Fermi-Liquid Approach for Superconducting Kondo Problems

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We present a Fermi liquid approach to superconducting Kondo problems applicable when the Kondo temperature is large compared to the superconducting gap. To illustrate the theory, we study the current-phase relation and the Andreev level spectrum for an Anderson impurity between two s-wave superconductors. In the particle-hole symmetric Kondo limit, we find a 4π periodic Andreev spectrum. The 4π periodicity persists under a small voltage bias which however causes an asymmetric distortion of Andreev levels. The latter distinguishes the present 4π effect from the one in topological Majorana junctions.

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Introduction.—The interplay between superconductivity and localized magnetic moments remains of central importance to modern condensed-matter physics. For instance, spin-fluctuation mediated pairing is encountered in a broad variety of unconventional superconducting materials [1,2]. Moreover, Yu-Shiba-Rusinov states induced by a magnetic impurity in a superconductor [3–5] can be responsible for Majorana bound states in magnetic atom chains deposited on superconducting substrates [6,7]. A paradigmatic example for superconducting Kondo problems is given by an Anderson dot in the magnetic regime (where it can realize a Kondo impurity) sandwiched between two conventional s-wave BCS superconductors [8–23], with experimental realizations available in nanoscale devices [24–30]. Numerical calculations [13,14,18,22] show that the low-temperature physics is governed by the ratio T_K/Δ, where Δ is the superconducting gap and T_K the Kondo temperature (for $\Delta = 0$). While the so-called π-junction regime with $T_K < \Delta$ is accessible by perturbative renormalization group (RG) methods [20,28], the complementary 0-junction regime with $T_K > \Delta$ has so far withstood analytical progress apart from an exact solution for $T_K/\Delta \to \infty$ [8] and different mean-field approximations [9–12,15–17,19]. In more general terms, the Kondo effect in a superconductor represents a long-standing open theoretical problem.

We here formulate a Fermi liquid theory for the Kondo effect in a superconductor which describes the regime $T_K \gg \Delta$ in a systematic and controlled manner. For the corresponding normal metal case, an elegant and asymptotically exact approach has been put forward by Nozières [31], cf. also Refs. [1,32–34]. His key insight was that the Kondo singlet formed by the impurity spin and the electron screening cloud can only be polarized, but not broken, near the strong-coupling fixed point. One then arrives at a Fermi liquid description by expanding the energy-dependent phase shifts for elastic quasiparticle scattering at low energies and by including residual local quasiparticle interactions [31–34]. We show below how those ideas can be extended to the superconducting case where, in particular, Andreev reflection (AR) processes turn out to be of key importance. Such processes can be fully captured by a boundary condition accounting both for AR and elastic scattering, cf. Eq. (7) below. For $\Delta = 0$, our approach becomes equivalent to Nozières' theory. It also reproduces the $T_K/\Delta \to \infty$ solution of Ref. [8]. For a Fermi liquid approach covering the opposite limit $T_K/\Delta \to 0$ in a normal-superconductor junction, see Ref. [35].

We illustrate our theory for an Anderson impurity between two s-wave BCS superconductors, see Fig. 1, by studying the Josephson current-phase relation (CPR), $I(\phi)$, as well as the Andreev level dynamics under a small bias voltage V. With minor modifications, our theory can be adapted to a plethora of interesting related problems, e.g., multiple Andreev reflection phenomena (so far studied only within mean-field schemes [10,12]), setups involving topological superconductors [36–40], or multi-terminal devices [21,41]. In the particle-hole symmetric Kondo

