Experimentally Robust Self-testing for Bipartite and Tripartite Entangled States

Wen-Hao Zhang, Geng Chen,* Xing-Xiang Peng, Xiang-Jun Ye, Peng Yin, Ya Xiao, Zhi-Bo Hou, Ze-Di Cheng, Yu-Chun Wu, Jin-Shi Xu, Chuan-Feng Li,† and Guang-Can Guo

CAS Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei, Anhui 230026, China and Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, Anhui 230026, China

(Received 7 April 2018; revised manuscript received 1 October 2018; published 13 December 2018)

Self-testing is a method with which a classical user can certify the state and measurements of quantum systems in a device-independent way. In particular, self-testing of entangled states is of great importance in quantum information processing. An understandable example is that the maximal violation of the Clauser-Horne-Shimony-Holt inequality necessarily implies that the bipartite system shares a singlet. One essential question in self-testing is that, when one observes a nonmaximum violation, how far is the tested state from the target state (which maximally violates a certain Bell inequality)? The answer to this question describes the robustness of the used self-testing criterion, which is highly important in a practical sense. Recently, J. Kaniewski derived two analytic self-testing bounds for bipartite and tripartite systems. In this Letter, we experimentally investigate these two bounds with high-quality two-qubit and three-qubit entanglement sources. The results show that these bounds are valid for various entangled states that we prepared. Thereby, a proof-of-concept demonstration of robust self-testing is achieved, which improves on the previous results significantly.

DOI: 10.1103/PhysRevLett.121.240402

Device-independent (DI) science, which is inspired by the requirements for secure quantum information processing, has attracted intense interest over the past decade [1]. In a DI approach, the only way to study the system is to perform local measurements and analyze the statistical results. Under the only assumptions of no signaling and the validity of quantum theory, it has been shown that it is possible to characterize the quantum devices in quantum key distribution [2,3], randomness generation [4], entanglement witness, and dimension witness [5] in a DI way. Especially, in some cases, one can certify uniquely the state and the measurements that are present in the devices, simply by querying the devices with classical inputs and observing the correlations in the classical outputs. This phenomenon is known as the concept of "self-testing" [6]. An explicit example is the fact that the maximal violation of the Clauser-Horne-Shimony-Holt (CHSH) Bell inequality [7,8] uniquely identifies the maximally entangled state of two qubits [9], namely, "singlet."

A general conclusion can be made that the violation of a Bell inequality [10] reveals the presence of entanglement. Concretely speaking, certain Bell correlations can be reproduced only by performing specific local measurements on a specific entangled state (up to local unitaries), and thus, the observation of such correlations allows one to characterize an unknown source of quantum states as well as the measurement devices [11] or observables [12–15] in a DI manner. Recently, criteria to self-test different forms of entangled states, i.e., multipartite entangled states [16], graph states [17], high-dimensional maximally entangled states [18,19], nonmaximally entangled states of two qubits [20,21], or arbitrary pure bipartite states [22,23] have also been proposed. The results of these works are limited to ideal scenarios in which the tested states are fully ideal. However, in practical quantum information processes, the state contained in the devices generally deviates from the ideal case due to the presence of errors. As a result, we cannot observe the maximal quantum violation in self-testing procedures. However, in such a case, it is still important to know how far the tested state is from $\Psi_{A'B'}$ (which maximally violates a certain Bell inequality), in other words, how robust the used self-testing criterion is considering realistic errors. Although self-testing statements of practically relevant robustness are highly significant, few theoretical results for them are known [12,24,25]. Self-testing has been used to estimate the quality of a large-scale integrated entanglement source [26]. However, it is a prerequisite to verify the practical reliability of a self-testing criterion before employing it, which is just the main result of this Letter.

Recently, Kaniewski had developed a new technique to prove analytic self-testing statements [27]. The new method can give rise to a family of operators to place a lower bound on the spectrum of these operators, and thus, immediately yields a self-testing robustness statement. The advantage of the new method is that it provides an explicit construction of the extraction channels in terms of the measurement operators. Previous methods, on the other hand, resort to a numerical optimization over a wide class of extraction maps and, hence, do not identify the optimal ones. This distinct advantage makes the given bound feasible to be experimentally implemented.

