Trigonometric Parity for Composite Higgs Models

Csaba Csáki,[1] Teng Ma,[2] and Jing Shu[2,3,4,5,*]
[1]*Department of Physics, LEPP, Cornell University, Ithaca, New York 14853, USA*
[2]*CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China*
[3]*School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China*
[4]*CAS Center for Excellence in Particle Physics, Beijing 100049, China*
[5]*Center for High Energy Physics, Peking University, Beijing 100871, China*

(Received 5 April 2018; revised manuscript received 19 August 2018; published 3 December 2018)

We identify trigonometric parity as the key ingredient behind models of neutral naturalness for the Higgs potential and show how to construct the minimal model realizing trigonometric parity. We show that any symmetric coset space readily includes such a trigonometric parity, which is simply a combination of a $\pi/2$ rotation along a broken direction and a Higgs parity transformation. The top sector can be extended such that this Z_2 remains intact, which ensures the cancelation of the quadratic divergences in the Higgs potential, yielding the simplest model of neutral naturalness.

DOI: 10.1103/PhysRevLett.121.231801

The possibility that the Higgs boson is not an elementary particle but a composite [1–7] remains one of the most fascinating explanations for the lightness of the Higgs boson. This can happen in models where the Higgs boson (in addition to being composite) was also a pseudo-Nambu-Goldstone boson (PNGB) of a global symmetry spontaneously broken at energy scale f, described by the coset G/H. The lightness of the Higgs boson also has a profound impact on the expected spectrum of beyond the standard model (BSM) particles. Natural models of EWSB will predict the existence of light top partners that cancel the bulk of the corrections to the Higgs potential. These can either be scalar top partners as in supersymmetric models, or fermionic top partners in composite Higgs models [7–12]. The fermionic top partners in turn will produce the "smoking gun" signals used for the LHC searches for colored top partners [13–17]. However, with the accumulated integrated luminosity already surpassing 60 fb^{-1}, our current bound on colored top partners is pushed up to 1–1.5 TeV. The ever increasing top partner bounds may make one wonder whether there are options where the nice features of composite Higgs models are maintained without the existence of colored top partners.

Twin Higgs (TH) models present another interesting direction for stabilizing the Higgs potential [18–22]. In this scenario an additional Z_2 discrete symmetry is responsible for the cancellation of the quadratic divergences. In TH models the Higgs boson is also identified as a PNGB, and the Z_2 symmetry manifests itself via the $s_h \leftrightarrow c_h$ exchange symmetry in the Higgs potential [18,21,23–25]. This Z_2 is very efficient at softening the Higgs potential and eliminating most of the sources for tuning: in addition to canceling the quadratic divergences, it also eliminates the so called double tuning leading to Higgs potentials with minimal tuning. Furthermore this Z_2 relates the top to the twin top, which is $\mathrm{SU}(3)_c$ color neutral, thus also evading the bounds from direct top partner searches [26–28]. While the TH framework is very attractive, the concrete models are not: the minimal SO(8)/SO(7) coset space is very large, leading to complicated models with very large representations. In these models the origin of the Z_2 exchange symmetry is not immediately obvious either.

In this Letter, we point out that the origin of the Z_2 symmetry responsible for the cancelation of quadratic divergences in composite Higgs models with color neutral top partners can be traced back to a simple and very generic discrete symmetry of the internal manifold describing the coset space. We argue that for any symmetric coset space the $s_h \leftrightarrow c_h$ exchange symmetry (which appears e.g., in twin Higgs models [29]) naturally emerges as a combination of Higgs parity with a $\pi/2$ rotation in the broken direction corresponding to the physical Higgs boson. We will show how to extend this "trigonometric Z_2 symmetry" such that it remains intact after the introduction of the top Yukawa couplings, which will provide a natural origin for the appearance of the color neutral top partners. The trigonometric Z_2 symmetry will relate the top and color neutral top partners to each other. For the gauge sector we will not assume a twin mechanism: indeed, in all TH models, the Z_2 twin parity is softly broken. We find the minimal model with custodial symmetry based the SO(6)/SO(5) coset [31,32], or, equivalently, the SU(4)/Sp(4) [33–36] coset, which has a simple UV completion from fermion condensation. If our procedure was applied to the SU(3)/SU(2) coset instead, we would obtain a model similar to Ref. [20], which was based on an extra dimensional construction. Finally, we also present some striking collider signatures, including six top final states.

