# Symmetry Breaking and Lattice Kirigami

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We consider an interacting quantum field theory on a curved two-dimensional manifold that we construct by geometrically deforming a flat hexagonal lattice by the insertion of a defect. Depending on how the deformation is done, the resulting geometry acquires a locally nonvanishing curvature that can be either positive or negative. Fields propagating on this background are forced to satisfy boundary conditions modulated by the geometry and that can be assimilated by a nondynamical gauge field. We present an explicit example where curvature and boundary conditions compete in altering the way symmetry breaking takes place, resulting in a surprising behavior of the order parameter in the vicinity of the defect. The effect described here is expected to be generic and of relevance in a variety of situations.

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*Introduction.*—The theory of quantum fields in curved space has produced remarkable results [1,2], particle production in gravitational fields [3] and black hole evaporation [4] being amongst its most celebrated offsprings. From a more general perspective, its semiclassical framework has established a highly nontrivial connection between thermodynamics, gravity, and quantum field theory (QFT), and it is at this crossroads where nontrivial manifestations of the geometrical and topological attributes of curved space on the quantum domain occur.

A particularly interesting corner of this intersection is that of QFTs featuring spontaneous symmetry breaking, where effects of curved space are expected to alter the way vacuum destabilization and phase transitions take place. In flat space the story has been known for a long time and the Coleman-Weinberg mechanism clarifies the way radiative corrections destabilize the vacuum, with symmetries being spontaneously broken as a result of quantum effects [5].

On curved backgrounds there are differences that are not difficult to anticipate. A first indication of how things change comes from the same Coleman-Weinberg mechanism that predicts a first-order phase transition from a broken to a restored symmetry phase in scalar electrodynamics when the scalar mass is increased [5]. When lifted to a weakly curved space, renormalization theory implies the appearance of a masslike contribution proportional to the Ricci curvature, thus causing an effective increase of the mass of the scalar. It is then natural to expect that the effect of a positive (negative) spacetime curvature would be to push the system *towards* a phase of unbroken (broken) symmetry.

Additional insight comes from spacetimes with horizons that are periodic in imaginary time. Green's functions on these backgrounds enjoy a periodicity with period set by the horizon size, analogous to thermal Green's functions for which the period is set by the inverse temperature. This leads to the expectation that for a sufficiently small horizon a transition from a broken to a symmetric phase may occur.

These arguments have been made quantitatively in a number of cases, with some initial discussions focusing on scalar fields and spatially homogeneous backgrounds (see Refs. [6–8], where direct computations have shown that a positive curvature does indeed assist symmetry restoration). Interestingly, it was also shown that the details of the scalar field theory (its conformal invariance or lack of it) were responsible for a change in the order of the curvature-induced phase transition [7]. Similar issues in relation to chiral symmetry breaking have also been discussed (see Ref. [9] for a review of earlier works). The situation is more complicated for inhomogeneous or topologically nontrivial backgrounds. A sample of early calculations can be found in Refs. [10–15]. A particularly interesting example is that of black holes discussed, for instance, in Refs. [14,15]. There it was shown that a spontaneously broken symmetry is locally restored near a (sufficiently hot) black hole (see also Refs. [16,17]). The interpretation is again that the strong gravitational gradient near the horizon is responsible for inducing symmetry restoration.

The natural playground for contemplating how spacetime topology and curvature might modify the stability of the vacuum in QFT has always been in the domain of early Universe cosmology. Recently, however, other areas of physics have been contributing to modernize the above questions and to formulate exciting new problems.

One such area is related to recent advances in condensed matter research, particularly in connection with layered materials. Graphene and other 2D materials are the most spectacular example of the sort, owing to geometrical versatility coupled with an emergent relativistic behavior of fermions [18–20]. In these examples the background geometry is the 2D lattice on top of which fluctuations propagate, and the relevance of curvature effects has already been appreciated [21–25]. Other interesting examples can be found in Refs. [26,27].

QCD physics is also fueling novel research in which the use of QFT in curved space is becoming more common. Interesting examples range from the generic remarks of Refs. [28,29] (and references therein) to the active area of strongly interacting fermions and chiral symmetry breaking in rotating backgrounds (see, for example, Refs. [30–36]) to applications of lattice QCD (see, for example, Refs. [37–39]). In these contexts a range of peculiar geometry-induced phenomena are expected to occur (e.g., condensate suppression or enhancement, new phases, changes in the critical points geography) whose relevance includes heavy ion collisions, transport phenomena, and compact stars.