For singlet self-testing with the CHSH inequality, this method can give a nearly optimal bound that improves on all the previously known results; for the tripartite Greenberger-Horne-Zeilinger (GHZ) state [28] self-testing with Mermin inequality [29], this method yields the first tight self-testing statement. In this Letter, we experimentally investigate these two bounds with high quality two-qubit and three-qubit entanglement sources, which can generate pure and mixed quantum states with an adjustable degree of entanglement. Under the fair sampling assumption, our Letter is one of the first proof-of-concept demonstrations of self-testing. The results show that these two bounds are valid for various entangled states prepared in this Letter. Furthermore, by preparing several example families of entangled states, we experimentally demonstrate robust self-testing processes which improve the previous known theories significantly, and thus, our Letter can be more instructive for the application of self-testing to new quantum techniques.

Theoretical framework.—We describe the self-testing scenario in detail: Alice and Bob share some quantum states in a blackbox-like device, and they want to identify the state through some measurement apparatus (MA). If these MAs can be trusted, they can perform tomography to precisely deduce the form of the shared state. Otherwise, their actions are limited to choosing the measurement setting and observing the outcome, and hence, the only information available to them is the conditional probability distribution $P(a,b|x,y)$ (i.e., the probability of observing outputs a, b for inputs x, y).

The self-testing statement can be quantified by the extractability $\Xi(\rho_{AB} \to \Psi_{A'B'})$ of the test state ρ_{AB} to a target state $\Psi_{A'B'}$, which can be defined as the maximum fidelity taking over all quantum channels (completely positive trace-preserving maps) of the correct input-output dimension [27]. In order to test the entanglement characteristics of ρ_{AB}, $\Psi_{A'B'}$ is assumed to be a state which achieves the maximal quantum violation. Theoretically, when the maximal quantum violation is observed in a self-testing scenario, the shared unknown state can be mapped to $\Psi_{A'B'}$ with the extractability to be 1. In practical quantum information processes, errors are unavoidable, and the robustness can be described by the lowest possible extractability $Q_{\Psi,B}(\beta)$ when one observes the violation of (at least) β on the Bell inequality B [27].

By constructing the local extraction channels Λ_A and Λ_B from the measurement parameters a and b [27], it can be proved

$$K(a,b) \geq \frac{4+5\sqrt{2}}{16} W_{\alpha,\beta}(a,b) - \frac{1+2\sqrt{2}}{4}. \qquad (1)$$

The Bell operator W is defined by the parameters a and b and written as $W(a,b) = \sum_{j,k\in\{0,1\}} (-1)^{jk} A_j(a) \otimes B_k(b)$. K is the fidelity operator constructed from a and b and written as $K \coloneqq (\Lambda_A(a) \otimes \Lambda_B(b))(\Psi_{A'B'})$ [27].

The lower (i.e., self-testing) and upper bounds can be deduced from Eq. (1) as

$$\frac{4+5\sqrt{2}}{16}\beta - \frac{1+2\sqrt{2}}{4} \leq Q_{\Psi_{AB},B_{\mathrm{CHSH}}}(\beta_{\mathrm{CHSH}}) < 0.5 + 0.5\frac{\beta-2}{2\sqrt{2}-2}. \qquad (2)$$

In a similar way, for the three-qubit scenario in which each of the three parties has a binary measurement operator and an extraction channel defined by a, b, c the operator inequality can be written as

$$K(a,b,c) \geq \frac{2+\sqrt{2}}{8} W(a,b,c) - \frac{1}{\sqrt{2}}. \qquad (3)$$

Surprisingly, using the Mermin inequality, a perfectly tight self-testing bound can be obtained [27] as

$$Q_{\Upsilon_{\mathrm{ABC}},B_{\mathrm{Mermin}}}(\beta) = \frac{1}{2} + \frac{1}{2}\frac{\beta - 2\sqrt{2}}{4-2\sqrt{2}}, \qquad (4)$$

where $\beta > 2\sqrt{2}$ to guarantee a nontrivial fidelity statement.

In a black-box scenario, certain Bell violation β may result from a strongly nonlocal state with a nonoptimal measurement setting, or a relatively weaker nonlocal state with an (nearly) optimal measurement setting. In this situation, self-testing bound aims to give a lowest possible extractability for an ensemble of states that achieve the violation value of (at least) β. Therefore, self-testing is essentially different from state tomography, which can precisely reconstruct the density matrix of the measured state. The more robust the self-testing bound is, the higher merit of entanglement can be guaranteed when observing a certain violation value. Previously, a violation exceeding 2.37 can only guarantee a nontrivial extractability (≥ 0.5) to the singlet, whereas the new bound can raise the extractability commitment ≥ 0.68. Meanwhile, the nontrivial threshold decreases to $\beta = 2.11$.

