transversal along the x or y axis induces bit flip. Biaxial DD protocol suppresses the transversal noise by π pulses along the z axis (denoted as Z pulses) and the longitudinal noise by π pulses along either the x or y axis (denoted as X or Y pulses, respectively) [12,36].

The Uni-DD protocol we present also suppresses noise in all three directions. Similar to the biaxial DD, Y pulses are employed to suppress dephasing from z-axis noise. Unlike the biaxial DD, a relatively strong bias magnetic field along the z axis is introduced to suppress transversal noises along the x or y axis (see Fig. 1). The main inspiration to our idea comes from the observation that a strong longitudinal magnetic field suppresses transverse fluctuating fields [45]. We further require the pulse delay τ and the effective Larmor precession frequency ω to satisfy $\omega\tau = n2\pi$, with n a positive integer. At this "magic" condition, the qubit processes an integer number of rounds in the bias magnetic field between Y pulses.

The Uni-DD protocol can be denoted in short hand as $[YU_\tau YU_\tau]^L$ for the L cycle with U_τ the precession operator, and the pulse delay τ satisfying the magic condition within the shortest decoherence time possible. The latter is often given by the inhomogeneous broadening induced lifetime T_2^* [46]. More details on the above results can be found in the Supplemental Material [47], where we show the number of pulses for the Uni-DD is $2L$, or about half of the $4L$ pulses required by the PDD. In the following, we consider two concrete examples illustrating that our Uni-DD protocol is capable of suppressing classical or quantum noise.

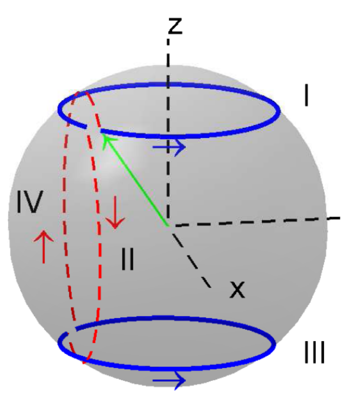

FIG. 1. The schematic illustration for one cycle of the Uni-DD $[YU_\tau YU_\tau]$ on the Bloch sphere. (I) A spin or qubit precesses around the total field composed of a large (longitudinal) bias and a small stochastic field for a duration τ (blue solid lines), (II) rotated by a hard Y pulse (red dashed lines), (III) precesses around the total field for another τ, (IV) and rotated by a second Y pulse. The green solid line denotes the initial spin or qubit state.

Suppressing classical stray magnetic fields in a spinor BEC.—As a model system decohered by classical noise, we consider a ferromagnetically interacting spin-1 atomic BEC under stray magnetic fields. Its full quantum state evolution is simulated including the Uni-DD protocol [48]. Such a model allows the condensate spin degrees of freedom to be treated in terms of a large collective spin $\mathbf{J}$ (with $J = 10^3$), which decohers by the stochastic rotations due to weak stray magnetic fields [49–51]. In the absence of the Uni-DD pulses, the model system is described by the Hamiltonian $H = c_2'\mathbf{J}^2 + \omega J_z + \gamma\mathbf{b}\cdot\mathbf{J}$, with c_2' the effective atomic spin exchange interaction strength, $\omega = \gamma B$ the Larmor frequency in the longitudinal bias magnetic field B (γ is the gyromagnetic ratio), and $\mathbf{b}$ the stray magnetic field stochastic in its direction and amplitude with a cutoff b_c ($b_{x,y,z} \in [-b_c, b_c]$). For simplicity, the chosen stray field distribution function mimics a white noise, with the probability density for each realization uniformly distributed in direction and amplitude over a limited range. We set $\hbar = 1$, $\gamma = 1$, and $b_c = 1$ for numerical simulations. The energy and the time units are b_c and b_c^{-1}, respectively. For the Uni-DD, we choose a pulse delay of $\tau = 0.05$. The Uni-DD Y pulses are assumed to be hard π pulses, i.e., a temporal delta function with zero pulse width [52,53].