First we present our essential new observation: a Z_2 symmetry useful for building TH-type models is readily present for every Goldstone boson as long as a Higgs parity V is maintained by the coset space. Such a Higgs parity automatically emerges for so-called symmetric coset spaces (which include most of the commonly used examples). The reason for the appearance of such a Z_2 symmetry is quite simple: whenever we have a broken symmetry, there is a shift symmetry on the corresponding pion π^i of the form $\pi^i/f \to \pi^i/f + \epsilon^i$. The effect of the Higgs parity is to simply reverse the sign of the pion $\pi^i \to -\pi^i$. Thus combining a $\pi/2$ rotation in the broken direction with Higgs parity will have the effect

$$\frac{\pi^i}{f} \to -\frac{\pi^i}{f} + \frac{\pi}{2}, \tag{1}$$

which, on the trigonometric functions, is equivalent to

$$\sin\frac{\pi^i}{f} \leftrightarrow \cos\frac{\pi^i}{f}. \tag{2}$$

We call this the trigonometric Z_2 symmetry which is exactly the type of exchange symmetry one needs for the TH models [18] to cancel the quadratic divergences and also further reduce the tuning of the Higgs potential. It is automatically contained in every symmetric coset space, e.g., $\mathrm{SO}(N+M)/\mathrm{SO}(N) \times \mathrm{SO}(M)$ and $\mathrm{SU}(N+M)/\mathrm{SU}(N) \times \mathrm{SU}(M) \times \mathrm{U}(1)$. Whether this symmetry will actually be realized on the Higgs potential will then depend on the structure of the explicit breaking terms. The task is to design the explicit breaking terms such that they break the general shift symmetry (in order to allow the generation of a Higgs potential) but maintain the Z_2 discrete subgroup of the shift symmetry identified above. Once this is achieved the generated Higgs potential will be automatically exchange symmetric.

As a simple and realistic illustration we present the top sector of the SO(6)/SO(5) coset space [37]. Another illustration based on the SU(3)/SU(2) coset leading to the model of Ref. [20] is provided in the Supplemental Material [39]. We will discuss the details of the gauge sector of the model later, for now all we need is that the SO(4) containing the $\mathrm{SU}(2)_L$ electroweak gauge group and $\mathrm{SU}(2)_R$ custodial symmetry of the SM are embedded in the first four components of the SO(6) and the vacuum expectation value breaking the global symmetry is chosen as $\mathcal{V} = (0,0,0,0,0,1)^T$. In this case, the PNGB matrix U corresponding to the physical Higgs boson will be given by

$$\mathrm{U} = \begin{pmatrix} \mathbb{1}_3 & & & \\ & c_h & & s_h \\ & & 1 & \\ & -s_h & & c_h \end{pmatrix}, \tag{3}$$

where $s_h = \sin(h/f)$ and $c_h = \cos(h/f)$. We can clearly see that the fourth and sixth rows and columns correspond to an $SO(2)$ rotation by angle h/f. As discussed above, the shift symmetry for the Higgs is exactly this (broken) SO(2) rotation. The explicit expression for the Z_2 trigonometric parity acting on the Higgs matrix [obtained by the combination of the SO(2) rotation by angle $\pi/2$ with the Higgs parity transformation $V = \mathrm{diag}(1,1,1,1,1,-1)$] is

$$P_1^h = P_{\pi/2} V = \begin{bmatrix} \mathbb{1}_3 & & & \\ & & & -1 \\ & & 1 & \\ & -1 & & \end{bmatrix}, \tag{4}$$

where $P_{\pi/2}$ is the SO(2) rotation by angle $\pi/2$ in the Higgs direction. The field U transforms as $\mathrm{U} \to P_1^h U V^\dagger$.

