Exact Spectral Form Factor in a Minimal Model of Many-Body Quantum Chaos

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The most general and versatile defining feature of quantum chaotic systems is that they possess an energy spectrum with correlations universally described by random matrix theory (RMT). This feature can be exhibited by systems with a well-defined classical limit as well as by systems with no classical correspondence, such as locally interacting spins or fermions. Despite great phenomenological success, a general mechanism explaining the emergence of RMT without reference to semiclassical concepts is still missing. Here we provide the example of a quantum many-body system with no semiclassical limit (no large parameter) where the emergence of RMT spectral correlations is proven exactly. Specifically, we consider a periodically driven Ising model and write the Fourier transform of spectral density's two-point function, the spectral form factor, in terms of a partition function of a two-dimensional classical Ising model featuring a space-time duality. We show that the self-dual cases provide a minimal model of many-body quantum chaos, where the spectral form factor is demonstrated to match RMT for all values of the integer time variable t in the thermodynamic limit. In particular, we rigorously prove RMT form factor for an odd t, while we formulate a precise conjecture for an even t. The results imply ergodicity for any finite amount of disorder in the longitudinal field, rigorously excluding the possibility of many-body localization. Our method provides a novel route for obtaining exact nonperturbative results in nonintegrable systems.

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The problem of finding a quantum analog of the classical concept of chaos has a long and fascinating history [1–3]. For systems with chaotic and ergodic classical limit, the quantum chaos conjecture [4–6] states that the statistical properties of energy spectrum are universal and given in terms of random matrix theory (RMT) [7], where all matrix elements of the Hamiltonian are considered to be independent Gaussian random variables. An analogous result for chaotic maps, or periodically driven (Floquet) systems, relates the statistics of quasienergy levels to circular ensembles of unitary random matrices [2,7]. This conjecture has been by now put on firm theoretical footing by clearly identifying contributions from periodic orbit theory and RMT for the simplest nontrivial measure of spectral correlations: the spectral form factor (SFF) [8–13]. This, however, has been rigorously proven only for a specific type of single-particle models: the incommensurate quantum graphs [14,15].

The situation is even less clear for nonintegrable many-body systems with simple, say clean and local, interactions, where evidence of RMT spectral correlations is abundant [16–19] but theoretical explanations are scarce. While for many-body systems of bosons with a large number of quanta per mode, or other models with small effective Planck's constant, a semiclassical reasoning may still be used [20–23], the intuition is completely lost and no methods have been known when it comes to fermionic or spin-$1/2$ systems. Very recently, a few steps of progress have been made. First, an analytic method analogous to the periodic orbit theory for spin-$1/2$ systems has been proposed in Ref. [24]. This method is able to establish RMT spectral fluctuations for long-ranged but non-mean-field nonintegrable spin chains; however, it fails in the important extreme case of local interactions. Second, it has been shown in Refs. [25,26] that Floquet local quantum circuits with Haar-random unitary gates have exact RMT SFF in the limit of large local Hilbert space dimension. Remarkably, in both cases, the Thouless time, where universal RMT behavior sets in, scales as the logarithm of the system size [24,26], which is consistent with the detailed numerical computations in Ref. [27].

In this Letter we make a crucial step forward by providing the example of a locally interacting many-body system with finite local Hilbert space for which the SFF exactly approaches the RMT prediction in the thermodynamic limit (TL) at all times. Thus, we identify the first nonperturbative exactly solvable model displaying scale-free many-body quantum chaos.

More specifically, we consider the Floquet Ising spin-$1/2$ chain with transverse and longitudinal fields, described by the following Hamiltonian [28,29]

$$H_{\rm KI}[\boldsymbol{h}; t] = H_{\rm I}[\boldsymbol{h}] + \delta_p(t) H_{\rm K}. \tag{1}$$

Here $\delta_p(t) = \sum_{m=-\infty}^{\infty} \delta(t-m)$ is the periodic delta function and we defined

$$H_{\rm I}[\boldsymbol{h}] \equiv \sum_{j=1}^{L} \{J\sigma_j^z\sigma_{j+1}^z + h_j\sigma_j^z\}, \qquad H_{\rm K} \equiv b\sum_{j=1}^{L}\sigma_j^x, \tag{2}$$

where we denote by L the volume of the system, σ_j^α, $\alpha \in \{x, y, z\}$, are the Pauli matrices at position j, and we impose $\sigma_{L+1}^\alpha = \sigma_1^\alpha$. The parameters J, b are, respectively, the coupling of the Ising chain and the transverse kick strength, while $\boldsymbol{h} = (h_1, \ldots, h_L)$ describes a position dependent longitudinal field. Here and in the following, vectors are indicated by bold latin letters. For generic values of the longitudinal fields $\boldsymbol{h}$ the only symmetry possessed by the Hamiltonian [Eq. (1)] is time reversal.