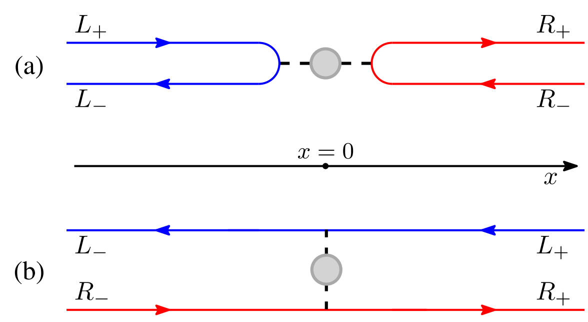

FIG. 1. Schematic setup. (a) Semi-infinite left and right ($j = L/R$, blue or red) superconducting leads at $x < 0/x > 0$, respectively, harbor one-dimensional (1D) right (+) and left (−) movers and are tunnel coupled (dashed lines) to an Anderson dot (shaded circle) at $x = 0$. (b) Unfolded representation with 1D chiral fermions.

limit of the Anderson model, we predict a 4π periodic Andreev level spectrum at low temperature $T \ll \Delta^3/T_K^2$, with zero-energy level crossings at $\phi = \pi (\mathrm{mod}\, 2\pi)$. Such a periodicity is also expected for topological Josephson junctions with Majorana states [37–40,42] (for experimental signatures, see Refs. [43–45]) and for other setups [46–48]. We find that under a small bias voltage V, the 4π periodicity persists. However, in contrast to all previously studied 4π periodic setups, the absorption and/or emission spectrum near the zero-energy crossings becomes asymmetric. This fact allows for experimental tests of the underlying mechanism.

Model.—We start with an Anderson dot tunnel-coupled to left and right superconducting leads ($j = L/R$), see Fig. 1(a). Writing $H = H_d + H_t + H_{\mathrm{leads}}$ with $H_d = \epsilon_d(n_\uparrow + n_\downarrow) + U n_\uparrow n_\downarrow$, where $n_\sigma = d_\sigma^\dagger d_\sigma$ for dot fermions d_σ, we have an interacting ($U > 0$) dot level at energy ϵ_d. For simplicity taking identical dot-lead tunnel couplings (t_0), the point-like tunneling Hamiltonian is $H_t = t_0 \sum_\sigma d_\sigma^\dagger b_\sigma(0) + \mathrm{H.c.}$, with symmetric combinations $b_\sigma(x)$ of 1D left and right lead fermion operators, cf. Eq. (2) below. Finally, H_{leads} describes s-wave BCS superconductor leads [49]. Each semi-infinite lead supports right and left movers, $\psi_{j,\sigma}^{(\pm)}(x) \sim e^{\pm i k_F x}$. In the equivalent unfolded representation in Fig. 1(b), we have infinite chiral leads containing only left- and right-moving field operators $\psi_{j,\sigma}(x)$ for lead $j = L/R$, respectively, $\psi_{L,\sigma}^{(\pm)}(x<0) = e^{\pm i k_F x}\psi_{L,\sigma}(\mp x)$ and $\psi_{R,\sigma}^{(\pm)}(x>0) = e^{\pm i k_F x}\psi_{R,\sigma}(\pm x)$. To simplify notation, we take the same absolute value Δ of the superconducting gap on both sides and put $\hbar = e = v_F = k_B = 1$ (the normal density of states is then just $1/\pi$), resulting in

$$H_{\mathrm{leads}} = \sum_{j=L/R=\pm} \int_{-\infty}^{\infty} dx \Bigg(\sum_{\sigma=\uparrow,\downarrow} \psi_{j,\sigma}^\dagger (\pm i \partial_x) \psi_{j,\sigma} + \Delta (e^{\mp i\phi/2} \psi_{j,\downarrow}(x) \psi_{j,\uparrow}(-x) + \mathrm{H.c.}) \Bigg), \quad (1)$$

where ϕ is the phase difference. Next, we switch to the linear combinations

$$\left\{ \begin{array}{c} a_\sigma(x) \\ b_\sigma(x) \end{array} \right\} = \frac{1}{\sqrt{2}} [\psi_{L,\sigma}(-x) \mp \psi_{R,\sigma}(x)], \quad (2)$$

representing incoming (outgoing) fermion states for $x < 0$ ($x > 0$). The a modes obey open boundary conditions corresponding to $a_\sigma(0^+) = a_\sigma(0^-)$, which for $t_0 = 0$ also apply to b modes.