Let us now consider the Yukawa couplings of the fermions. We embed the third generation SM quark doublet Q_L in the fundamental representation of SO(6) while the right-handed top t_R is assumed to be an SO(6) singlet. The explicit expression for Q_L using the standard embedding is $\Psi_{Q_L} = (b_L, -ib_L, t_L, it_L, 0, 0)^T/\sqrt{2}$.

The top Yukawa coupling will be of the form

$$y_t \bar{\Psi}_{Q_L} \Sigma t_R + \mathrm{H.c.}, \tag{5}$$

where $\Sigma = U\mathcal{V}$ is the linearly realized Sigma field and transforms as $\Sigma \to g\Sigma$ for any $g \in SO(6)$. Since the Higgs is composite there will be additional form factors showing up in Eq. (5) that however play no role in the following argument, thus for simplicity we will suppress them for now. To extend the Z_2 trigonometric parity to the Yukawa couplings we must introduce the twin tops $\tilde{t}_{L,R}$ and an appropriate extension of the Z_2 parity involving the exchange of the ordinary and the twin tops. Due to the form of the embedding of Q_L into Ψ_{Q_L} we can see that the twin top also needs to be embedded into multiple components of the SO(6) vector. This is the underlying reason why SO(6) is the smallest global symmetry where the trigonometric Z_2 can be implemented. Since the action of the parity on the Higgs field involves exchanging the fourth and sixth components, we embed the left handed twin top into the sixth component of an SO(6) vector. However, the embedding of the ordinary top contains t_L twice, so we expect that the proper embedding of the twin top into an SO(6) vector will also contain $\tilde{t}$ twice. Thus we conclude that, in order to realize the exchange symmetry between t and $\tilde{t}$ (which contains the P_1^h operation), the embedding for $\tilde{t}_L$ must be

$$\Psi_{\tilde{t}_L} = (0,0,0,0,\tilde{t}_L, i\tilde{t}_L)^T/\sqrt{2}, \tag{6}$$

while $\tilde{t}_R$ is also a singlet under SO(6). We note that since we do not assume the existence of a twin $SU(2)_L$ gauge symmetry, $\tilde{t}_L$ and $\tilde{b}_L$ do not have to be in the same multiplet. We can now extend the Yukawa sector to include the twin top Yukawa coupling as well:

$$y_t \bar{\Psi}_{Q_L} \Sigma t_R + \tilde{y}_t \bar{\Psi}_{\tilde{t}_L} \Sigma \tilde{t}_R + \text{H.c.}. \tag{7}$$

If $y_t = \tilde{y}_t$ this Lagrangian will be invariant under the trigonometric parity

$$\Psi_{Q_L} \leftrightarrow P\Psi_{\tilde{t}_L}, \qquad t_R \leftrightarrow \tilde{t}_R, \qquad \Sigma \to P\Sigma \tag{8}$$

where P is the parity operator implementing the exchange of t_L and $\tilde{t}_L$

$$P = -P_0 P_1^h = \begin{pmatrix} -1 & & & & & \\ & -1 & & & & \\ & & & & 1 & \\ & & & & & 1 \\ & & 1 & & & \\ & & & 1 & & \end{pmatrix}. \tag{9}$$

In the above decomposition, P_0 is the operator exchanging the third and fifth components (and which acts trivially on the Higgs PNGB matrix) and also $[P_0, P_1^h] = 0$. Since this is a symmetry of the Lagrangian, the Higgs potential generated by these interactions must also be invariant under the trigonometric parity P. The action of P on the Higgs sector is $s_h \leftrightarrow c_h$, hence the Higgs potential will necessarily be invariant under this exchange symmetry.

