

Quantum Non-Markovian Processes Break Conditional Past-Future Independence

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For classical Markovian stochastic systems, past and future events become statistically independent when conditioned to a given state at the present time. Memory non-Markovian effects break this condition, inducing a nonvanishing conditional past-future correlation. Here, this classical memory indicator is extended to a quantum regime, which provides an operational definition of quantum non-Markovianity based on a minimal set of three time-ordered quantum system measurements and postselection. The detection of memory effects through the measurement scheme is univocally related to departures from Born-Markov and white noise approximations in quantum and classical environments, respectively.

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The definition of Markovianity and non-Markovianity in a quantum regime has changed in time. Given that the physical essence of a classical (memoryless) Markov approximation leads to a local-in-time evolution for a probability density [1], accordingly a quantum Markovian regime was originally associated to local-in-time (nonunitary) density matrix evolutions [2,3]. Hence, approximations that guarantee this property, such as the well-known Born-Markov and white noise approximations [2–5], were related to quantum Markovianity. The underlying assumptions in these approximations are a weak system-environment coupling while the environment fluctuations define the minor timescale of the problem. Consequently, departure from these physical conditions was associated to a quantum non-Markovian regime [3,6].

In the last years the previous paradigm changed drastically. The more general local-in-time evolutions that preserve the density matrix properties (usually known as Lindblad equations) are established by the rigorous theory of quantum dynamical semigroups [7]. General behavioral properties of the system propagator and different quantum information measures can be established in this context. Thus, in the last years quantum non-Markovianity has been defined by departures from these “canonical behaviors” [8,9]. While strong progress have been made on this basis [8–24], some undesirable aspects have emerged. For example, in these novel approaches dynamical departures from a Born-Markov approximation may be included in a Markovian regime. This incongruence is present in almost all proposals. On the other hand, in an incoherent limit, the classical notion of Markovianity may not be recovered. Given that quantum systems are intrinsically perturbed by measurement processes, a lack of an equivalent operational (measurement-based) definition is also usual.

The aim of this work is to introduce an alternative approach to quantum non-Markovianity that surpasses all previous drawbacks, which in turn is consistent with the former (physical) notion of quantum Markovianity. The proposal relies on postselection techniques [25] and retrodicted quantum measurements [26,27], formalisms that allow inferring the state of a quantum system in the past. Thus, the present approach brings an active and fundamental area of research [25–35] into contact with the characterization of memory effects in open quantum system dynamics.

A notable progress in the formulation of quantum memory indicators consistent with classical non-Markovianity was introduced in Ref. [36]. Based on the usual definition of classical Markovianity in terms of conditional probability distributions [1] an operational based “process tensor” formalism defines quantum non-Markovianity. The main theoretical component of the present approach is similar but relies on an alternative and equivalent formulation of classical Markovianity: the statistical independence of past and future system events when conditioned to a given state at the present time [37]. Hence, here a hierarchical set of conditional past-future (CPF) correlations indicate departure from a classical Markovian regime. The quantum extension of this alternative formulation leads to an operational definition of quantum non-Markovianity based on a *minimal* set of three time-ordered successive measurements performed solely on the quantum system. Postselection introduces the conditional character of the quantum measurement scheme. Furthermore, a nonvanishing CPF correlation, which has the same meaning and (average) structure as in a classical regime, becomes a univocal indicator of departures from Born-Markov and white noise approximations in quantum and classical environments respectively. Analytical

solutions of relevant system-environment interaction models support the formalism and conclusions.