The focus of this Letter is to reconsider the role of the background geometry in altering the stability of the vacuum. We will argue that, contrary to expectation, increasing the spatial curvature does not necessarily push the system closer to a phase of restored symmetry, and we shall present an explicit example of this. Although the example is nontrivial, it is amenable to simple explanation and anticipates the possibility of appearance of exotic changes in the phase behavior of interacting QFTs with a number of interesting implications.

*Model and geometry.*—For concreteness, we shall consider a class of (2 + 1)-D QFT of the Hubbard type, whose Hamiltonian $\mathsf{H} = \mathsf{H}_0 + \mathsf{H}_I$ is the sum of a free part,

$$\mathsf{H}_0 = -t \sum_{\mathbf{r},i,\sigma=\pm} u^\dagger_\sigma(\mathbf{r}) v_\sigma(\mathbf{r} + \mathbf{b}_i) + \text{H.c.},$$

plus an interacting sector,

$$\mathsf{H}_I = \frac{U}{4} \sum_{\mathbf{r},\sigma,\sigma',i} [n_\sigma(\mathbf{r}) n_{\sigma'}(\mathbf{r}) + n_\sigma(\mathbf{r} + \mathbf{b}_i) n_{\sigma'}(\mathbf{r} + \mathbf{b}_i)].$$

The above QFT is defined on a lattice that we assume to be flat with hexagonal cells, generated by linear combinations of basis vectors (see Fig. 1; $\mathbf{r}$ span a triangular sublattice

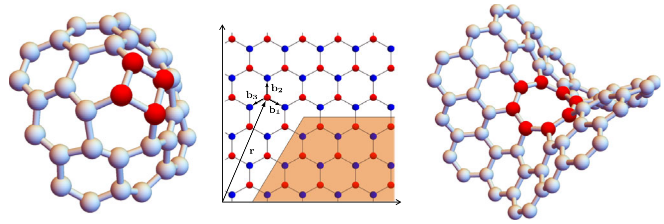


FIG. 1. (Center panel) The flat hexagonal lattice with a $2\pi/3$ section highlighted. (Left panel) Subtracting a $2\pi/3$ section, one obtains a positive curved cone with a $n_s = 4$ sides defect. (Right panel) Adding a $2\pi/3$ section, instead, generates a negative curved saddle geometry with $n_s = 8$.

and the vectors $\mathbf{b}_i$, $i = 1, 2, 3$, connect the atom in $\mathbf{r}$ with the nearest neighbors). The annihilation operators of the two sublattices are $u$ and $v$, and $n_\sigma$ is the number operator. $U$ and $t$ are, respectively, the hopping and the interaction constant (both $> 0$).

The above model, routinely used to describe many of the properties of graphene [18,40], allows for the possibility of locally introducing a curvature by inserting a defect in the lattice.

Performing a lattice simulation should be feasible. Here, we propose a geometrical perspective on how symmetry breaking is altered by lattice deformations, and we will be concerned with the continuum limit, which we implement by separately taking the continuum limit of the flat space field theory and of the lattice. Both limits are known, and this way of proceeding guarantees that the free-field limit (see, for example, Refs. [22,24]) is safely recovered.

The type of symmetry that we wish to discuss is associated with the bipartite nature of the honeycomb lattice that the Hubbard model captures in the magnetization (defined below). Since our goal here is to scrutinize the effect of curvature on the spontaneous breakdown of the above sublattice symmetry, our first task is to covariantize the model to curved space. For this it is convenient to work with the continuum Lagrangian counterpart that can be obtained, using standard methods, by expressing the original Hamiltonian in terms of the SU(2) vector $\mathbf{S} = \sum_{\sigma,\sigma'} u^\dagger_\sigma(\mathbf{r}) \vec{\tau}_{\sigma,\sigma'} u_{\sigma'}(\mathbf{r})/2$, where $\vec{\tau}$ is a vector with the Pauli matrices as components, and then to proceed by means of a Hubbard-Stratonovich transformation. To keep our treatment as simple as possible, we will assume a scalar order parameter, motivated by a rotational anisotropy favoring symmetry breaking along the $z$ axes (for graphene this could be due to the presence of a substrate and related spin-orbit coupling enhancement [41,42]). This allows us to gap out the Goldstone modes that can be straightforwardly included in a more involved treatment. Choosing an auxiliary field $\phi$ that breaks the $\mathbb{Z}_2$ and the discrete sublattice symmetry, the Hamiltonian can be mapped, at low energies, onto the following (2 + 1)-dimensional theory:

$$\mathcal{L} = \bar{\psi}_\sigma \imath \partial\!\!\!/ \psi_\sigma + (\sigma \bar{\psi}_\sigma \phi \psi_\sigma) + \frac{\phi^2}{2\lambda}, \qquad (1)$$

where the first term is a free Dirac contribution and the remaining terms describe the interaction sector. The summation over repeated spin indices $\sigma = \pm$ is understood, and the four-component spinors $\psi_\sigma$ are arranged as $\psi_\sigma^T = (\psi_\sigma^{A1}, \psi_\sigma^{B1}, \psi_\sigma^{A2}, \psi_\sigma^{B2})$, with $\psi_\sigma^{IJ}(x) = (a/v_F) \int [(d^2 p)/(2\pi)^2] e^{-\imath \mathbf{p} \cdot \mathbf{x}} z_\sigma^{IJ}(\mathbf{p})$, and where $z_\sigma^{I,J}(\mathbf{p}) = z_\sigma^I[\mathbf{K}_J + (a/v_F)\mathbf{p}]$ represents the sublattice annihilation operators ($z^A = u$, $z^B = v$) near the two Dirac cones $\mathbf{K}_{J=1,2}$ of the dispersion relation. The spatial coordinates were rescaled by $\mathbf{x} = \mathbf{r}/v_F$, where $v_F = \frac{3}{2} ta$ is the Fermi velocity and $a$ is the lattice spacing. The coupling constant $\lambda$ is proportional to the interaction strength $\lambda \propto U$ up to an unimportant factor, dependent on the particular regularization of the low energy theory.

The exchange of the sublattices can then be implemented by the simultaneous exchange $x_2 \to -x_2$ ($p_2 \to p_2$), leaving intact the Dirac points and the spin, and the Lagrangian (1) invariant as long as $\phi$ vanishes. The order parameter for the above symmetry is $\phi = 2\lambda \langle \bar{\psi}_- \psi_- - \bar{\psi}_+ \psi_+ \rangle$, and it describes the staggered magnetization; i.e., $\phi \neq 0$ indicates broken symmetry. The same expression (1) is obtained following the general decomposition outlined in Ref. [43].

Kirigami is a variation of origami that includes cutting [44]. Utilizing a kirigami procedure, we introduce a spatial curvature in the model by inserting a disclination that warps the lattice locally. If we wish to isolate the interplay between quantum effects and geometry, we need to preserve the bipartite nature of the lattice at tree level to avoid frustration. This requirement restricts the allowed deformations to those induced by defects with an even number of sides, as these are the only that preserve the above symmetry classically. Inserting a defect with $n_s < 6$ sides into a hexagonal lattice generates a deficit angle and a curvature that is locally positive (Fig. 1). By contrast, adding a defect with $n_s > 6$ generates an excess angle and a locally negative curvature (Fig. 1). These lattice structures form the basis of chiral curved polyaromatic systems [45].

The lattice plays the role of geometry, and its continuum limit is that of a manifold with a regularized conical singularity (see, for instance, Ref. [22,24]). The Riemannian geometry of such manifolds has been studied since, at least, Ref. [46]. References [47,48] give details and additional bibliographies on the topic. Here, to model such a localized curvature, we use a Euclidean parametrization for the metric tensor

$$ds^2 = d\tau^2 + dr^2 + \alpha^2 r^2 d\varphi^2, \qquad (2)$$

with $r \geq 0$ and $0 \leq \varphi < 2\pi$ being the polar coordinates centered at the apex. Setting $\tilde{\varphi} = \alpha\varphi$, one sees that the metric is that of flat space with $0 \leq \tilde{\varphi} < 2\pi\alpha$. If $\alpha < 1$, then $\gamma = 2\pi - 2\pi\alpha$ describes a deficit angle. Removing the deficit angle and identifying the two sides results in a cone with opening angle $2 \arcsin \alpha$. The closer to unity $\alpha$ is, the flatter the cone. If $\alpha > 1$, then the deficit angle becomes an excess angle.