The complete discussion of the fermion sector requires to also include the form factors capturing the effects of compositeness. Based on the $SO(6)$ symmetry transformation properties of the fields Ψ_{Q_L}, $\Psi_{\tilde{t}_L} \to g\Psi_{Q_L}$, $g\Psi_{\tilde{t}_L}$ and $\Sigma \to g\Sigma$, the general low-energy effective Lagrangian of the top—twin top—Higgs sector after integrating out the heavy fields must be of the form [40] [where we used $(\Sigma\Sigma^\dagger)^2 = 1$ to truncate the series in Σ]:

$$\begin{aligned}\mathcal{L} = {} & \bar{\Psi}_{Q_L} \not{p} (\Pi_0^q(p) + \Pi_1^q(p)\Sigma\Sigma^\dagger)\Psi_{Q_L} + \bar{t}_R \not{p} \Pi_0^t(p) t_R \\ & + M_1^t(p)\bar{\Psi}_{Q_L}\Sigma t_R + \bar{\Psi}_{\tilde{t}_L} \not{p} (\tilde{\Pi}_0^q(p) + \tilde{\Pi}_1^q(p)\Sigma\Sigma^\dagger)\Psi_{\tilde{t}_L} \\ & + \bar{\tilde{t}}_R \not{p} \tilde{\Pi}_0^t(p)\tilde{t}_R + \tilde{M}_1^t(p)\bar{\Psi}_{\tilde{t}_L}\Sigma\tilde{t}_R, \end{aligned} \tag{10}$$

where $\Pi_{0,1}^q(\tilde{\Pi}_{0,1}^q)$, $\Pi_0^t(\tilde{\Pi}_0^t)$, and $M_1^t(\tilde{M}_1^t)$ are the form factors encoding the effect of the strong dynamics. We can see that there is an additional requirement for the Z_2 exchange symmetry: the form factors in the visible and twin sectors should be equal:

$$\begin{aligned} &\Pi_{0,1}^q(p) = \tilde{\Pi}_{0,1}^q(p), \qquad \Pi_0^t(p) = \tilde{\Pi}_0^t(p), \\ &M_1^t(p) = \tilde{M}_1^t(p), \end{aligned} \tag{11}$$

which is expressing the requirement that the structure of the underlying strong dynamics should also be Z_2 symmetric. We will require in addition the condition that QCD and mirror QCD should be Z_2 symmetric

$$SU(3)_c \leftrightarrow SU(3)'_c, \tag{12}$$

otherwise QCD running effects will be different in the visible and the twin sectors, which could lead to significant (two-loop) corrections to the Higgs mass. Once the form factor relations Eq. (11) are satisfied, the effective Lagrangian has a global $\mathrm{SO}(6) \times \mathrm{SU}(6)$ invariance where QCD and twin QCD are contained in the SU(6): $\mathrm{SU}(3)_c \times \mathrm{SU}(3)'_c \subset \mathrm{SU}(6)$ and the top doublet Q_L (singlet t_R) together with its twin partner are embedded in $(\mathbf{6},\mathbf{6})$ $[(\mathbf{1},\mathbf{6})]$ of $\mathrm{SO}(6) \times \mathrm{SU}(6)$. The Z_2 symmetry of this effective Lagrangian can be easily confirmed if it is written in terms of SM quarks and hidden fermion $\tilde{t}$.

If the quadratic divergence is proportional to $s_h^2 + c_h^2$, then it will be independent of the Higgs field and the quadratic divergences are eliminated. However in principle it could also be proportional to $s_h^4 + c_h^4$, which is still exchange symmetric but would remain quadratically divergent. Which of these situations we encounter will depend on the representations chosen for the embedding for the top and twin tops. The divergent contributions in the Higgs potential can depend only on the form factors $\Pi_1^q(\tilde{\Pi}_1^q)$ since partial compositeness implies that the form factors $M_1^t(\tilde{M}_1^t)$ are $\propto(1/\Lambda)$. For a simple representation like Eq. (10), there will only be a $\Sigma\Sigma^\dagger$ insertion, which depends at most on s_h^2 and c_h^2. We conclude that exchange symmetry in addition with choosing simple group representations will be sufficient for eliminating the quadratic divergences from the Higgs potential generated by the top sector.