Conditional past-future independence.—The observation of a classical stochastic system at three successive times $t_x < t_y < t_z$ yields the outcomes $x \rightarrow y \rightarrow z$ (see Fig. 1). For a Markov process, the joint probability distribution $P(z, y, x)$ of a particular sequence can be written as $P(z, y, x) = P(z|y)P(y|x)P(x)$ [1], where $P(x)$ is the probability of the first event and, in general, $P(b|a)$ is the conditional probability of b given a . From here and the Bayes rule, the conditional probability $P(z, x|y)$ of future (z) and past (x) events *given* the present state y is [37]

$$P(z, x|y) = P(z|y)P(x|y). \quad (1)$$

Thus, for a classical Markovian process past and future events become statistically independent when conditioned to a given (fixed) intermediate state. This property can be corroborated through a conditional past-future correlation, which is defined as

$$C_{\text{pf}} \equiv \langle O_z O_x \rangle_y - \langle O_z \rangle_y \langle O_x \rangle_y, \quad (2)$$

where O is a quantity or property related to each system state [1], $C_{\text{pf}} = \sum_{zx} [P(z, x|y) - P(z|y)P(x|y)] O_z O_x$. In here, indexes x and z run over all possible outcomes occurring at times t_x and t_z , respectively. On the other hand, the y index can be any fixed particular value from all possible outcomes of the second observation. Markovian processes lead to $C_{\text{pf}} = 0$, whatever the conditional state y is. Given that, in general, $P(z, x|y) = P(z|y, x)P(x|y)$, it follows that non-Markovian effects break CPF independence and are present whenever $C_{\text{pf}} \neq 0$. Higher conditional objects are discussed below [Eq. (11)].

Markovianity of quantum measurements.—The previous memory indicator can be extended to a quantum regime. In a first step, it is shown that successive quantum measurement processes fulfill CPF independence. Hence, a completely isolated quantum system is considered, whose own evolution between measurements is disregarded. Three consecutive generalized quantum measurements, which in general are different and arbitrary, deliver the successive random outcomes $x \rightarrow y \rightarrow z$. The corresponding measurement operators [5] are $x \leftrightarrow \Omega_x$, $y \leftrightarrow \Omega_y$, $z \leftrightarrow \Omega_z$ (Fig. 1) and satisfy $\sum_x \Omega_x^\dagger \Omega_x = \sum_y \Omega_y^\dagger \Omega_y = \sum_z \Omega_z^\dagger \Omega_z = I$, where I is the identity matrix and the sum indexes run over all possible outcomes at each stage.

CPF independence entails the calculation of $P(z, x|y) = P(z|y, x)P(x|y)$ [Eq. (2)]. Given that x is in the past of y , $P(x|y)$ is a retrodicted quantum probability. Thus, it can be written in terms of the measurement operator Ω_x and the “past quantum state” $\Xi \equiv (\rho_0, E_y)$, where ρ_0 is the initial density matrix and $E_y \equiv \Omega_y^\dagger \Omega_y$ is the effect operator [27,32]. On the other hand, $P(z|y, x)$ is a standard predictive quantum probability. Hence,

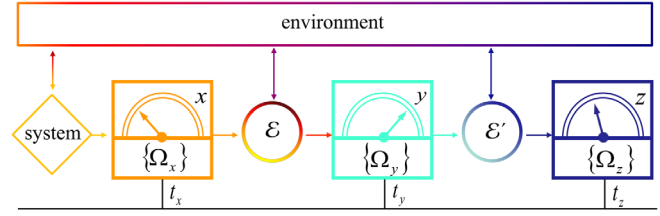


FIG. 1. Measurement scheme. At times $t_x < t_y < t_z$ an open system is subjected to three measurement processes whose random outcomes are $x \rightarrow y \rightarrow z$. A set of operators $\{\Omega_x\}$, $\{\Omega_y\}$, and $\{\Omega_z\}$ define the measurement processes in a quantum regime. \mathcal{E} and \mathcal{E}' are the system propagators between consecutive measurements.