The simulations are carried out for four initial condensate spin directions, respectively along the x, y, z, and $-z$ axes, and the initial quantum state is either a coherent spin state (CSS) or a squeezed spin state (SSS). The worst performing case among the four is reported as a benchmark. We monitor the normalized spin average j/J with $j = \sqrt{\langle J_x\rangle^2 + \langle J_y\rangle^2 + \langle J_z\rangle^2}$ for an initial CSS and the squeezing parameter $\xi^2 = 2\ \min\{\Delta J_x^2, \Delta J_y^2, \Delta J_z^2\}/J$ with $\Delta J_\alpha^2 = \langle J_\alpha^2\rangle - \langle J_\alpha\rangle^2$ ($\alpha = x, y, z$) for an initial SSS during the Uni-DD [54–56]. In the absence of noises, the normalized spin average and the squeezing parameter should stay at unit and 0.000 91, respectively. Therefore, their respective rate of deterioration in the presence of noise manifests how fast the initial quantum system is decohered. A slower rate of deterioration due to the Uni-DD pulses than under pure free evolution (FE) indicates noise suppression.

The coherence time of the condensate spin in an initial CSS is clearly seen prolonged by 2 orders of magnitude as shown in Figs. 2(a) and 2(b), if the magic condition $\omega\tau = 2\pi$ is satisfied. For the more "quantum" initial SSS, which is highly entangled and strongly correlated and thus expected to be more fragile or sensitive to noise, the coherence time is also seen prolonged by 2 orders of magnitude under the same magic condition [Figs. 2(c) and 2(d)] [57].

Enhanced understanding is gained by calculating analytically the evolution operator for a unit cycle of Uni-DD: $U_{2\tau} = [YU_\tau YU_\tau]$. Following the Fer expansion under the magic condition $\omega\tau = 2\pi$, we find $U_\tau \approx \exp(-i\tau c_2' J^2)\exp(-i\tau H_{F,1})\exp(-i\tau H_{F,0})$ with $H_{F,0} = \gamma b_z J_z$ and $H_{F,1} = \gamma^2 (b_z/\omega)(b_x J_x + b_y J_y) + J_z(b_x^2 + b_y^2)/(2\omega)$ [58,59]. By further employing the Magnus expansion, we

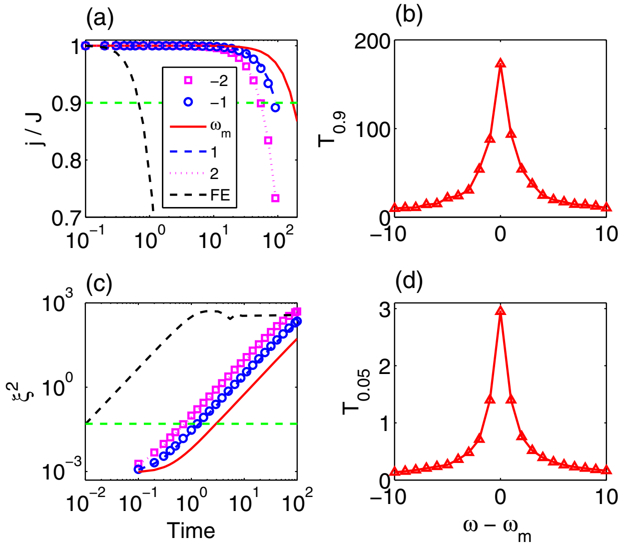

FIG. 2. Suppressing classical noise with Uni-DD in a spinor atomic BEC for $\tau = 0.05$. (a) The evolution of the spin average under Uni-DD cycles for an initial CSS in the worst case at $\omega - \omega_m = -2$ (magenta squares), -1 (blue circles), 0 (red solid line), 1 (blue dashed line), and 2 (magenta dotted line). The magic condition requires $\omega_m = 2\pi/\tau \approx 126$. The FE results are presented for easier comparisons. The horizontal green dashed line denotes how the characteristic time $T_{0.9}$, where $j/J = 0.9$, is extracted. (b) Dependence of the enhanced coherence time on the Larmor frequency (or the bias field). A peak occurs at the magic condition $\omega_m \tau = 2\pi$. (c) Same as (a) except for the squeezing parameter ξ^2 with an initial SSS. The horizontal green dashed line is for $T_{0.05}$ where $\xi^2 = 0.05$. (d) Same as (b) except for $T_{0.05}$ with an initial SSS.