We can now complete the SO(6)/SO(5) model by presenting the gauge (Goldstone) sector. One important point to emphasize is that the price of choosing the minimal coset SO(6)/SO(5) suitable for implementing the Z_2 symmetry in the top sector is that the gauge sector will not be Z_2 symmetric—one can see that clearly from the embedding of the twin top into $\Psi_{\tilde{t}_L}$ in Eq. (6). As a consequence the gauge contribution to the Higgs potential will be significant. Besides being minimal another important advantage of the SO(6)/SO(5) coset is that it is automorphic to SU(4)/Sp(4), which has a simple fermionic UV completion. This UV completion can render the Higgs potential from the gauge sector finite and small, which makes it one of the more desirable models of neutral naturalness.

The SO(6)/SO(5) coset corresponds to five NGBs parametrized by h_i and η with $i = 1, 2, 3, 4$. The SO(4) of the first four components of the SO(6) corresponds to $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ of the SM electroweak group. The four NGBs h_i form a quartet of the custodial symmetry SO(4), identified as the Higgs doublet, while η is a singlet of custodial symmetry. As for the SO(5)/SO(4) minimal composite Higgs model, we gauge $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y \subset \mathrm{SU}(2)_R$ to provide the electroweak gauge symmetries, and, in addition, we also gauge the $\mathrm{SO}(2)_\eta$ subgroup, corresponding to the rotations of the last two components of an SO(6) vector. This $\mathrm{SO}(2)_\eta$ is the broken direction providing the additional singlet Goldstone η. Since we gauge this direction, the η will be eaten by the corresponding massive gauge boson. In unitary gauge, only the physical Higgs boson h remains, which is described by the nonlinear Sigma field U in Eq. (3). Note that the quantum numbers of the five Goldstones are identical to those in the quirky little Higgs model of Ref. [20] based on SU(3)/SU(2), resulting in a very similar phenomenology in the light scalar sector. However, the gauge boson and fermion sectors are quite different due to the different global symmetries.

The gauge interaction of the PNGB fields is most conveniently written in terms of the Σ field and the leading Goldstone Lagrangian is then given by $\mathcal{L} = f^2(D_\mu\Sigma)^T D^\mu\Sigma/2$, where $D_\mu = \partial_\mu - igW_\mu^a T_L^a - ig' B_\mu T_R^3 - ig_1 B'_\mu T_\eta$. After electroweak symmetry breaking, $\langle h\rangle \neq 0$, the masses of SM and hidden gauge bosons are

$$m_W^2 = \frac{g^2 f^2 \xi}{4}, \qquad m_Z^2 = \frac{m_W^2}{\cos^2\theta_W}, \qquad m_{B'}^2 = \frac{g_1^2 f^2(1-\xi)}{2}, \tag{13}$$

where θ_W is the usual weak mixing angle and $\xi \equiv s_h^2$.

One of the main advantages of the minimal model presented above is that it has a simple UV completion. This is based on the fact that locally the cosets SO(6)/SO(5) and SU(4)/Sp(4) are isomorphic and the SU(4)/Sp(4) coset can be realized via fermion condensation in a UV complete hypercolor theory. Here we briefly sketch the essential elements of this UV completion, using the SU(4)/Sp(4) language. In order to realize the SU(4)/Sp(4) breaking pattern, we introduce four Weyl fermions ψ_i with $i = 1$, 2, 3, 4 [34,35]. These preons will transform in the fundamental representation of the hypercolor gauge group Sp(2N) [or alternatively could also be in the spinor representation of a different hypercolor gauge group $SO(2N+1)$] [36]. In this Letter, we only focus the Sp(2N) case. The electroweak gauge symmetries as well as the extra $\mathrm{U}(1)_\eta \cong \mathrm{SO}(2)_\eta$ are embedded in the global symmetry in the following way: the fermions (ψ_1, ψ_2) are arranged into an $\mathrm{SU}(2)_L$ doublet while the other two fermions, ψ_3 and ψ_4, are $\mathrm{SU}(2)_R$ doublets and

TABLE I. The quantum number of the Weyl fermion preons under gauge symmetries $\mathrm{Sp}(2N) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{SU}(3)_c \times \mathrm{U}(1)_\eta$. Square boxes indicate that the Weyl fermion preons are in the fundamental representation.