Experimental results.—The bounds from Ref. [27] exhibit excellent robustness to noise, so they are perfectly suited to be applied in an actual experiment. This is precisely the motivation for the current Letter.

The above two analytic self-testing bounds in inequalities (2) and (4) can be experimentally studied by the setup in Fig. 1. The setup consists of two main parts. One part is responsible for the generation of the polarization-entangled photon pairs, and the other part is used to generate various kinds of pure or mixed two-qubit and three-qubit states. In the first part, polarization-entangled photons are generated in the state $\cos\theta|00\rangle + \sin\theta|11\rangle$ (0 and 1 denote the horizontally and vertically polarized components, respectively) and θ is controlled by the pumping polarization [30].

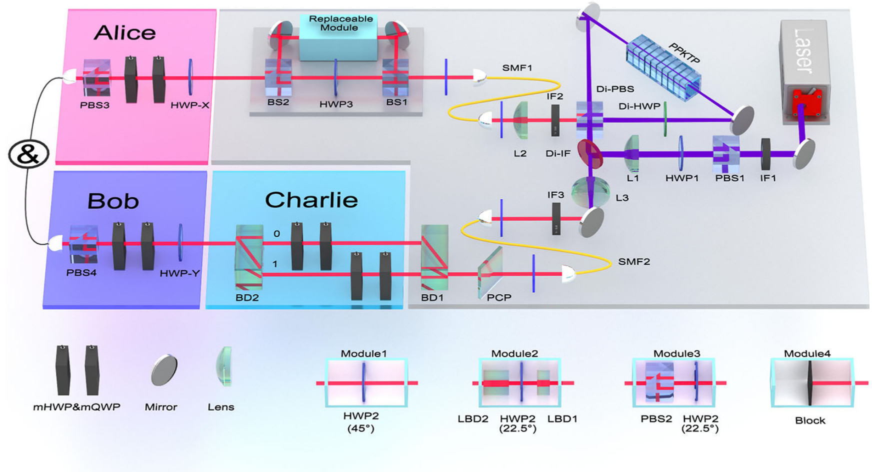

FIG. 1. Experimental setup. A periodically poled $KTiOPO_4$ (PPKTP) nonlinear crystal placed inside a phase-stable Sagnac interferometer (SI) is pumped by a single mode 405.4 nm laser to produce polarization-entangled photon pairs at 810.8 nm. Bandpass filters at 810 nm and long-pass filters are used to block the pump light. One photon will pass through a sufficiently unbalanced (to make the two light paths fully decoherent) Mach-Zehnder interferometer (MZI) constructed by two 50/50 nonpolarizing beam splitters BS1 and BS2 when mixed states must be generated. By introducing replaceable modules on the long arm of this unbalanced MZI, different categories of states can be prepared. The other photon will pass through a balanced MZI constructed by beam displacers BD1 and BD2 when we need to generate tripartite states. One polarization beam splitter (PBS), a motorized half-wave plate (mHWP), and a motorized quarter-wave plate (mQWP) are used to perform the projection measurements on each qubit. BS, beam splitter; L, lense; Dia, diaphragm, Di, dichroic; IF, interfering filter; LBD, longtitudinal beam displacer; PCP, phase compensation plate.

These photon pairs are then sent into the second part by two single mode fibers, and the polarization is maintained by two HWPs before and after each fiber. A process tomography shows that the polarization maintaining fidelity is 0.997 ± 0.0007 [31]. For singlet state self-testing, the overall efficiency from creation to detection of entangled photons is $\simeq 0.11$ for both sides.

For the generation of two-qubit states, Charlie does nothing and can be neglected. Alice and Bob can perform measurement operators K and W with random (a, b), and we would like to verify that inequality (2) cannot be violated for all the tested scenarios. $W(a, b)$ can be decomposed into $A0$, $A1$, $B0$, $B1$ to be realized by a QWP-HWP array, which is in the form of

$$A(B)_r = \cos a(b)\sigma_x + (-1)^r \sin a(b)\sigma_z, \qquad (5)$$

with $r \in 0, 1$. $K(a, b)$ consists of mixtures of Pauli matrices $\sigma_i \otimes \sigma_i$ ($i \in 1, 2, 3, 4$), and its expected value can be directly measured and is $\langle K(a, b)\rangle$.