In the magnetic regime, $U \gg \max(\Delta, |t_0|^2)$ and $-U < \epsilon_d < 0$, the impurity corresponds to a spin-1/2 operator $\mathbf{S}$, with the particle-hole symmetric Kondo limit at $\epsilon_d = -U/2$. A Schrieffer-Wolff transformation yields $H \to H_{\mathrm{leads}} + H_K$, where H_K contains a potential scattering term (for $\epsilon_d \neq -U/2$) and an exchange term with coupling $J > 0$ between $\mathbf{S}$ and the spin density of b fermions at $x = 0$ [1]. Importantly, a modes always decouple from the impurity and thus can be integrated out exactly. Using the imaginary-time functional integral approach [49], b modes are then governed by the action $S_b + \int d\tau H_K(\tau)$, where $S_b = -\sum_{k,\omega} \tilde{\Psi}^\dagger(k,\omega) G^{-1}(k,\omega) \tilde{\Psi}(k,\omega)$ with fermion Matsubara frequencies ω and the Nambu spinor

$$\Psi(x,\tau) = \begin{pmatrix} b_\uparrow(x,\tau) \\ b_\downarrow^\dagger(-x,\tau) \end{pmatrix} \sim \sum_{k,\omega} e^{i(kx-\omega\tau)} \tilde{\Psi}(k,\omega). \quad (3)$$

Here and below, $\tilde{\Psi}(\omega)$ refers to the frequency representation of a time-dependent spinor $\Psi(\tau)$. After taking into account the pairing-induced bulk coupling between a and b fermions, the free ($t_0 = 0$) Green's function (GF) appearing in S_b is given by [cf. Eq. (1)],

$$G(k,\omega) = -\frac{i\omega + k\tau_z + \Delta \cos(\phi/2)\tau_x}{k^2 + \omega^2 + \Delta^2}, \quad (4)$$

where Pauli matrices $\tau_{x,z}$ act in Nambu space.

Weak-coupling regime.—At high energy scales, the dynamics is restricted to the Hilbert subspace respecting open boundary conditions. Integrating also over the bulk $b_\sigma(x \neq 0)$ modes, we obtain $S_b = -\sum_\omega \tilde{\Psi}^\dagger(\omega) G_0^{-1}(\omega) \tilde{\Psi}(\omega)$ with $\Psi(\tau) = \Psi(0,\tau)$ and

$$G_0(\omega) = \int \frac{dk}{2\pi} G(k,\omega) = -\frac{i\omega + \Delta\cos(\phi/2)\tau_x}{2\sqrt{\omega^2 + \Delta^2}}. \quad (5)$$

Standard energy-shell integration [49] then yields the one-loop RG equations

$$\frac{dJ}{d\ell} = \frac{J^2}{\pi\sqrt{1+\delta^2}}, \qquad \frac{d\mathcal{Q}}{d\ell} = -\frac{3}{4\pi} \frac{\delta \cos(\phi/2)}{\sqrt{1+\delta^2}} J^2, \quad (6)$$

where $\delta(\ell) = \Delta/D(\ell)$. As the effective bandwidth $D(\ell) = e^{-\ell} D$ decreases with increasing RG flow parameter ℓ, a local pairing term, $H_{\mathrm{AR}} = \mathcal{Q} b_\downarrow(0) b_\uparrow(0) + \mathrm{H.c.}$, is generated by AR processes. In fact, for $\phi \neq \pi$ (mod 2π), the growing exchange coupling $J(\ell)$ drives $\mathcal{Q}(\ell)$ toward strong coupling, resulting in Kondo-enhanced AR [20,28]. Note that $\mathcal{Q}(\ell) \sim \cos(\phi/2)$ throughout the flow. However, the RG approach breaks down at energies below $T_K \simeq D e^{-\pi/J}$, where one enters the strong-coupling regime.