obtain $U_{2\tau} \approx \exp(-i2\tau c_2' J^2) \exp[-i2\tau\gamma^2 (b_z/\omega) b_y J_y]$ to the leading nonzero order [9,10,59]. The derivation details can be found in the Supplemental Material [47], which includes Refs. [60–62]. Compared to the corresponding FE operator $U_{\rm FE} = \exp(-i2\tau c_2' J^2) \exp[-i2\tau\gamma(b_x J_x + b_y J_y + b_z J_z)]$ without the bias magnetic field, the effective coupling strength between the condensate spin and the stray magnetic field is seen to be reduced by a factor of b_z/B, which can become much smaller. Thus, noise suppression of the Uni-DD protocol is rooted in the bias field's suppression of the transversal fluctuation field and augmented by the cancellation of the longitudinal fluctuation field from the Y pulses. This is quite different from the nominal biaxial DD protocol, which often relies on the smallness of the pulse delay.

Suppressing nuclear spin quantum noise in a QD.—Unlike the classical environment described by stochastic complex fields, a proper description for a quantum environment must deal with environment operators and their correlations. To illustrate the power of the Uni-DD, we choose a gate-defined GaAs semiconductor QD system which is well described by a central spin model with the electron spin decohered by the surrounding nuclear spins [19,30,63–65]. To further simplify the problem, we assume the electron spin ($\mathbf{S}$) as well as all nuclear spins ($\mathbf{I}_k$) are spin-1/2. The coupling A_k between $\mathbf{S}$ and $\mathbf{I}_k$ results from the Fermi contact hyperfine interaction [46,66]. The Hamiltonian for the model system of N nuclear spins without Uni-DD takes the form $H = \mathbf{S}\cdot\sum_{k=1}^{N} A_k \mathbf{I}_k + \sum_{i<j}^{N} \Gamma_{ij}(\mathbf{I}_i \cdot \mathbf{I}_j - 3\mathbf{I}_{iz}\cdot\mathbf{I}_{jz})$. In general, A_k is proportional to the local density of the electron at the position of the kth nucleon. In this work, it is modeled as in Ref. [67] for $N = 4\times 5$ nuclear spins, by $A_k \propto \exp[-(x-x_0)^2/w_x^2 - (y-y_0)^2/w_y^2]$, a 2D Gaussian form with effective widths $w_x/a_x = 3/2$ and $w_y/a_y = 2$ and a shifted center $x_0/a_x = 0.1$ and $y_0/a_y = 0.2$, leading to the final values of A_k ranging between 0.309 and 0.960. Γ_{ij} accounts for the magnetic dipolar interaction of nearest neighbor nuclear spins and is randomly distributed between 0 and 0.01.

It is well known for a GaAs QD system that the short time FE quantum dynamics agrees well with the classical (quasistatic bath approximation) fluctuation model, but the long time dynamics gradually deviates, implicating the important role played by quantum correlations within the coupled central spin system [46,68,69]. We expect our Uni-DD would prolong the coherent dynamics, and quantum simulations are thus carried out to fully account for the quantum corrections.