Since the curvature of conical manifolds diverges at the apex, some regularization is necessary to deal with the singular behavior. Here, we will regulate the geometry by replacing the singular space with a sequence of regular manifolds as was done in Refs. [47,48]. Calculations are done in the regularized geometry, and results in the original singular space can be obtained as a limit once the regularization is removed at the end. This procedure can be implemented by replacing the original metric (2) with the following regular one:

$$d\tilde{s}^2 = d\tau^2 + f_\epsilon(r) dr^2 + \alpha^2 r^2 d\varphi^2, \qquad (3)$$

where $\epsilon$ represents a regularization parameter and $f_\epsilon(r)$ is a smooth function satisfying the following properties: (a) $\lim_{\epsilon\to 0} f_\epsilon(r) = 1$, (b) $f_\epsilon(r) \approx 1$ for $r \gg \epsilon$, (c) $f_\epsilon(r) = \text{const}$ for $r = 0$. Notice that while the limit of $\epsilon \to 0$ removes the regularization, in a comparison with a lattice simulation, $\epsilon$ is related to the lattice spacing acquiring the role of a physical cutoff. Alternatively, one can deal with the singular structure from the beginning at the price of having to deal with a more complex form of the heat kernel [49–51]. (See the Supplemental Material [52] for additional details.)

The Lagrangian (1) is extended to curved space by a minimal covariantization procedure, i.e., letting metric, derivatives, and $\gamma$ matrices go to the corresponding quantities in curved space (see Ref. [53]).

Finally, we take into account the boundary conditions along the cut where the two sides of the lattice have been glued. It is not difficult to realize that for a generic even-sided defect, the sublattice symmetry is preserved and the fermion wave function, after circulating around the defect, satisfies the following boundary condition: $\psi(r, \varphi + 2\pi) = -\exp[i(6 - n_s)\pi\gamma_5/2]\psi(r, \varphi)$. (We follow Ref. [22] and choose to work in the standard planar representation of the $\gamma$ matrices.) A way to incorporate these boundary conditions is by reexpressing the fields as $\psi'(r, \varphi) = \exp[-i\varphi(6 - n_s)\gamma_5/4]\psi(r, \varphi)$, and by noticing that the primed fields obey the standard periodicity condition $\psi'(r, \varphi + 2\pi) = -\psi'(r, \varphi)$. Using the above redefinition in the Lagrangian has the effect of introducing a nondynamical gauge connection $\mathcal{A}_\mu = -\delta_\mu^\varphi (6 - n_s)\gamma_5/4$. This term will be crucial in altering the way symmetry breaking takes place.

*Methods and results.*—We can now examine whether the geometry and the associated boundary conditions favor a phase of broken or restored symmetry in the region where curvature attains a positive ($n_s = 4$) or negative (even $n_s > 6$) value. As motivated at the beginning, since the curvature in the regularized case increases (decreases) as we approach the defect for $n_s = 4$ ($n_s = 8, 10, \ldots$), the expectation is that curvature should favor symmetry restoration in the vicinity of the defect for $n_s = 4$, while for $n_s$ even and larger than 6 broken symmetry should be favored.

Below we shall address this question by computing the effective action for the order parameter $\phi$ and by numerically solving the associated effective equations. Our analysis follows the large-$N$ deformation of Ref. [43], where we pass from 2 to $N$ flavors of the Dirac fields. The general form of the effective action is

$$\tilde{\Gamma}[\phi] = -\int d^3x \sqrt{\tilde{g}} \frac{\phi^2}{2\lambda} + \frac{1}{2} \sum_{p=\pm} \log\det\left(\tilde{\Box} + \frac{\tilde{R}}{4} + \phi_p^2\right),$$

where $\phi_\pm^2 = \phi^2 \pm \sqrt{\tilde{g}^{rr}}\phi'$ and the d'Alembertian is calculated from the spinor covariant derivative $\tilde{D}_\nu = \tilde{\nabla}_\nu + i\mathcal{A}_\nu$. The tildes indicate quantities computed in the regularized metric (3). We use heat-kernel and zeta regularization techniques to perform the computation of the determinant (see Refs. [2,54–57] for general discussions and Refs. [16,53] for similar calculations in curved space). For the convenience of the reader, the computation of the determinant is described in the Supplemental Material [52].