The results for different singlets, partially entangled and mixed bipartite states are shown in Figs. 2(a)–2(c). It is clear for all the tested states and randomly chosen $\{a, b\}$, the data points are all above the red line, exactly satisfying the operator inequality (1). As a reasonable inference, the self-testing bound represented by left side of the inequality (2) is valid for all tested scenarios.

The self-testing bound can also be directly tested by measuring the relation between the Bell violation β and the extractability Ξ, and hence to reveal how tight the bound is in a visual representation. Considering the fact that for a certain violation β, the lowest possible extractability is reached by the state of which the ultimate violation is just β, we measure both the maximal violation and the extractability of various of entangled states, as shown in Fig. 2(d).

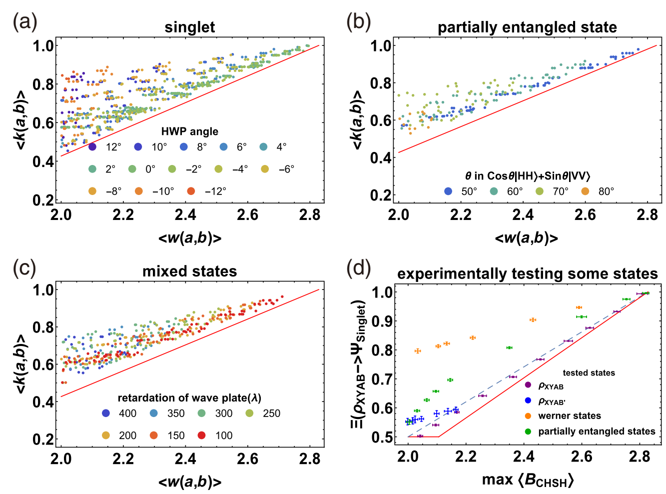

FIG. 2. Experimental test of operator inequality (1) with (a) different singlet states obtained by rotating HWP3 to the angles shown in the figure; (b) partially entangled states characterized by the value of θ shown in the figure; (c) mixed states obtained by inserting wave plates of different levels of retardation. With randomly sampled $\{a, b\}$, all data points are above the given bound. (d) Experimental self-testing of four special categories of entangled states, with the red and blue dashed lines representing the lower (self-testing) and upper bounds in inequality (2), respectively.

In particular, a category of registered two-qubit states ρ_{XYAB} can approach the lower bound, providing evidence that the robustness bound is nearly optimal [see Supplemental Material [32] for details].

Replacing τ_{AB}^{11} in ρ_{XYAB} with a Werner state, another category of registered entangled states $\rho_{\mathrm{XYAB}'}$ are tested with unaltered extraction strategy [see Supplemental Material [32] for details]. Besides, these Werner states have also been tested individually. The partially entangled states are prepared by rotating HWP1 before the SI, and thus, the generated two-qubit states are in the form of $\cos\theta|00\rangle + \sin\theta|11\rangle$. By varying θ, a category of partially entangled states are tested. For these four categories of states, the extractability Ξ are measured and all the data points are above the self-testing bound (red line), which are exactly consistent with inequality (2). Each category consists of states with the violation above and below 2.11, and thus, nontrivial and trivial self-testing conclusions can be made accordingly.

Considering all the above results, we experimentally prove that the given robustness bound for the self-testing of bipartite states is valid for all the states tested in this Letter. Furthermore, we show that the given bound is nearly optimal to implement a robust self-testing procedure by identifying the states falling in the narrow band between the upper and lower (i.e., self-testing) bounds. Although the given CHSH self-testing bound is not tight, i.e., for violations between 2.00 and 2.11, we currently cannot certify entanglement, but in the future it is possible to formulate a stronger self-testing statement. However, according to the experiment results for ρ_{XYAB}, any improved self-testing bound cannot surpass the current upper bound, shown as the blue dashed line in Fig. 2(d).