The Floquet operator generated by Eq. (1) reads as

$$U_{\rm KI}[\boldsymbol{h}] = T\exp\left(-i\int_0^1 ds H_{\rm KI}[\boldsymbol{h};s]\right) = e^{-iH_{\rm K}}e^{-iH_{\rm I}[\boldsymbol{h}]}. \tag{3}$$

In Floquet systems it is customary to introduce quasienergies $\{\varphi_n\}$ defined as the phases of the eigenvalues of the Floquet operator. The quasienergies take values in the interval $[0, 2\pi]$ and their number is equal to the dimension of the Hilbert space $\mathcal{N} = 2^L$. The quasienergy distribution function can then be written as $\rho(\varepsilon) = (2\pi/\mathcal{N})\sum_n \delta(\varepsilon - \varphi_n)$. It is instructive to consider the connected two-point function of $\rho(\varepsilon)$, defined as [30]

$$r(\nu) = \frac{1}{2\pi}\int_0^{2\pi} d\varepsilon \rho\left(\varepsilon + \frac{\nu}{2}\right)\rho\left(\varepsilon - \frac{\nu}{2}\right) - 1. \tag{4}$$

The Fourier transform of this quantity, known as the spectral form factor, is the main object of our study

$$K(t) = \frac{\mathcal{N}^2}{2\pi}\int_0^{2\pi} d\nu e^{i\nu t} r(\nu) = \sum_{m,n} e^{i(\varphi_m - \varphi_n)t} - \mathcal{N}^2\delta_{t,0}. \tag{5}$$

This object can be efficiently calculated in the context of RMT. Since our system is time reversal invariant, the RMT prediction relevant to our case is that of the circular orthogonal ensemble, $K_{\rm COE}(t) = 2t - t\ln(1 + 2t/\mathcal{N})$ for $0 < t < \mathcal{N}$ [7]. SFF represents an extremely efficient and sensitive diagnostic tool for determining the spectral properties of a system. Any significant deviation from RMT is an indicator of nonergodicity. For example, for integrable or localized systems, spectral fluctuations are conjectured to be Poissonian [31] and SFF is drastically different, $K(t) = \mathcal{N}$ for all $t > 0$.

Floquet SFF is defined for integer times t only (multiples of driving period), and $t > 0$ admits a simple representation in terms of the Floquet operator [Eq. (3)]

$$K(t) = |{\rm tr}(U_{\rm KI}^t[\boldsymbol{h}])|^2. \tag{6}$$

The trace of the Floquet operator can be thought of as the partition function of a two dimensional classical Ising model defined on a periodic rectangular lattice of size $t \times L$

$$\begin{aligned}{\rm tr}(U_{\rm KI}^t[\boldsymbol{h}]) &= \sum_{\{\boldsymbol{s}_\tau\}}\prod_{\tau=1}^{t}\langle \boldsymbol{s}_{\tau+1}|e^{-iH_{\rm K}}e^{-iH_{\rm I}[\boldsymbol{h}]}|\boldsymbol{s}_\tau\rangle \\ &= [(\sin 2b)/(2i)]^{Lt/2}\sum_{\{s_{\tau,j}\}} e^{-i\mathcal{E}[\{s_{\tau,j}\},\boldsymbol{h}]}. \end{aligned}\tag{7}$$

Here the configurations are specified by $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_t\} \equiv \{s_{\tau,j}\}$, where $s_{\tau,j} \in \{\pm 1 \equiv \uparrow\downarrow\}$ for all τ, j, and can be regarded as classical spin variables, $|\boldsymbol{s}\rangle$ is such that $\sigma_j^z|\boldsymbol{s}\rangle = s_j|\boldsymbol{s}\rangle$ and the energy of a configuration reads as

$$\mathcal{E}[\{s_{\tau,j}\},\boldsymbol{h}] = \sum_{\tau=1}^{t}\sum_{j=1}^{L}(Js_{\tau,j}s_{\tau,j+1} + J's_{\tau,j}s_{\tau+1,j} + h_j s_{\tau,j}) \tag{8}$$

where $J' = -\pi/4 - (i/2)\log\tan b$. Note that the Boltzmann weights of this model are generically complex.