$$P(z, x|y) = \text{Tr}[\Omega_z^\dagger \Omega_z \rho_y] \frac{\text{Tr}[E_y \Omega_x \rho_0 \Omega_x^\dagger]}{\sum_{x'} \text{Tr}[E_y \Omega_{x'} \rho_0 \Omega_{x'}^\dagger]}, \quad (3)$$

where the first and second factors correspond to $P(z|y, x)$ and $P(x|y)$, respectively [38]. Furthermore, $\text{Tr}[\bullet]$ is the trace operation, while ρ_y is the system state after the y measurement. When the y measurement is a *projective* one, $\Omega_y = |y\rangle\langle y|$, being associated to an Hermitian operator $\mathcal{O}_y = \sum_y \mathcal{O}_y |y\rangle\langle y|$, it follows $\rho_y = |y\rangle\langle y|$. This state only depends on the outcome y , while being independent of any former outcome x . Thus, CPF independence is fulfilled naturally [Eq. (1)]. Introducing a “causal break” [36] or “preparation” [39], this property is also valid for non-projective y measurements [$\Omega_y^\dagger \Omega_y \neq \Omega_y$] [38].

Quantum Markovian dynamics.—In general, the system evolves between consecutive measurement events. Its dynamics is defined as Markovian if, for arbitrary measurement processes, it does not break CPF independence. This condition is preserved when the system propagator does not depend on past measurement outcomes. Propagator independence of future outcomes is guaranteed by causality. Hence, from Eq. (3) the CPF probability reads

$$P(z, x|y) = \text{Tr}(\Omega_z^\dagger \Omega_z \mathcal{E}'[\rho_y]) \frac{\text{Tr}(E_y \mathcal{E}[\Omega_x \rho_0 \Omega_x^\dagger])}{\sum_{x'} \text{Tr}(E_y \mathcal{E}[\Omega_{x'} \rho_0 \Omega_{x'}^\dagger])}, \quad (4)$$

where $\mathcal{E} = \mathcal{E}(t_y, t_x)$ and $\mathcal{E}' = \mathcal{E}'(t_z, t_y)$ are the (measurement independent) system density matrix propagators between consecutive events (Fig. 1). The fulfillment of condition (4) provides an explicit measurement-based criteria for defining quantum Markovianity, which similarly to classical systems, leads to a vanishing CPF correlation (2). In particular, a unitary dynamics is Markovian.

Quantum system-environment models.—Consider a system (s) interacting with its environment (e), with total Hamiltonian H_T . The dynamics is set by the propagator

$$\mathcal{E}_t = \exp(t\mathcal{L}_{se}), \quad \mathcal{L}_{se}[\bullet] = -i[H_T, \bullet]. \quad (5)$$

The system density matrix is $\rho_t = \text{Tr}_e(\mathcal{E}_t[\rho_0^{se}])$, where ρ_0^{se} is the initial system-environment state. For measurements that only provide information about system observables, the proposed scheme (Fig. 1) allows us to characterizing departures of the system partial dynamics from a Markovian regime. The probabilities calculus is almost the same as for Eq. (4) [38]. In particular, after the second measurement (y) the bipartite state ρ^{se} suffers the disruptive transformation $\rho^{se} \rightarrow \rho_y \otimes \sigma_e^{yx}$. Thus, the system and the environment become uncorrelated. This property is always granted by projective measurements. The system state ρ_y does not depend on the past measurement outcome x , while the marginal bath state σ_e^{yx} does. It is given by

$$\sigma_e^{yx} = \frac{\text{Tr}_s(E_y \mathcal{E}_t[\Omega_x \rho_0^{se} \Omega_x^\dagger])}{\text{Tr}_{se}(E_y \mathcal{E}_t[\Omega_x \rho_0^{se} \Omega_x^\dagger])}. \quad (6)$$

The CPF probability, similarly to Eq. (4), is [38]

$$P(z, x|y) = \text{Tr}_{se}(\Omega_z^\dagger \Omega_z \mathcal{E}_\tau[\rho_y \otimes \sigma_e^{yx}]) \times \frac{\text{Tr}_{se}(E_y \mathcal{E}_t[\Omega_x \rho_0^{se} \Omega_x^\dagger])}{\sum_{x'} \text{Tr}_{se}(E_y \mathcal{E}_t[\Omega_{x'} \rho_0^{se} \Omega_{x'}^\dagger])}, \quad (7)$$

where $t \equiv t_y - t_x$ and $\tau \equiv t_z - t_y$ are the time intervals between consecutive measurements. Given the dependence of the environment state σ_e^{yx} on the first measurement (x), here CPF independence is broken in general. The properties of this departure can be quantified with the CPF correlation (2), $C_{\text{pf}} \rightarrow C_{\text{pf}}(t, \tau)$, which can be obtained from the previous expression and the system observables definition.