	Sp(2N)	$\mathrm{SU}(2)_L$	$U(1)_Y$	$\mathrm{SU}(3)_c$	$U(1)_\eta$
(ψ_1, ψ_2)	□	□	0	1	1
ψ_3	□	1	−1/2	1	−1
ψ_4	□	1	1/2	1	−1

these two doublets have equal and opposite $\mathrm{U}(1)_\eta$ charges which are listed in Table I.

Thus if the Sp(2N) hypercolor group confines and the fermionic preons condense $\langle\psi_i\psi_j\rangle \neq 0$, similar to the QCD quark condensates, the SU(4) global symmetry will be broken to its Sp(4) subgroup, producing five NGBs. If the vacuum has the form

$$V = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}, \tag{14}$$

the electroweak symmetries will be left unbroken by the preon condensates [while $U(1)_\eta$ will be one of the broken directions]. For simplicity, we assume that the preons ψ_i are all massless, so there are no additional contributions to the Higgs potential. One can also introduce colored (twin) preons, either scalars or fermions, to give a UV complete implementation of partial compositeness (along with the corresponding twin sectors). The details of those models are beyond the scope of this current Letter, but will be presented in a future publication.

Let us now discuss the structure of the Higgs potential. The contributions from the gauge and top sectors can be parametrized as

$$V = -\gamma s_h^2 + \beta s_h^4, \tag{15}$$

where $\gamma = \gamma_f - \gamma_g$, $\beta = \gamma_f$, and γ_g and γ_f are the contributions from the gauge and fermion sectors. Similar to Ref. [40], the potential from the fermion sector has a vacuum at $\xi = 0.5$ due to the Z_2 symmetry. In order to reduce ξ to experimentally allowed values $\xi \ll 1$, the contribution from the gauge sector must be included and a cancellation between gauge and fermionic contributions in the s_h^2 term $\gamma_f \approx \gamma_g$ must be imposed.

To estimate the gauge contributions, one can find a low-energy effective action for the gauge sector analogous to Eq. (10). It turns out that in the gauge sector there is a single form factor relevant for the Higgs potential. Furthermore, in a particular fermionic UV completion this form factor will have very soft UV behavior implying two sum rules on the mass spectrum of spin-1 resonances and their decay constants [41,42]. These sum rules will ensure the finiteness of the gauge contributions to the Higgs potential, which can be used to achieve the cancelation between γ_f and γ_g. As in Ref. [40], the tuning in this model will be around the minimal tuning $\Delta \simeq 1/\xi$. The Higgs potential from top sector is quartic in the top Yukawa coupling, $\gamma_f \sim \mathcal{O}(y_t^4)$, hence by power counting, it is not explicitly dependent on the (colored) top partner masses. The explicit expression at leading order is Ref. [43]

$$V_f \simeq c' \frac{N_c f^4}{16\pi^2} y_t^4 [-s_h^2 + s_h^4], \tag{16}$$

where c' is an order one dimensionless constant and N_c is the number of QCD colors. This expression shows explicitly that the Higgs mass does not linearly depend on the colored top partner mass, and heavy colored top partners can be achieved without increasing the tuning, just like in the composite twin Higgs models based on SO(8)/SO(7) [23–25]. Since the Higgs potential is suppressed at $\mathcal{O}(y_t^4)$, a light Higgs can be easily produced without much tuning [44].

The most interesting new prediction of this model is a characteristic six top signal. In the UV completion of the model there will also be some heavy colored top partners (in addition to the color neutral top partners responsible for cutting off the top contributions to the Higgs potential), which are electroweak singlets and charged under the $U(1)_\eta$. These colored top partners mix with our SM top after electroweak symmetry breaking. By rotating into the physical mass basis, these resonances (denoted by t') can decay into three tops through the $U(1)_\eta$ gauge interactions via $t' \to B'_\mu t \to t\bar{t}t$. So when they are pair produced at the LHC, a striking signal corresponding to six top final states is predicted. The background for the six top signals is very small, so this channel can impose significant bounds on heavy colored top partners. At present, there are no LHC searches for six tops, and projections from other searches like black holes [45,46], multilepton SUSY [47], etc. are loose. A well designed strategy to search for six or multiple tops produced in LHC will be presented elsewhere [48].