Observing that Eq. (8) couples only "spins" on neighboring sites in both t and L directions, the partition function [Eq. (7)] can be written both as the trace of a transfer matrix propagating in the time direction and as the trace of a transfer matrix propagating in the space direction. This reveals the known duality transformation of the kicked Ising model [32,33]. The transfer matrix in the time direction is clearly given by $U_{\rm KI}[\boldsymbol{h}]$, while the transfer matrix "in space," $\tilde{U}_{\rm KI}[\mathbf{h}_j]$, is given by the same algebraic form [Eqs. (2) and (3)] exchanging J and J' but acting on a spin chain of t sites. Moreover, it acts at a nonstationary homogeneous field $\mathbf{h}_j = h_j\boldsymbol{\epsilon}$, where $\boldsymbol{\epsilon} = (1, \ldots, 1)$ is a t-component constant vector. In other words, we have the identity

$${\rm tr}(U_{\rm KI}^t[\boldsymbol{h}]) = {\rm tr}\left(\prod_{j=1}^{L}\tilde{U}_{\rm KI}[h_j\boldsymbol{\epsilon}]\right). \tag{9}$$

Here $U_{\rm KI}[\boldsymbol{h}]$ acts on $\mathcal{H}_L = (\mathbb{C}^2)^{\otimes L}$ and $\tilde{U}_{\rm KI}[h_j\boldsymbol{\epsilon}]$ acts on $\mathcal{H}_t = (\mathbb{C}^2)^{\otimes t}$. Note that $\tilde{U}_{\rm KI}[h_j\boldsymbol{\epsilon}]$ is generically nonunitary: it becomes unitary only for $|J| = |b| = (\pi/4)$ where $J' = \pm(\pi/4)$. We call these points of parameter space the "self dual points" and from now we focus on these.

The SFF is known to be non-self-averaging [34]. This means that $K(t)$ computed in a single system, i.e., for fixed parameters J, b, $\boldsymbol{h}$, does not generically reproduce the ensemble average. In order to compare to RMT predictions we then need to average over an ensemble of similar systems. Here we consider a very natural form of averaging by introducing disorder (which we may switch off at the end of calculation): we assume that the longitudinal magnetic fields at different spatial points h_j are independently distributed Gaussian variables with the mean value $\bar{h}$ and variance $\sigma^2 > 0$, and we average over their distribution. In other words, we consider

$$\bar{K}(t) \equiv \mathbb{E}_{\boldsymbol{h}}[K(t)] = \mathbb{E}_{\boldsymbol{h}}[{\rm tr}(U_{\rm KI}^t[\boldsymbol{h}]){\rm tr}(U_{\rm KI}^t[\boldsymbol{h}])^*], \tag{10}$$

where the symbol $\mathbb{E}_{\boldsymbol{h}}[\cdot]$ denotes the average over the longitudinal fields

$$\mathbb{E}_{\boldsymbol{h}}[f(\boldsymbol{h})] = \int_{-\infty}^{\infty} f(\boldsymbol{h}) \prod_{j=1}^{L} e^{-(h_j-\bar{h})^2/2\sigma^2} \frac{dh_j}{\sqrt{2\pi}\sigma}. \quad (11)$$

The average in Eq. (10) mixes two copies of the classical Ising model [Eq. (8)] with complex conjugate couplings. After rewriting in terms of dual transfer matrices [Eq. (9)], and noting that $|\mathrm{tr}U|^2 = \mathrm{tr}(U \otimes U^*)$, we see that the average factorizes row by row, and local averaging results in translationally invariant coupling between two periodic rows of t spins at the same spatial point. The resulting averaged SFF can again be interpreted as the trace of an appropriate transfer matrix in spatial direction (Fig. 1)

$$\bar{K}(t) = \mathrm{tr}(\mathbb{T}^L), \quad (12)$$

where the transfer matrix acts on $\mathcal{H}_t \otimes \mathcal{H}_t$ and reads as [35]

$$\mathbb{T} \equiv \mathbb{E}_h(\tilde{U}_{\mathrm{KI}}[h\boldsymbol{\epsilon}] \otimes \tilde{U}_{\mathrm{KI}}[h\boldsymbol{\epsilon}]^*) = (\tilde{U}_{\mathrm{KI}} \otimes \tilde{U}^*_{\mathrm{KI}})\mathbb{O}_\sigma. \quad (13)$$