Born-Markov approximation.—A Markovian regime, defined by the measurement-based condition (4), is approached when the initial bipartite state is separable, $\rho_0^{se} = \rho_0 \otimes \sigma_e$, and for arbitrary time t ,

$$\mathcal{E}_t[\Omega_x \rho_0^{se} \Omega_x^\dagger] \simeq \tilde{\rho}_x(t) \otimes \sigma_e, \quad (8)$$

where $\tilde{\rho}_x(0) = \Omega_x \rho_0 \Omega_x^\dagger$. Indeed, under this approximation the bath state is (approximately) unperturbed during the total evolution, $\sigma_e^{yx} \simeq \sigma_e$ [see Eq. (6)], implying $C_{\text{pf}}(t, \tau) \simeq 0$. Therefore, the CPF correlation measures and quantifies departures with respect to the standard Born-Markov approximation. In fact, the separability constraint (8) is valid when the conditions under which it applies are fulfilled [4].

Classical environment fluctuations.—Instead of a unitary bipartite evolution [Eq. (5)], the open system dynamics may be described by a quantum Liouville operator $\mathcal{L}_{st}(t)$ modulated by classical noise fluctuations,

$$\frac{d}{dt} \rho_t^{\text{st}} = -i \mathcal{L}_{st}(t) [\rho_t^{\text{st}}]. \quad (9)$$

The system density operator $\rho_t = \overline{\rho_t^{\text{st}}}$ follows by averaging over realizations of $\mathcal{L}_{st}(t)$ [overbar symbol]. The CPF probability can straightforwardly be written as

$$P(z, x|y) = \overline{P_{\text{st}}(z, x|y)}, \quad (10)$$

where the classical average is restricted to the y outcome and the “stochastic probability” $P_{\text{st}}(z, x|y)$ follows from Eq. (4) under the replacements $\mathcal{E} \rightarrow \exp[-i \int_0^t dt' \mathcal{L}_{st}(t')]$ and $\mathcal{E}' \rightarrow \exp[-i \int_t^{t+\tau} dt' \mathcal{L}_{st}(t')]$. Non-Markovian effects are then related to the correlation between both intermediate propagators, while white fluctuations lead to a Markovian dynamics [38]. The model (9) not only covers the case of stochastic Hamiltonian evolutions [40] but also quantum-classical hybrid arrangements [27,33] where, in general, the incoherent and quantum systems may affect each other [41].

CPF correlation properties.—Similarly to classical systems, a non-Markovian regime is defined by the condition $C_{\text{pf}}(t, \tau) \geq 0$. In general $C_{\text{pf}}(t, \tau) \neq C_{\text{pf}}(\tau, t)$. From Eq. (6) [and Eq. (10)] it follows $\lim_{\tau \rightarrow 0} C_{\text{pf}}(t, \tau) = 0$ and $\lim_{t \rightarrow 0} C_{\text{pf}}(t, \tau) = 0$, this last condition being only valid when the system and the environment are uncorrelated at the initial time. If the environment fluctuations have a finite correlation time τ_c [3], $C_{\text{pf}}(t, \tau) \simeq 0$ if $t \gg \tau_c$ or $\tau \gg \tau_c$. Thus, $\lim_{\tau \rightarrow \infty} C_{\text{pf}}(t, \tau) = \lim_{t \rightarrow \infty} C_{\text{pf}}(t, \tau) = \lim_{t \rightarrow \infty} C_{\text{pf}}(t, ct) = 0$, $\forall c > 0$. In an experimental setup $C_{\text{pf}}(t, \tau)$ follows straightforwardly by performing a statistical average with a *postselected* subensemble of realizations $x \rightarrow y' \rightarrow z$, where y' is the chosen conditional fixed value. Contrarily to classical systems, the condition $C_{\text{pf}}(t, \tau) \neq 0$ may depends on the chosen measurement observables. This richer behavior in turn gives a deeper characterization of memory effects in quantum systems.