By introducing Charlie, we can generate various tripartite entangled states and measure the corresponding $\langle K(a,b,c)\rangle$ and $\langle W(a,b,c)\rangle$. In the experiment, the simulated tripartite states are generated by introducing the spatial mode of one photon, which is realized by BD1 and BD2 in Fig. 1. All these data points are well above the lower bound described by inequality (3), as shown in Fig. 3(a).

For tripartite self-testing, we prepare a category of states which are mixtures of a GHZ state Υ_{ABC} and ν_{ABC}, where $\nu_{\mathrm{ABC}} = |0\rangle_{\mathrm{A}}|\Phi\rangle_{\mathrm{BC}}$ with Φ_{BC} to be equivalent to the singlet state (up to local unitary). Concretely, for photons passing the short arm of the unbalanced MZI, Υ_{ABC} is produced. Although for photons passing the long arm with module 3, they are firstly rotated to be diagonally polarized and only the 0 component is then selected by PBS2 to produce ν_{ABC}. By changing the mixture weight, a series of mixed states are generated with fidelity $\simeq 0.99$ and the measured results are shown in Fig. 3(b). The data points almost fall on the lower bound (red line) considering the error bars, indicating the given bound for the tripartite scenario is provably tight for performing a robust self-testing procedure.

Device-independent certifications require no-signaling constraints on the devices [33], which can be tested through

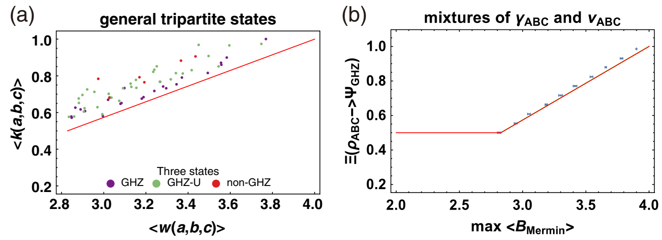

FIG. 3. Experimental test of the self-testing bound for tripartite scenarios. (a) For several typical tripartite entangled states, the mean values of both the fidelity operator K and the Mermin operator W are measured, with random sampling on (a,b,c). All the data points are above the bound (red line) given by operator inequality (3). (b) For mixtures of Υ_{ABC} and ν_{ABC} with different weights, the maximum violations of Mermin inequality and corresponding extractabilities are measured. All the dots almost fall on the lower bound (red line), indicating the given bound is tight.

the influence on the one side from the measurements of all the other sides [10]. For example, for a tripartite state, the measurement settings are x and y, z on Alice, Bob and Charlie sides respectively, with the corresponding outcomes to be a, b, c. As a result, the mean value of $\langle A_x\rangle$ can be calculated by summing all the possibilities on b and c. The no-signaling constraint requires that $\langle A_x\rangle$ remains constant when (y,z) is changed to (y',z'), which can be written as

$$\begin{aligned}\langle A_x\rangle = &\sum_{b=0}^{1}\sum_{c=0}^{1}(P(a=1,b,c|x,y,z)\\ &- P(a=-1,b,c|x,y,z)), \qquad (6)\\ = &\sum_{b=0}^{1}\sum_{c=0}^{1}(P(a=1,b,c|x,y',z')\\ &- P(a=-1,b,c|x,y',z')) \quad \text{for all}\, x,y,y',z,z'. \qquad (7)\end{aligned}$$

Similar requirements need to be satisfied for all joint measurement settings appearing in the CHSH and Mermin Bell inequalities, and the results are shown in Fig. 4. When the tested states are the maximally entangled

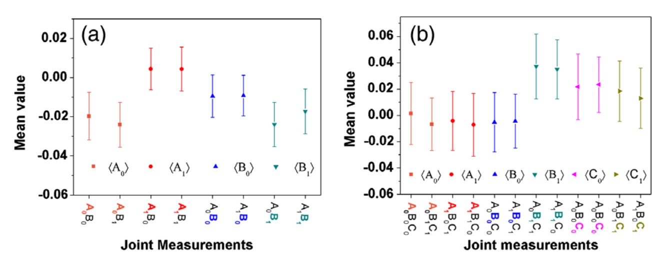

FIG. 4. No-signaling tests for measuring the (a) CHSH inequality with singlet state and (b) Mermin inequality with the GHZ state. The mean value of each operator is invariant within one standard deviation, confirming the no-signaling characteristics of the used devices.

singlet and GHZ state, the mean values of an arbitrary observable are approximately identical for different joint measurement settings appearing in the inequalities, which exactly satisfies the no-signaling constraint.