Here $\tilde{U}_{\mathrm{KI}} \equiv \tilde{U}_{\mathrm{KI}}[\bar{h}\boldsymbol{\epsilon}]$ and the local Gaussian average is encoded in the following positive symmetric matrix

$$\mathbb{O}_\sigma = \exp\left[-\frac{1}{2}\sigma^2(M_z \otimes \mathbb{1} - \mathbb{1} \otimes M_z)^2\right], \quad (14)$$

where $M_\alpha \equiv \sum_{\tau=1}^{t} \sigma_\tau^\alpha$ for $\alpha \in \{x, y, z\}$. Note that, because of $\mathbb{O}_\sigma$, the matrix $\mathbb{T}$ is a nonunitary contraction.

The disorder averaged SFF $\bar{K}(t)$ can be computed numerically by evaluating Eq. (6) for several values of the longitudinal fields and then taking the average [Eq. (11)]. This can be done for small systems up to very large times, see Fig. 2. Here, however, we follow a different route. Numerical data provide strong evidence for the validity of RMT for any fixed t, and any $\sigma > 0$, in the TL $L \to \infty$. Indeed, the RMT prediction applies also for $t \ll \mathcal{N}$ when the system behaves as if it were effectively of infinite size. We then consider the TL and use Eq. (12) to analytically compute $\bar{K}(t)$. This is done in two steps: (i) we map the seeming formidable problem of computing $\bar{K}(t)$ in our nonintegrable many-body system into a simple problem in operator algebra; (ii) we solve the latter.

A numerical investigation indicates that, as long as $\sigma \neq 0$, the spectral gap $\Delta = 1 - \max_{\lambda \in \mathrm{eigenvalues}(\mathbb{T})}^{|\lambda|<1} |\lambda|$ remains finite for all mean fields and times, see Fig. 3. Therefore, the TL of the averaged SFF is entirely determined by the eigenvalues of $\mathbb{T}$ with largest magnitude. To find all such eigenvalues it is useful to exploit the following property [36], which is a consequence of the contractive nature of $\mathbb{O}_\sigma$ and of the unitarity of $\tilde{U}_{\mathrm{KI}}$.

Property 1.—(i) the eigenvalues of $\mathbb{T}$ have at most unit magnitude; (ii) even if $\mathbb{T}$ is generically not guaranteed to be diagonalizable, the algebraic and geometric multiplicites of any eigenvalue of magnitude 1 coincide.

Let us then construct all eigenvectors $|A\rangle$ of $\mathbb{T}$ of unimodular eigenvalues. First, we note that all such $|A\rangle$ lie in the eigenspace of $\mathbb{O}_\sigma$ with unit eigenvalue. This is seen by expanding $\langle A|\mathbb{T}^\dagger\mathbb{T}|A\rangle = 1$ in an eigenbasis of $\mathbb{O}_\sigma$

$$1 = \langle A|\mathbb{T}^\dagger\mathbb{T}|A\rangle = \langle A|\mathbb{O}_\sigma^2|A\rangle = \sum_n |\langle A|n\rangle|^2 o_{\sigma,n}^2, \quad (15)$$

where $0 < o_{\sigma,n} \leq 1$ are the eigenvalues of $\mathbb{O}_\sigma$. Since $|A\rangle$ is normalized and $\{|n\rangle\}$ is complete, this is possible only if $\langle A|n\rangle = 0$ for all $o_{\sigma,n} < 1$. In other words, $|A\rangle$ is a linear combination of eigenvectors of $\mathbb{O}_\sigma$ with unit eigenvalue, namely $\mathbb{O}_\sigma|A\rangle = |A\rangle$. Using the exponential form (14) of