We have presented a novel approach to the Z_2 parity necessary to construct TH-type models. Our key observation was that, for arbitrary symmetric G/H coset spaces, a Z_2 "trigonometric parity" emerges, which can remain unbroken after introducing the matter fields. Once the twin partners are introduced in a manner that preserves the trigonometric parity, the Higgs potential will have the $s_h \leftrightarrow c_h$ symmetry, which renders the Higgs potential free of quadratic divergences. We constructed a concrete model based on the minimal coset SO(6)/SO(5), and examined its Higgs potential, tuning, phenomenology, and the fermionic UV completion based on SU(4)/Sp(4), yielding the simplest model of neutral naturalness. Our techniques used for arriving at this model open a vast new field of unexplored model building, which we expect will lead to many more fruitful results in the future.

C. C. and J. S. thank the Mainz Institute for Theoretical Physics for its hospitality while this work was in progress. C. C. also thanks the Aspen Center for Physics—supported in part by NSF PHY-1607611—for its hospitality while working on this project. C. C. is supported in part by the NSF Grant No. PHY-1719877. J. S. is supported by the NSFC under Grants No. 11647601, No. 11690022, No. 11675243, No. 11851302, and No. 11761141011 and also supported by the Strategic Priority Research Program of the Chinese Academy of Sciences under Grants No. XDB21010200 and No. XDB23000000. T. M. is supported in part by Project No. Y6Y2581B11 supported by National Postdoctoral Program for Innovative Talents.

*Corresponding author.
jshu@itp.ac.cn

[1] D. B. Kaplan and H. Georgi, Phys. Lett. **136B**, 183 (1984).
[2] H. Georgi and D. B. Kaplan, Phys. Lett. **145B**, 216 (1984).
[3] M. J. Dugan, H. Georgi, and D. B. Kaplan, Nucl. Phys. **B254**, 299 (1985).
[4] G. F. Giudice, C. Grojean, A. Pomarol, and R. Rattazzi, J. High Energy Phys. 06 (2007) 045.
[5] For a review, see B. Bellazzini, C. Csaki, and J. Serra, Eur. Phys. J. C **74**, 2766 (2014); G. Panico and A. Wulzer, Lect. Notes Phys. **913**, 1 (2016).
[6] K. Agashe, R. Contino, and A. Pomarol, Nucl. Phys. **B719**, 165 (2005).
[7] N. Arkani-Hamed, A. G. Cohen, E. Katz, and A. E. Nelson, J. High Energy Phys. 07 (2002) 034.
[8] R. Contino, L. Da Rold, and A. Pomarol, Phys. Rev. D **75**, 055014 (2007).
[9] O. Matsedonskyi, G. Panico, and A. Wulzer, J. High Energy Phys. 01 (2013) 164.
[10] M. Redi and A. Tesi, J. High Energy Phys. 10 (2012) 166.
[11] D. Marzocca, M. Serone, and J. Shu, J. High Energy Phys. 08 (2012) 013.
[12] A. Pomarol and F. Riva, J. High Energy Phys. 08 (2012) 135.
[13] G. Burdman, M. Perelstein, and A. Pierce, Phys. Rev. Lett. **90**, 241802 (2003); **92**, 049903(E) (2004).
[14] M. Perelstein, M. E. Peskin, and A. Pierce, Phys. Rev. D **69**, 075002 (2004).
[15] T. Han, H. E. Logan, and L. T. Wang, J. High Energy Phys. 01 (2006) 099.
[16] A. De Simone, O. Matsedonskyi, R. Rattazzi, and A. Wulzer, J. High Energy Phys. 04 (2013) 004.
[17] J. Li, D. Liu, and J. Shu, J. High Energy Phys. 11 (2013) 047.
[18] Z. Chacko, H. S. Goh, and R. Harnik, Phys. Rev. Lett. **96**, 231802 (2006).
[19] Z. Chacko, H. S. Goh, and R. Harnik, J. High Energy Phys. 01 (2006) 108; G. Burdman, Z. Chacko, H. S. Goh, and R. Harnik, J. High Energy Phys. 02 (2007) 009.
[20] H. Cai, H. C. Cheng, and J. Terning, J. High Energy Phys. 05 (2009) 045.
[21] N. Craig, A. Katz, M. Strassler, and R. Sundrum, J. High Energy Phys. 07 (2015) 105.