To implement a practical device-independent self-testing, one has to achieve high efficiency in detecting entangled particles so as to close the detection loophole [34]. For CHSH Bell inequality, an overall efficiency above 0.828 is required. Prospectively, this requirement can be met by improving the detector performance and the coupling efficiency. Specifically, employing superconducting nanowire single-photon detectors and setting the beam waists properly with respect to that of the pump beam, one can increase the overall efficiency significantly toward a detection loophole-free self-testing.

Discussion.—Although the scientific community pursues construction of perfect correlations with which one can uniquely (up to local isometries) infer the appearance of a certain ideal state, self-testing robustness statement is of significant importance from a practical point of view. Similar statements are mainly inspired by the fact that the realistic states may deviate from the ideally entangled states and cannot violate the utilized Bell inequality maximally. In these cases, the robustness statements are able to provide a quantitative description of the entanglement characteristics for the tested states. As a specific example, the concrete form of the shared states contained in the entanglement sources may not be important, and rather, only, a guarantee of the quality of the entanglement is desired. In this scenario, simply by querying the devices with classical inputs and observing the correlations in the classical outputs, one can immediately obtain a minimum fidelity to the ideal state according to the robustness bound. The two (nearly) optimal analytic robustness bounds, which are applicable for bipartite and tripartite systems, are tested in the present experiment. The results clearly confirm the validity of these two bounds; thereby, robust self-testing processes are achieved in this experiment. Our Letter is instructive for practical self-testing tasks, such as placing a lower bound on the distillable entanglement of an unknown state.

This work was supported by the National Key Research and Development Program of China (Grants No. 2016YFA0302700 and No. 2017YFA0304100), National Natural Science Foundation of China (Grants No. 11874344, No. 61835004, No. 61327901, No. 11774335, No. 91536219, and No. 11821404), Key Research Program of Frontier Sciences, Chinese Academy of Sciences (Grant No. QYZDY-SSW-SLH003), Anhui Initiative in Quantum Information Technologies (Grants No. AHY020100 and No. AHY060300), the Fundamental Research Funds for the Central Universities (Grants No. WK2030020019 and No. WK2470000026).

*chengeng@ustc.edu.cn
†cfli@ustc.edu.cn

[1] V. Scarani, The device-independent outlook on quantum physics, Acta Phys. Slovaca **62**, 347 (2012).
[2] A. Acín, N. Gisin, and L. Masanes, From Bell's Theorem to Secure Quantum Key Distribution, Phys. Rev. Lett. **97**, 120405 (2006).
[3] A. Acín, N. Brunner, N. Gisin, S. Massar, S. Pironio, and V. Scarani, Device-Independent Security of Quantum Cryptography Against Collective Attacks, Phys. Rev. Lett. **98**, 230501 (2007).
[4] S. Pironio *et al.*, Random numbers certified by Bell's theorem, Nature (London) **464**, 1021 (2010).
[5] R. Gallego, N. Brunner, C. Hadley, and A. Acín, Device-Independent Tests of Classical and Quantum Dimensions, Phys. Rev. Lett. **105**, 230501 (2010).
[6] D. Mayers and A. Yao, Self testing quantum apparatus, Quantum Inf. Comput. **4**, 273 (2004).
[7] J. S. Bell, On the Einstein-Podolsky-Rosen paradox, Physics **1**, 195 (1964).
[8] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Proposed Experiment to Test Local Hidden-Variable Theories, Phys. Rev. Lett. **23**, 880 (1969).
[9] S. Popescu and D. Rohrlich, Which states violate Bell's inequality maximally?, Phys. Lett. A **169**, 411 (1992).
[10] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, and S. Wehner, Bell nonlocality, Rev. Mod. Phys. **86**, 419 (2014).
[11] E. S. Gómez *et al.*, Device-Independent Certification of a Nonprojective Qubit Measurement, Phys. Rev. Lett. **117**, 260401 (2016).
[12] J. Kaniewski, Self-testing of binary observables based on commutation, Phys. Rev. A **95**, 062323 (2017).
[13] A. Tavakoli, J. Kaniewski, T. Vertesi, D. Rosset, and N. Brunner, Self-testing quantum states and measurements in the prepare-and-measure scenario, arXiv: 1801.08520.
[14] I. Šupić, R. Augusiak, A. Salavrakos, and A. Acín, Self-testing protocols based on the chained Bell inequalities, New J. Phys. **18**, 035013 (2016).
[15] M. McKague and M. Mosca, Generalized self-testing and the security of the 6-state protocol, *Theory of Quantum Computation, Communication, and Cryptography: 5th Conference*, Lecture Notes in Computer Sciences Vol. 6519 (Springer, Berlin, Heidelberg, 2011), pp. 113–130.
[16] Y. C. Liang, D. Rosset, J. D. Bancal, G. Pütz, T. J. Barnea, and N. Gisin, Family of Bell-Like Inequalities as Device-Independent Witnesses for Entanglement Depth, Phys. Rev. Lett. **114**, 190401 (2015).
[17] M. McKague, Self-testing graph states, *Theory of Quantum Computation, Communication, and Cryptography*, Lecture Notes in Computer Science Vol. 6745 (Springer, Berlin Heidelberg, 2014), pp. 104–120.
[18] W. Slofstra, Lower bounds on the entanglement needed to play XOR non-local games, J. Math. Phys. (N.Y.) **52**, 102202 (2011).
[19] A. Salavrakos, R. Augusiak, J. Tura, P. Wittek, A. Acín, and Stefano Pironio, Bell Inequalities Tailored to Maximally Entangled States, Phys. Rev. Lett. **119**, 040402 (2017).