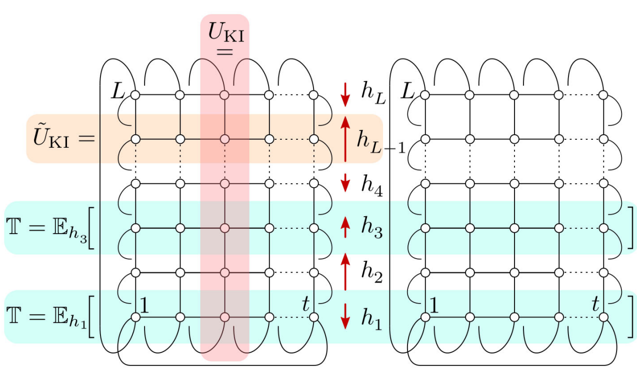

FIG. 1. Pictorial representation of $\bar{K}(t)$. The average over h_j produces a transfer matrix $\mathbb{T}$ for all $j = 1, \ldots, L$. Each column and row of the first lattice correspond respectively to the transfer matrix $U_{\mathrm{KI}}[\boldsymbol{h}]$ and the dual transfer matrix $\tilde{U}_{\mathrm{KI}}[h_j\boldsymbol{\epsilon}]$. Each column and row of the second lattice correspond, respectively, to the complex conjugate transfer matrix $U_{\mathrm{KI}}[\boldsymbol{h}]^*$ and the complex conjugate dual transfer matrix $\tilde{U}_{\mathrm{KI}}[h_j\boldsymbol{\epsilon}]^*$.

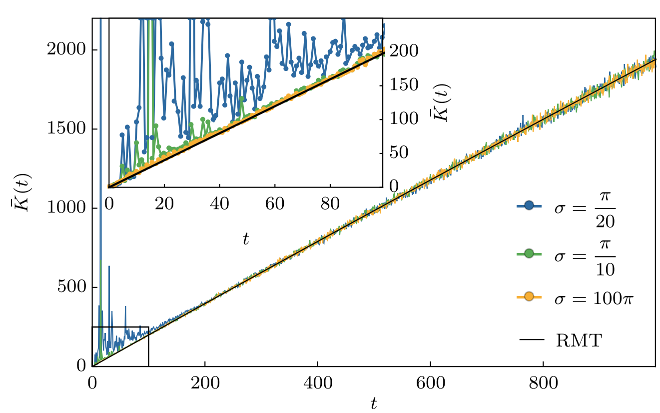

FIG. 2. SFF in the disordered kicked Ising model for $J = b = (\pi/4)$, $L = 15$, and $\bar{h} = 0.6$. The figure compares the time evolution of the SFF for different widths σ of the disorder distribution. Inset: short-time window. The large-time fluctuations are due to the finite number ($N = 9490$) of disorder realizations.

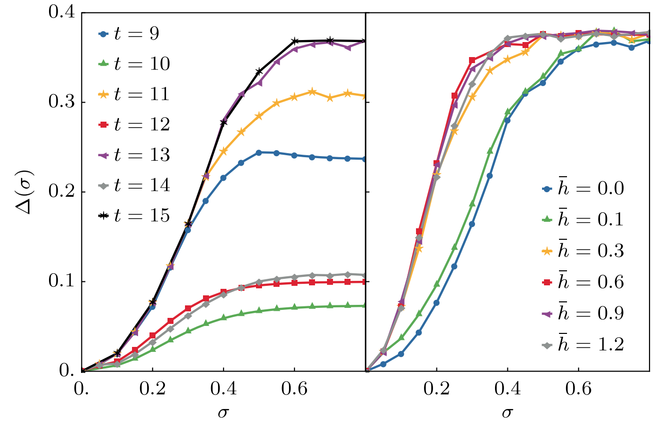

FIG. 3. Spectral gap of $\mathbb{T}$ as a function of the disorder strength σ. The left panel shows $\Delta(\sigma)$ for $\bar{h}=0$ and different values of t: we observe a clear even-odd effect in the data, but, in both cases, the gap approaches a finite limiting curve for large t. The right panel shows $\Delta(\sigma)$ for $t=13$ and different values of $\bar{h}$.