Strong-coupling theory.—In the deep Kondo regime, the impurity spin is almost perfectly screened by the leads. To implement the Fermi liquid approach for the normal case, it is convenient to employ a scattering state formalism where the leading effects due to the polarizable Kondo singlet come from energy-dependent phase shifts and residual interaction corrections [31–34]. For the superconducting case, we also need to include AR processes. This is achieved below by describing both AR and elastic scattering in a unified manner through a simple yet general boundary condition. To that end, by performing a Wick rotation, $i\omega \to E$, with energy E relative to the chemical potential μ, we define $\tilde{\Psi}_\pm(E) = \tilde{\Psi}(x = 0^\pm, E)$ from the Nambu spinor (3) taken at $x = 0^\pm$. Arbitrary elastic scattering and AR processes are then captured by the boundary condition

$$\tilde{\Psi}_+(E) = e^{2i\hat{\eta}(E)}\tilde{\Psi}_-(E), \quad \hat{\eta}(E) = \begin{pmatrix} \eta_\uparrow(E) & \eta_a(E) \\ -\eta_a^*(E) & \eta_\downarrow(-E) \end{pmatrix}, \tag{7}$$

where the Nambu matrix $\hat{\eta}(E)$ has the most general form allowed by Hermiticity of the self-energy $\hat{\Sigma}(E)$ in Eq. (8) below. While the real functions $\eta_{\uparrow,\downarrow}(E)$ are energy-dependent phase shifts precisely as in the normal case, the complex-valued function $\eta_a(E)$ describes AR.

Next, Eq. (7) is linked to the retarded response of bulk modes, $\tilde{\Psi}_\pm(E) = \sum_k e^{ik0^\pm} G^R(k, E)\tilde{\Psi}(E)$, to an effective boundary field, $\tilde{\Psi}(E)$, living at $x = 0$. Using the retarded GFs $G^R(k, E)$ and $G_0^R(E)$ obtained by Wick rotation from Eqs. (4) and (5), respectively, we find $\tilde{\Psi}_\pm(E) = [G_0^R(E) \mp (i/2)\tau_z]\tilde{\Psi}(E)$. Here, the τ_z term originates from the respective τ_z term in Eq. (4). One can thereby write Eq. (7) as equation of motion for the boundary spinor,

$$[G_0^R(E) + \hat{\Sigma}(E)]\tilde{\Psi}(E) = 0, \quad \hat{\Sigma}(E) = \frac{1}{2}\cot[\hat{\eta}(E)]\tau_z. \tag{8}$$

Finally passing back to imaginary time and rescaling $\Psi(\tau) = (1/\sqrt{2})[b_\uparrow(\tau), b_\downarrow^\dagger(\tau)]^T$, the strong-coupling action is given by [cf. Eqs. (5) and (8)]

$$\begin{aligned} S_{\mathrm{SC}}[\Psi] &= -\sum_\omega \tilde{\Psi}^\dagger(\omega)\mathcal{G}^{-1}(\omega)\tilde{\Psi}(\omega) + S_I, \\ \mathcal{G}^{-1}(\omega) &= \mathcal{G}_0^{-1}(\omega) - \cot[\hat{\eta}(i\omega)]\tau_z, \\ \mathcal{G}_0^{-1}(\omega) &= -2G_0(\omega), \end{aligned} \tag{9}$$

while S_I describes residual interaction corrections addressed below. We emphasize that our self-energy formulation of AR and elastic scattering processes in Eq. (9) is completely general.