Starting from the electron spin initially pointing along x, y, z, and $-z$ axis, respectively, the system evolution is simulated quantum mechanically with the Chebyshev polynomial expansion method [70]. The initial nuclear spin state is a fully mixed state which is approximated numerically by a random pure state [2,71]. The evolution is recorded, and the fidelity, which calibrates the survival probability of the initial electron spin state, $F(t) = \mathrm{Tr}\{\rho_e(0)\mathrm{Tr}_n[\rho(t)]\}$, is calculated, where $\mathrm{Tr}_n[\rho(t)]$ is the reduced electron spin state after tracing out all nuclear spins. The worst case fidelity, $F_w = \min_{\{\rho_e(0)\}}(F)$, among the four initial states is easily identified and used as a benchmark and shown in Fig. 3. Compared to FE, the coherence time of the electron spin is prolonged by up to 2 orders of magnitude with Uni-DD, depending on whether the condition $\omega\tau = 2\pi$ is satisfied or not [Fig. 3(a)]. Compared to PDD, the Uni-DD is also found to be superior. The enhanced coherence time implicates successful decoupling of the electron spin from its surrounding nuclear spins. More interestingly, the magic condition exhibits a resonance, around which the characteristic coherence time of the Uni-DD protocol $T_{0.9}$ versus the Larmor frequency ω of the bias field ($\omega = \gamma B$) is shown in Fig. 3(b).

The decoupling by the Uni-DD protocol under the magic condition for the QD model can again be proven analytically by following the Fer expansion of U_τ with the average Hamiltonian theory based on the Magnus expansion. Although the average Hamiltonian theory does not directly apply since the convergence condition $|H|\tau \ll 1$ is violated, its application becomes possible after adopting the Fer expansion of U_τ in a rotating reference frame defined by the bias field [58,59]. In fact, Fer expansion is applicable at

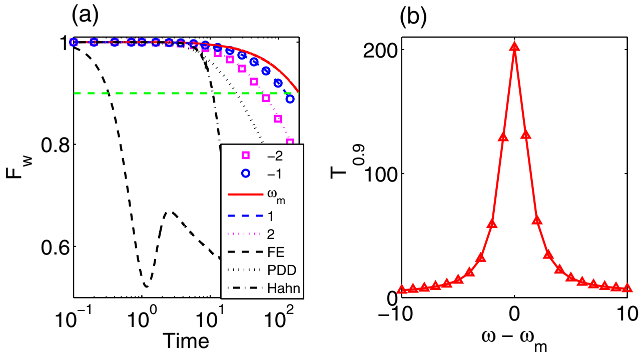

FIG. 3. Suppression of quantum noise by the Uni-DD in a GaAs QD for $\tau = 0.05$. (a) The evolution of fidelity under Uni-DD cycles in the worst case for $\omega - \omega_m = -2$ (magenta squares), -1 (blue circles), 0 (red solid line), 1 (blue dashed line), and 2 (magenta dotted line). The FE (black dashed line), Hahn echo (black dash-dotted line), and PDD (black dotted line) results are also presented for comparisons. The horizontal green dashed line denotes how the characteristic time $T_{0.9}$ is extracted. (b) Dependence of the prolonged coherence time on the Larmor frequency (or the bias field), exhibiting a peak at the magic condition $\omega_m \tau = 2\pi$.

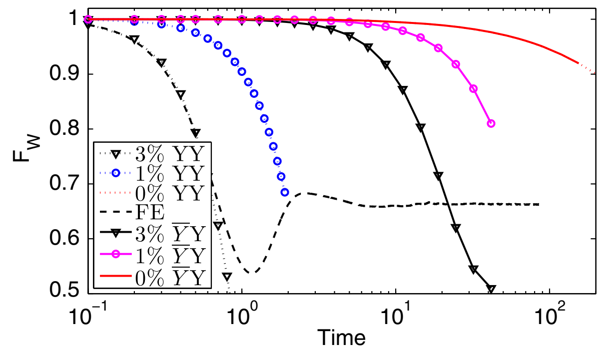

FIG. 4. Robustness of the Uni-DD against rotation angle errors for $[YU_\tau YU_\tau]$ with $\varepsilon = 3\%$ (black dotted line with triangles), 1% (blue dotted line with circles), 0% (red dotted line) and for $[\bar{Y}U_\tau YU_\tau]$ with $\varepsilon = 3\%$ (black solid line with triangles), 1% (blue solid line with circles), 0% (red solid line, coinciding with the red dotted line). The FE results are also presented for comparisons.