[20] T. H. Yang and M. Navascués, Robust self-testing of unknown quantum systems into any entangled two-qubit states, Phys. Rev. A **87**, 050102 (2013).

[21] C. Bamps and S. Pironio, Sum-of-squares decompositions for a family of Clauser-Horne-Shimony-Holt-like inequalities and their application to self-testing, Phys. Rev. A **91**, 052111 (2015).

[22] A. Coladangelo, K. T. Goh, and V. Scarani, All pure bipartite entangled states can be self-tested, Nat. Commun. **8**, 15485 (2017).

[23] J. Bowles, I. Šupić, D. Cavalcanti, and A. Acín, Device-Independent Entanglement Certification of all Entangled States, Phys. Rev. Lett. **121**, 180503 (2018).

[24] C. E. Bardyn, T. C. H. Liew, S. Massar, M. McKague, and V. Scarani, Device-independent state estimation based on Bell's inequalities, Phys. Rev. A **80**, 062327 (2009).

[25] J.-D. Bancal, M. Navascués, V. Scarani, T. Vértesi, and T. H. Yang, Physical characterization of quantum devices from nonlocal correlations, Phys. Rev. A **91**, 022115 (2015).

[26] J. W. Wang *et al.*, Multidimensional quantum entanglement with large-scale integrated optics, Science **360**, 285 (2018).

[27] J. Kaniewski, Analytic and Nearly Optimal Self-Testing Bounds for the Clauser-Horne-Shimony-Holt and Mermin Inequalities, Phys. Rev. Lett. **117**, 070402 (2016).

[28] D. M. Greenberger, M. A. Horne, and A. Zeilinger, in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, edited by M. Kafatos (Kluwer, Dordrecht, 1989), pp. 69–72.

[29] N. D. Mermin, Extreme Quantum Entanglement in a Superposition Of Macroscopically Distinct State, Phys. Rev. Lett. **65**, 1838 (1990).

[30] A. Fedrizzi, T. Herbst, A. Poppe, T. Jennewein, and A.Zeilinger, A wavelength-tunable fibercoupled source of narrowband entangled photons, Opt. Express **15**, 15377 (2007).

[31] J. M. Chow, J. M. Gambetta, L. Tornberg, J. Koch, L. S. Bishop, A. A. Houck, B. R. Johnson, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf, Randomized Benchmarking and Process Tomography for Gate Errors in a Solid-State Qubit, Phys. Rev. Lett. **102**, 090502 (2009).

[32] See Supplementary Material athttp://link.aps.org/supplemental/10.1103/PhysRevLett.121.240402 for details on the experimental preparation of tested states.

[33] B. Hensen *et al.*, Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres, Nature (London) **526**, 682 (2015).

[34] M. Giustina *et al.*, Bell violation using entangled photons without the fair-sampling assumption, Nature (London) **497**, 227 (2013).