In order to arrive at a low-energy Fermi liquid theory, we now expand $\hat{\eta}(E)$ in powers of $|E|/T_K \ll 1$ and $\Delta/T_K \ll 1$. Using the spin symmetry of the problem and noting that conventional even-frequency pairing generated from Eq. (1) implies $\eta_a(-E) = \eta_a(E)$, we find

$$\begin{aligned} \eta_\uparrow(E) &= \eta_\downarrow(E) = \eta_F + \alpha_1 E + \alpha_2 E^2 + \cdots, \\ \eta_a(E) &= \Delta(\beta_1 + \beta_3 E^2 + \cdots), \end{aligned} \tag{10}$$

where η_F is the quasiparticle phase shift at the Fermi energy for $\Delta = 0$. The Fermi liquid parameters α_n and β_n scale as $1/T_K^n$, where the α_n determine the elastic scattering phase shifts [31,34] and the complex-valued β_n depend on the phase difference ϕ (see below). Keeping all terms up to order $1/T_K^2$, and using the renormalized parameters $\tilde{\alpha}_n = \alpha_n/\sin^2\eta_F$ and $\tilde{\beta}_n = \beta_n/\sin^2\eta_F$, we arrive at

$$\begin{aligned} \mathcal{G}^{-1}(\omega) &= \mathcal{G}_0^{-1}(\omega) - \begin{pmatrix} \lambda(\omega) - i\tilde{\alpha}_1\omega & \tilde{\beta}_1\Delta \\ \tilde{\beta}_1^*\Delta & -\lambda(\omega) - i\tilde{\alpha}_1\omega \end{pmatrix}, \\ \lambda(\omega) &= \cot\eta_F\left(1 - \frac{\alpha_1^2\omega^2 + |\beta_1|^2\Delta^2}{\sin^2\eta_F}\right) + \tilde{\alpha}_2\omega^2. \end{aligned} \tag{11}$$

Further simplifications arise in the Kondo limit, where particle-hole symmetry (which is not broken by pairing terms) imposes the condition $\tau_x e^{2i\hat{\eta}(E)}\tau_x = e^{-2i\hat{\eta}(E)}$ [50], resulting in $\eta_F = \pi/2$, $\alpha_2 = 0$, and $\beta_1 = \beta_1^*$. In the Kondo limit, we thus have $\lambda(\omega) = 0$ in Eq. (11).

Residual interaction processes.—We now turn to S_I in Eq. (9). Keeping all terms up to order $1/T_K^2$, this action contribution has the general form

$$S_I = \frac{1}{2}\sum_{\sigma=\uparrow,\downarrow}\int d\tau b_{-\sigma}^\dagger b_{-\sigma} b_\sigma^\dagger(\tilde{u}_1 - \tilde{u}_2\partial_\tau)b_\sigma, \tag{12}$$

with expansion parameters $\tilde{u}_n \sim 1/T_K^n$ (where $\tilde{u}_1 \geq 0$). Defining normal ordering and averages $\langle\cdots\rangle_0$ with respect to the BCS ground state for $\mathcal{G}_0(\omega)$, cf. Eq. (9), it is convenient to express Eq. (12) by virtue of Wick's theorem as $S_I = \langle S_I\rangle_0 + \tilde{S}_I + S_I^H$, where $\tilde{S}_I$ is the normal-ordered form of Eq. (12) and S_I^H represents Hartree terms which can be accounted for via the $\hat{\eta}(E)$ expansion in Eq. (10). Up to order $1/T_K^2$, with $u_n = \tilde{u}_n \sin^2\eta_F$, we find

$$\begin{aligned} \eta_\sigma(E) &= \eta_F + \alpha_1 E + \alpha_2 E^2 - (u_1 + u_2 E)\delta N_{-\sigma}, \\ \eta_a(E) &= \beta_1\Delta + u_1\delta Q, \end{aligned} \tag{13}$$

where δN_σ and δQ are self-consistent Hartree parameters for local density and pairing fluctuations, respectively. Again invoking spin symmetry, $\delta N_\uparrow = \delta N_\downarrow$, Eq. (13) implies that Hartree terms can indeed be included by renormalizing α_n and β_n. We assume henceforth that this renormalization has already been carried out. Moreover, since the Kondo singularity is tied to the Fermi level, the phase shifts $\eta_\sigma(E)$ must be independent of the chemical potential μ [31,34]. This fact implies that one can derive relations between Fermi liquid parameters without having to specify δN_σ or δQ [31,34]. In particular, in the Kondo limit, $\partial_\mu\eta_F = 0$ and $\alpha_2 = u_2 = 0$ imply the well-known identity $u_1 = \pi\alpha_1$ [31] and $\partial_\mu\alpha_1 = 0$.