Higher order CPF correlations.—The CPF correlation (2) can be generalized by increasing the number of observations, $x \rightarrow y_1 \rightarrow y_2 \rightarrow \dots \rightarrow y_n \rightarrow z$. An n -order CPF correlation is defined as

$$C_{\text{pf}}^{(n)} = \sum_{zx} [P(z, x|y) - P(z|y)P(x|y)] O_z O_x, \quad (11)$$

where $\mathbf{y} \equiv y_n, \dots, y_2, y_1$. Given that $P(z, x|y_n \dots y_1) = P(z|y_n \dots y_1, x)P(x|y_n \dots y_1)$, $C_{\text{pf}}^{(n)}$ is sensitive to memory effects that may only appear in these higher conditional objects. For example, it may happen that $C_{\text{pf}}^{(k)} = 0 \forall k < n$ and $C_{\text{pf}}^{(n)} \neq 0$. Classical Markovian processes fulfill $C_{\text{pf}}^{(n)} = 0 \forall n$. Thus, higher order CPF correlations provide an overall check of the definition of classical Markovianity in terms of conditional probabilities. This property guarantees the consistency of the present formalism with Ref. [36]. In fact, $C_{\text{pf}}^{(n)}$ can also be extended and calculated

in a quantum regime [38] (previous expressions correspond to $n = 1$). Nevertheless, in contrast to classical systems, given the degrees of freedom provided by the measurement operators, for a wide class of quantum dynamics [Eqs. (5) and (9)] it is expected that $C_{\text{pf}}^{(1)} \neq 0$ [38]. Thus, memory effects can be analyzed over the basis of a minimal three quantum-measurements scheme (Fig. 1). The next results support this conclusion.

Dephasing spin bath.—As a first example we consider a paradigmatic model of decoherence [42–44]: a qubit system interacting with an N -spin bath via the microscopic interaction Hamiltonian

$$H_T = \sigma_z \otimes \sum_{k=1}^N g_k \sigma_z^{(k)}. \quad (12)$$

Here, σ_z is the system Pauli matrix in the \hat{z} direction (Bloch sphere) [45], whose eigenvectors are denoted as $|\pm\rangle$. On the other hand, $\sigma_z^{(k)}$ is \hat{z} -Pauli matrix corresponding to the k spin. Its eigenvectors are denoted by $|\uparrow\rangle_k$ and $|\downarrow\rangle_k$. $\{g_k\}$ are real coupling constants.

As is well known, the model (12) admits an exact solution [42–44]. Assuming a separable pure initial condition $\rho_0^{se} = |\Psi_0^{se}\rangle\langle\Psi_0^{se}|$, where

$$|\Psi_0^{se}\rangle = (a|+\rangle + b|-\rangle) \otimes \sum_{k=1}^N (\alpha_k |\uparrow\rangle_k + \beta_k |\downarrow\rangle_k), \quad (13)$$

the system density matrix reads $\rho_t = |a|^2 |+\rangle\langle+| + |b|^2 |-\rangle\langle-| + ab^* c_t |+\rangle\langle-| + a^* b c_t^* |-\rangle\langle+|$. Its evolution can then be written as

$$\frac{d\rho_t}{dt} = \frac{-i}{2} \omega(t) [\sigma_z, \rho_t] + \gamma(t) \frac{1}{2} (\sigma_z \rho_t \sigma_z - \rho_t), \quad (14)$$

where the time dependent frequency $\omega(t)$ and decay rate $\gamma(t)$ follow from $\gamma(t) + i\omega(t) = -(1/c_t)(d/dt)c_t$. The system coherence behavior,

$$c_t = \prod_{k=1}^N (|\alpha_k|^2 e^{+i2g_k t} + |\beta_k|^2 e^{-i2g_k t}), \quad (15)$$

depends on the initial bath state and coupling constants.