$\mathbb{O}_\sigma$, we see that this condition means that all $|A\rangle$ are in the kernel of $M_z \otimes \mathbb{1} - \mathbb{1} \otimes M_z$. Putting all together, we have that the eigenvectors $|A\rangle$ associated to unimodular eigenvalues must satisfy

$$(\tilde{U}_{\rm KI} \otimes \tilde{U}^*_{\rm KI})|A\rangle = e^{i\phi}|A\rangle, \qquad \phi \in [0, 2\pi],$$
$$(M_z \otimes \mathbb{1} - \mathbb{1} \otimes M_z)|A\rangle = 0. \qquad (16)$$

These conditions can be turned into equations for operators over $\mathcal{H}_t$ as follows. Denoting by $\{|n\rangle\}$ a basis of $\mathcal{H}_t$, we can expand a generic vector in $\mathcal{H}_t \otimes \mathcal{H}_t$ as

$$|A\rangle = \sum_{n,m} A_{n,m}|n\rangle \otimes |m\rangle^*, \qquad (17)$$

where the 2^{2t} complex numbers $\{A_{n,m}\}$ are interpreted as the matrix elements of an operator A: $\langle n|A|m\rangle = A_{n,m}$. The operator A is in one-to-one correspondence with the state $|A\rangle$ and we can rewrite the conditions of Eq. (16) as follows

$$[A, M_z] = 0, \qquad \tilde{U}_{\rm KI} A \tilde{U}^\dagger_{\rm KI} = e^{i\phi}A. \qquad (18)$$

After some simple manipulations, [36] we find

Property 2.—The relations of Eq. (18) are equivalent to

$$UAU^\dagger = e^{i\phi}A, \qquad [A, M_\alpha] = 0, \qquad \alpha \in \{x, y, z\}. \quad (19)$$

Here we defined the unitary operator U as

$$U = \exp\left[i\frac{\pi}{4}\sum_{\tau=1}^{t}(\sigma^z_\tau \sigma^z_{\tau+1} - \mathbb{1})\right]. \qquad (20)$$

TABLE I. Number of eigenvalues 1 and -1 of the transfer matrix $\mathbb{T}$ determined via exact diagonalization for $t \le 17$.

t	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$\#_{+1}$	2	5	7	9	13	14	18	18	22	22	25	26	29	30	33	34
$\#_{-1}$	0	0	0	0	2	0	0	0	2	0	0	0	0	0	0	0

The goal of step (i) is then achieved: the calculation of $\bar{K}(t)$ is reduced to finding all linearly independent matrices A that fulfil Eq. (19) for some ϕ. Let us now consider the latter algebraic problem.

The property $U^2 = \mathbb{1}$ implies $\phi \in \{0, \pi\}$. Namely, the unimodular eigenvalues of $\mathbb{T}$ are either 1 or -1. By exact numerical diagonalization of $\mathbb{T}$ we find that the eigenvalues -1 are much rarer than $+1$ and are observed only for small systems (see Table I). In particular, for an odd t we have the following additional simplification [36]:

Property 3.—$\phi = 0$ for odd t.

For odd t we then need to determine all linearly independent matrices A commuting with the set $\mathcal{M} = \{U, M_x, M_y, M_z\}$. A subset of all possible operators commuting with $\mathcal{M}$ is found by considering the common symmetries: reflection R and one-site shift Π on a periodic chain of t sites

$$\Pi = \prod_{\tau=1}^{t-1} P_{\tau,\tau+1}, \qquad R = \prod_{\tau=1}^{\lfloor t/2 \rfloor} P_{\tau,t+1-\tau}. \qquad (21)$$

Here $P_{\tau,\omega} = \frac{1}{2}\mathbb{1} + \frac{1}{2}\sum_\alpha \sigma^\alpha_\tau \sigma^\alpha_\omega$ is the elementary permutation operator (transposition). These operators generate the so called dihedral group (see, e.g., Ref. [37])

$$\mathcal{G}_t = \{\Pi^n R^m; n \in \{0, \ldots, t-1\}, m \in \{0, 1\}\}, \qquad (22)$$

which is the symmetry group of a polygon with t vertices. All elements of $\mathcal{G}_t$ commute with $\mathcal{M}$ and we have [36]

Property 4.—The number of linearly independent elements of $\mathcal{G}_t$ is $2t$ for $t \ge 6$, $2t-1$ for $t \in \{1, 3, 4, 5\}$, and 2 for $t = 2$.