treating long time quantum evolution beyond the convergence radius of the widely used Magus expansion [9,10,59]. After a straightforward derivation at the magic condition $\omega\tau = 2\pi$, we find $U_\tau \approx \exp[-i\tau H_{F,1}]\exp[-i\tau H_{F,0}]$, where $H_{F,0} = S_z h_z$ and $H_{F,1} = S_x(h_z h_x + h_x h_z)/(2\omega) + S_y(h_z h_y + h_y h_z)/(2\omega) + S_z(h_x^2 + h_y^2)/(2\omega) + i(h_x h_y - h_y h_x)/(4\omega)$ with the quantum Overhauser field operator $h_{\alpha\in\{x,y,z\}} = \sum_{k=1}^{N} A_k I_{k\alpha}$. For one Uni-DD cycle, the evolution operator reduces to $U_{2\tau} = [YU_\tau YU_\tau] \approx \exp\{-i2\tau[S_y(h_z h_y + h_y h_z)/(2\omega) + i(h_x h_y - h_y h_x)/(4\omega)]\}$, as shown in detail in the Supplemental Material [47], where Refs. [72–74] are included. Similar to the classical noise example considered earlier, one sees immediately that the relatively strong bias field suppresses the relaxation effect of the transversal quantum noise and the Y pulses suppress the dephasing effect of the longitudinal quantum noise [75].

Robustness of the Uni-DD against rotation angle errors.—To estimate the robustness of the Uni-DD protocol, we consider rotation angle error ε of the Y pulses, i.e., assuming an imperfect rotation angle of the Y pulse $(1-\varepsilon)\pi$. As shown in Fig. 4, even a small $\varepsilon = 1\%$ causes the worst case fidelity F_w to drop significantly. Such a sensitive dependence on the rotation angle can be remedied by replacing one of the Y pulses with a $\bar{Y}$ pulse, which rotates along the $-y$ direction with the same imperfect angle $(1-\varepsilon)\pi$. This idea is behind the Carr-Purcell-Meiboom-Gill (CPMG) protocol which improves greatly the robustness of the Carr-Purcell protocol [11,76,77]. Remarkably, the modified Uni-DD protocol likewise shows strong robustness against the rotation angle error, as illustrated in Fig. 4. Even for $\varepsilon = 3\%$, the coherence time remains prolonged by an order of magnitude. For a smaller $\varepsilon = 1\%$, the Uni-DD protocol is seen to almost reach the same outcome as in the perfect pulse case of $\varepsilon = 0$.

Finally, we note the Uni-DD protocol we discuss differs fundamentally from the single pulse Hahn echo by requiring the magic condition. Such a condition in the Uni-DD allows the bias field to stay as low as possible while keeping the noise suppression effect comparable or even better than in the Hahn echo, as shown in Fig. 3(a) (and more in the Supplemental Material [47]). By adopting symmetrization and concatenation, more advanced DD protocols based on Uni-DD may be developed with improved performances. Our preliminary investigations into the performance comparisons of the Uni-DD with the standard symmetrized DD, the second level concatenated DD, the nonequidistant concatenated Uhrig DD, and the quadratic DD are presented in the Supplemental Material [47], with Ref. [78] included. More efforts could be devoted to explore systematically these protocols in order to find the most suitable one for a specific experiment [79].

In conclusion, we propose a Uni-DD protocol for suppressing the decoherence of an open quantum system from its environment. Compared to the nominal biaxial PDD, the Uni-DD achieves the same degree of noise suppression with half the number of control pulses. We demonstrate with numerical and analytical calculations the efficacy of the Uni-DD under the magic condition $\omega\tau = 2\pi$ in suppressing the classical stray fields in a spinor BEC and in suppressing the quantum nuclear spin noises in a GaAs QD. Our results point to alternative low-cost DD techniques which may find wide applications in quantum computing and quantum information processing, NMR and magnetic resonance imaging, as well as quantum precision measurements beyond the standard quantum limit.

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