Measurement scheme and CPF correlation.—In order to check non-Markovian effects, the three measurements (Fig. 1) are chosen as projective ones, being performed in \hat{x} direction. The outcomes of each measurement are then $x = \pm 1$, $y = \pm 1$, $z = \pm 1$, which in turn define the system operators values in Eq. (2), $O_z = z$ and $O_x = x$. The measurement operators are $\{\Omega_x\} = \{\Omega_y\} = \{\Omega_z\} = |\hat{x}_{\pm}\rangle\langle\hat{x}_{\pm}|$, where $|\hat{x}_{\pm}\rangle = (|+\rangle \pm |-\rangle)/\sqrt{2}$.

All calculations leading to the CPF probability (7) can be performed in an exact way [46]. Assuming, for simplicity, that the system begin in the state $|+\rangle$ ($a = 1, b = 0$),

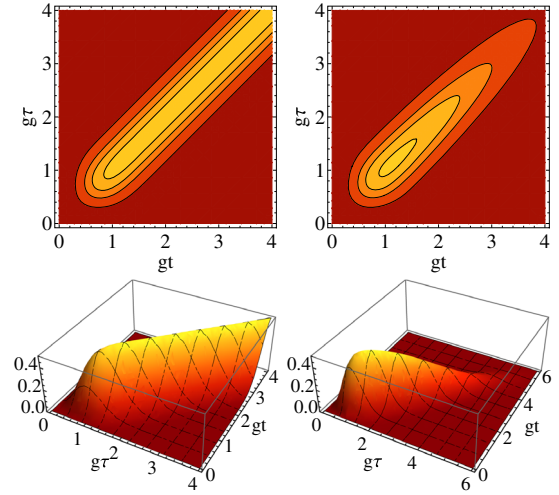


FIG. 2. Left panels: CPF correlation (17) for the spin bath model (12), with coupling $g_k = g/\sqrt{N}$. The parameters of the initial condition (13) are $a = 1$, $b = 0$, $\alpha_k = \beta_k = 1/2$, and $N = 50$. Right panels: CPF correlation for the stochastic Hamiltonian model (19), with noise correlation $\overline{\xi_t \xi_{t'}} = g^2 \exp[-|t - t'|/\tau_c]$. The system starts at the same initial condition. The parameters are $\tau_c g = 100$. For $\tau_c \rightarrow \infty$, the left panels are recovered.

$$P(zx|y) = \frac{1}{4} [1 + xyf(t) + zyf(\tau) + zxf(t, \tau)], \quad (16)$$

where $f(t) = \text{Re}[c_t]$ gives the coherence decay and $f(t, \tau) = [f(t + \tau) + f(t - \tau)]/2$. From here, it follows $\langle O_z \rangle_y = yf(\tau)$, $\langle O_x \rangle_y = yf(t)$, and $\langle O_z O_x \rangle_y = f(t, \tau)$. The exact expression for the CPF correlation (2) then is

$$C_{\text{pf}}(t, \tau) = f(t, \tau) - f(t)f(\tau), \quad (17)$$

which, due to the symmetries, here is independent of the conditional value $y = \pm 1$.