[22] N. Craig, S. Knapen, and P. Longhi, Phys. Rev. Lett. **114**, 061803 (2015).

[23] M. Geller and O. Telem, Phys. Rev. Lett. **114**, 191801 (2015); C. Csaki, M. Geller, O. Telem, and A. Weiler, J. High Energy Phys. 09 (2016) 146.

[24] M. Low, A. Tesi, and L. T. Wang, Phys. Rev. D **91**, 095012 (2015).

[25] R. Barbieri, D. Greco, R. Rattazzi, and A. Wulzer, J. High Energy Phys. 08 (2015) 161.

[26] ATLAS Collaboration, CERN Report No. ATLAS-CONF-2016-104, 2016.

[27] ATLAS Collaboration, CERN Report No. ATLAS-CONF-2016-101, 2016.

[28] ATLAS Collaboration, CERN Report No. ATLAS-CONF-2016-102, 2016.

[29] While this Z_2 exchange symmetry is at the heart of neutral naturalness models with PNGB Higgs models, other mechanisms, such as folded supersymmetry [30] can realize neutral naturalness without trigonometric parity.

[30] G. Burdman, Z. Chacko, H. S. Goh, and R. Harnik, J. High Energy Phys. 02 (2007) 009.

[31] B. Gripaios, A. Pomarol, F. Riva, and J. Serra, J. High Energy Phys. 04 (2009) 070.

[32] M. Frigerio, A. Pomarol, F. Riva, and A. Urbano, J. High Energy Phys. 07 (2012) 015;

[33] P. Batra and Z. Chacko, Phys. Rev. D **79**, 095012 (2009);

[34] J. Galloway, J. A. Evans, M. A. Luty, and R. A. Tacchi, J. High Energy Phys. 10 (2010) 086.

[35] G. Cacciapaglia and F. Sannino, J. High Energy Phys. 04 (2014) 111.

[36] G. Ferretti and D. Karateev, J. High Energy Phys. 03 (2014) 077.

[37] A similar setup has been presented independently in Ref. [38].

[38] J. Serra and R. Torre, Phys. Rev. D **97**, 035017 (2018).

[39] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.121.231801 for the trigonometric parity of $\mathrm{SU}(3)/\mathrm{SU2}) \times \mathrm{U}(1)$, the Quirky Little Higgs.

[40] C. Csaki, T. Ma, and J. Shu, Phys. Rev. Lett. **119**, 131803 (2017).

[41] S. Weinberg, Phys. Rev. Lett. **18**, 507 (1967).

[42] R. Contino, arXiv:1005.4269.

[43] G. Panico, M. Redi, A. Tesi, and A. Wulzer, J. High Energy Phys. 03 (2013) 051.

[44] To see this, we can explicitly estimate its mass: $m_h^2/m_t^2 \simeq c' y_t^2 N_c (1-\xi)/2\pi^2 \approx 0.07(1-\xi)c'$. So $m_h = 125$ GeV can be naturally produced for a natural value $c' \approx 7$.

[45] ATLAS Collaboration, CERN Report No. ATLAS-CONF-2015-043, 2015.

[46] G. Aad *et al.* (ATLAS Collaboration), J. High Energy Phys. 03 (2016) 026.

[47] G. Aad *et al.* (ATLAS Collaboration), Phys. Rev. D **86**, 092002 (2012).

[48] H. Y. Han, L. Huang, T. Ma, J. Shu, and T. Tait and Y. C. Wu, Many tops at the LHC (to be published).