The results are illustrated in Fig. 2 for cases with locally positive ($n_s = 4$) and negative curvature ($n_s = 8$). The asymptotic value of the coupling constant should be fixed by imposing renormalization conditions (see Ref. [9]) to adjust its value to specific physical situations. Here, we have varied its value to encompass cases in which symmetry is either broken or close to the critical value far from the defect (see Fig. 2). Numerical solutions indicate that $\phi$ develops a spatial dependence and attains a value near the defect that is larger than its asymptotic value for any $n_s \neq 6$, signaling that curvature, irrespectively of its sign, encourages an ordered phase near the defect.

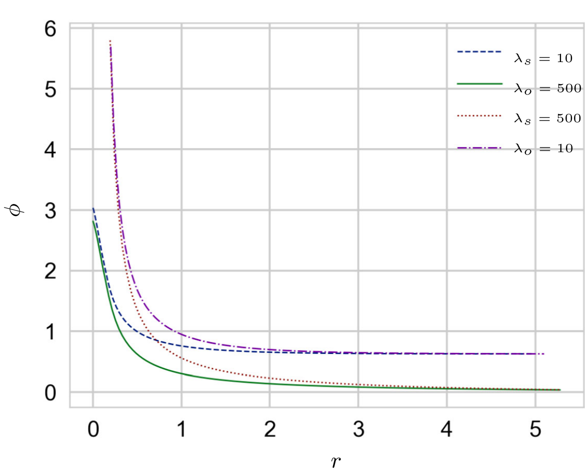


FIG. 2. Order parameter profile as the defect is approached. The values of $\lambda_{s,o}$ are given in units of the renormalization scale ($s$, square; $o$, octagon). $\epsilon$ has been varied between 0.1 and 0.01.

The competition between curvature and gauge field in deforming the order parameter near the defect can be understood in light of the arguments of Ref. [28]. When the background is curved, a masslike correction proportional to scalar curvature ($+\tilde{R}/4$) appears in the determinant. The additional gauge field alters the determinant by an amount $-n_s^2/r^2$, as it follows by squaring the Dirac operator and using the Lichnerowicz formula (interestingly, the number of sides of the defect can be interpreted as an effective charge). Thus, the total correction is proportional to $+\tilde{R}/4 - n_s^2/r^2$, and its contribution to the effective action can be fully resummed (see the Supplemental Material [52]). Its effect is to induce a local change that locally shifts the effective potential (upwards when the correction is positive, downwards when the correction is negative). The term $-n_s^2/r^2$ always dominates near the defect, and it is enhanced for $n_s > 6$ when $\tilde{R} < 0$.

*Conclusions.*—In this Letter we have looked at symmetry breaking in the Hubbard model on a curved 2D manifold constructed by taking the continuum limit of a flat hexagonal lattice deformed by insertion of a defect. The two important features of the story turn out to be the increasing (or decreasing) curvature near the defect and the boundary conditions along the cut. The latter can be assimilated by a nondynamical gauge field. As an example, we have considered the staggered magnetization, the order parameter associated with the discrete sublattice symmetry. Numerical results have shown a surprising increase of the order parameter as the locally curved region is approached. This behavior has been explained by the competition between the effect of curvature and of the emergent gauge field (i.e., boundary conditions) modulated by the conical structure.

What we have seen here should be generic and should be expected to occur for different QFT phases and different lattice structures as long as the same geometrical traits are maintained. Here, we have ignored fluctuations of the order parameter; thus, the system is always in the ordered state. However, the way $\tilde{R}$ and the emergent gauge field appear in the determinant suggests an analogous suppression to what we find here of the onset of a metallic phase. An intriguing possibility is to consider a multidefect configuration and look for arrangements where the competition between the geometry-induced gauge fields and curvature can be adjusted to produce specific order parameter profiles.

The effect of curvature on the spontaneous breaking of sublattice symmetry could be used to shed light on the question regarding the semimetal-insulator phase transition in graphene: even though graphene is predicted to be very close to the transition point [40,58], no experimental signature of the insulating behavior has been found in flat graphene so far [59]. Another promising route in graphene would be the combination of curvature with adatom adsorption in order to enhance symmetry breaking, in particular of magnetic order [60].

Cosmic strings in cosmology [61] or their condensed matter analogue (in liquid crystals, for example [62,63]) offer a straightforward application of our results. In this case the effect discussed here should locally trigger fermion condensation and offer a mechanism for inducing a superconducting phase at the string.

Finally, it is tempting to relate the present ideas to gravity at the Planck scale, where spacetime may be discrete and the presence of defects could cause local changes in the lattice geometry and topology similar to those described here, triggering a form of graviton condensation in the vicinity of these spacetime glitches.

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