A non-Markovian quantum dynamical semigroup.—As is well known [43,44], the model (12) may lead to Gaussian system decay behaviors. For example, taking $g_k = (1/\sqrt{N})g$, $\alpha_k = \beta_k = 1/2$, for $N \gg 1$ it follows $c_t \simeq \exp[-2(gt)^2]$ (behavior valid before the unitary recurrence time). Thus, $\omega(t) = 0$ and $\gamma(t) \simeq 4g^2 t$. This positive time-dependent rate leads to a time-dependent Lindblad semigroup [Eq. (14)] that in almost all proposed non-Markovian measure schemes [8,9] is classified as a Markovian evolution. In contrast, here due to strong departures from condition (8), the CPF correlation does not vanish. In fact, for $N \gg 1$, it can be approximated as

$$C_{\text{pf}}(t, \tau) \simeq \frac{e^{-2g^2(t+\tau)^2} + e^{-2g^2(t-\tau)^2}}{2} - e^{-2g^2(t^2+\tau^2)}. \quad (18)$$

In Fig. 2 (left panels) we plot $C_{\text{pf}}(t, \tau)$, which is very well fitted by the previous expression. The symmetry

$C_{\text{pf}}(t, \tau) = C_{\text{pf}}(\tau, t)$ is a consequence of the chosen environment initial conditions. Furthermore, for increasing equal time intervals $C_{\text{pf}}(t, t) \simeq 1/2$. This property indicates that the bath correlation does not decay (vanishes) in time (infinite bath correlation time).

Stochastic Hamiltonian.—An alternative decoherence model, which mimics the interaction with a spin bath [47], is given by a stochastic Hamiltonian evolution

$$\mathcal{L}_{st}(t)[\bullet] = -i\xi_t[\sigma_z, \bullet], \quad (19)$$

where ξ_t is a classical noise [Eq. (9)]. The density matrix evolution is also defined by Eq. (14), where now

$$c_t = \exp\left(-2i \int_0^t dt' \xi(t')\right). \quad (20)$$

The CPF probability (10) can also be calculated in an exact way [46]. It can be written as in Eq. (16), where similarly $f(t) = \text{Re}[c_t]$ [Eq. (20)] while $f(t, \tau) = \text{Re}(\exp[-2i \int_0^t dt' \xi(t')])\text{Re}(\exp[-2i \int_t^{t+\tau} dt' \xi(t')])$.

Taking a Gaussian noise with $\xi_t = 0$ and correlation $\overline{\xi_t \xi_{t'}} = g^2 \exp[-|t - t'|/\tau_c]$, Eq. (14) is defined with $\omega(t) = 0$ and $\gamma(t) = 4g^2 \tau_c (1 - e^{-t/\tau_c}) > 0$, providing a second example of a non-Markovian time-dependent quantum semigroup. In particular, in the limit $\tau_c \rightarrow \infty$, the same Gaussian behavior is recovered, $c_t = \exp[-2(gt)^2]$. Thus, the CPF correlation is exactly given by Eq. (18) [left panels in Fig. 2]. On the other hand, taking $\gamma_w/2 = g^2 \tau_c$ as a constant parameter, in the limit $\tau_c \rightarrow 0$, a Markovian regime is achieved, $C_{\text{pf}}(t, \tau) \rightarrow 0$, with $c_t = \exp[-2\gamma_w t]$. In Fig. 2 (right panels), we also plot $C_{\text{pf}}(t, \tau)$ for a finite τ_c . All expected characteristics corresponding to a finite bath correlation time are developed.

Conclusions.—Similarly to classical systems, a quantum (memoryless) Markovian regime was defined by the statistical independence of past and future events when conditioned to a present system state. Thus, a minimal set of three time-ordered quantum measurements leads to an operational (measurement-based) definition of quantum non-Markovianity. Postselection gives the conditional character of the measurement scheme. Its associated CPF correlation is a direct and univocal indicator of departures from Born-Markov and white noise approximations.

The proposed scheme leads to a powerful theoretical and experimental basis for the study of memory effects in open quantum systems. Its capacity for characterizing the underlying physical origin of memory effects was established by studying different dephasing mechanisms that admit an exact treatment. The conditional character of the measurement scheme opens an interesting way to describe quantum memory effects by means of recent theoretical and experimental advances in retrodicted quantum measurement processes [25–35].

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