We thus have a lower bound on the number of independent matrices A fulfilling Eq. (19) and hence on the value of the averaged SFF for odd t. Our main result is to show that such lower bound is also an upper bound, namely

Theorem 1.—For odd t, any A simultaneously commuting with all elements of $\{U, M_x, M_y, M_z\}$ is of the form

$$A = \sum_{n=0}^{t-1}\sum_{m=0}^{1} a_{n,m}\Pi^n R^m, \qquad a_{n,m} \in \mathbb{C}. \qquad (23)$$

See Ref. [36] for a proof. As the number of such linearly independent A is the multiplicity of eigenvalue 1 of $\mathbb{T}$, and since there is a finite gap between unit circle and the rest of the spectrum, we have

$$\lim_{L\to\infty}\bar{K}(t)=\begin{cases}2t-1, & t\le 5\\ 2t, & t\ge 7\,,\end{cases}\qquad t\text{ odd.}\qquad(24)$$

For even t the situation is more complicated. In this case, we identify an additional independent operator besides $\mathcal{G}_t$ spanning the commutant of $\mathcal{M}$. This operator can be written as a projector $|\psi\rangle\langle\psi|$, where we introduced a t spin singlet state

$$|\psi\rangle=\frac{1}{2^t}\prod_{\tau=1}^{t/2}(1-P_{\tau,\tau+t/2})|\underbrace{\downarrow,\ldots,\downarrow}_{t/2},\underbrace{\uparrow,\ldots,\uparrow}_{t/2}\rangle,\qquad(25)$$

satisfying $U|\psi\rangle=-|\psi\rangle$, $M_{x,y,z}|\psi\rangle=0$, $\Pi|\psi\rangle=-|\psi\rangle$, $R|\psi\rangle=(-1)^{t/2}|\psi\rangle$. Moreover, for $t\in\{8,10\}$ we identify the second additional operator commuting with the set $\mathcal{M}$ [36]. Finally, for $t\in\{6,10\}$ we construct two operators satisfying Eq. (19) with eigenphase $\phi=\pi$ [36]. All these additional operators, except Eq. (25), appear to be a short-time fluke and are observed only for t smaller than 11. We are then lead to conjecture

$$\lim_{L\to\infty}\bar{K}(t)=2t+1,\qquad t>11,\qquad t\text{ even.}\qquad(26)$$

This conjecture, together with the exact result [Eq. (24)], is in agreement with exact diagonalization of $\mathbb{T}$ on chains of length $t\le 17$, see Tab. I.

The results of Eqs. (24) and (26) are remarkable: we fully recovered two-point RMT spectral fluctuations (in the TL) in a simple nonintegrable spin-1/2 chain with local interactions. A key step of our calculation was to average over the distribution of independent longitudinal fields $\boldsymbol{h}$. This average introduces a finite gap in the spectrum of the transfer matrix $\mathbb{T}$ and selects the $2t$ "universal" eigenvalues out of the exponentially many eigenvalues of $\mathbb{T}$. Note that any nonvanishing σ is sufficient for this astonishing simplification to occur. Moreover, after the TL is taken, there is no additional dependence of the result on the disorder variance σ^2, we can then consider the limit $\sigma\to 0$ corresponding to a clean system. Finally, our result does not depend on the particular distribution of the longitudinal fields, as long as they are independent and identically distributed random variables; a different choice modifies the form of Eq. (14) but not the TL result. Since our analysis is carried out in the TL, it is unable to access RMT physics at timescales growing with L, such as level repulsion emerging at $t\sim 2^L$.

Our proof of ergodicity pertains to some special, self-dual, points in the parameter space of the system. At these points the system is "maximally ergodic" as the Thouless time does not grow with L. We have numerically verified the stability of the ergodic behavior under perturbations around the self-dual points. In this case, however, the Thouless time becomes an increasing function of L, as expected in generic chaotic systems.

A striking consequence of our result is a rigorous proof of nonexistence of many-body localization [38–40] at any self-dual point in our model ($J,b\in\{\pm(\pi/4)\}$) for any amount of uncorrelated disorder in the longitudinal field. Indeed, knowing that $K(t)=2t$ for an odd $t\ge 7$ is enough to exclude localization which should be connected to Poissonian behavior.

The technique developed gives a new way of analytically treating nonintegrable systems and suggests immediate applications in several directions. First, one can apply it to compute the bipartite entanglement entropy dynamics starting from a random separable state, testing recent conjectures [41–43] on its universal linear behavior in ergodic systems. Moreover, our method can be used to rigorously approach ETH by studying averages and higher moments of distributions of expectation values of local observables. Finally, one can use our technique to evaluate dynamical correlation functions of local observables. A preliminary analysis shows that, at the self-dual point in the TL, they vanish for all $t\ge 1$, consistently with an L-independent Thouless time.

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