Current-phase relation.—The CPR follows as a phase derivative of the free energy,

$$I(\phi) = 2\partial_\phi F = I_A(\phi) + I^{(1)}_{\rm int}(\phi) + I^{(2)}_{\rm int}(\phi), \qquad (14)$$

where $I_A(\phi) = -2T\sum_\omega \partial_\phi \ln\det \mathcal{G}^{-1}(\omega)$ is the Andreev bound state (ABS) contribution, see Eq. (11). In particular, the ABS spectrum follows by solving $\det[\mathcal{G}^{-1}(-iE)] = 0$ for subgap energies, $|E| < \Delta$. Keeping terms up to order $1/T_K$, where $\lambda(-iE) = \lambda = \cot\eta_F$ [cf. Eq. (11)], this condition reads

$$\frac{E^2}{\Delta^2} = \frac{|\cos(\phi/2) - \tilde\beta_1\sqrt{\Delta^2 - E^2}|^2 + \lambda^2}{(1 + \tilde\alpha_1\sqrt{\Delta^2 - E^2})^2 + \lambda^2}. \qquad (15)$$

In the Kondo limit (with $\lambda = 0$), Eq. (15) holds up to order $1/T_K^2$.

The leading interaction contribution to the CPR, see Eq. (14), follows from $\langle S_I\rangle_0$ [50],

$$I^{(1)}_{\rm int}(\phi) = \delta I_c \sin\phi, \qquad \delta I_c \simeq -\frac{\tilde u_1\Delta^2}{4\pi^2}\ln^2(T_K/\Delta). \qquad (16)$$

As expected in the presence of repulsive quasiparticle interactions, we obtain a decrease of the critical current, $\delta I_c < 0$, where $|\delta I_c| \sim (\Delta^2/T_K)\ln^2(T_K/\Delta)$ contains a logarithmic enhancement factor. Finally, $I^{(2)}_{\rm int}$ describes higher-order interaction corrections to the CPR due to $\tilde S_I$. To order $1/T_K^2$, we obtain [50]

$$I^{(2)}_{\rm int}(\phi) \approx \tilde u_1^2\Delta^3\left(\sin\phi + \frac{1}{2}\sin(2\phi)\right), \qquad (17)$$

where the $\sin(2\phi)$ term describes coherent tunneling processes involving two Cooper pairs.

Let us then turn to the dominant ABS contribution, see Eq. (15), where the ϕ dependence of the AR coupling $\tilde\beta_1$ follows from Eq. (6), $\tilde\beta_1(\phi) = \gamma\cos(\phi/2)$, with constant $\gamma \sim 1/T_K$. (i) For $T_K/\Delta \to \infty$, all Fermi liquid parameters and thus also the interaction corrections (16) and (17) can be dropped. Solutions to Eq. (15) are then given by $E = \pm\Delta\sqrt{1 - \mathcal{T}\sin^2(\phi/2)}$ with the junction transparency $\mathcal{T} = \sin^2\eta_F = 1/(1 + \lambda^2)$. We thus readily recover the results of Ref. [8]. (ii) Including $1/T_K$ corrections, see Fig. 2, Eq. (15) predicts a 4π periodic ABS spectrum in the Kondo limit ($\lambda = 0$), with zero-energy ABS crossings at $\phi = \pi(\mathrm{mod}\,2\pi)$). For $\lambda \neq 0$, we instead have avoided crossings with gap $E_g \simeq 2\sqrt{1 - \mathcal{T}}\Delta$, and thus obtain a conventional 2π periodic spectrum. (iii) Fermi liquid corrections imply a detachment of ABSs from quasiparticle continuum states at $\phi = 0(\mathrm{mod}\,2\pi)$. The detachment gap, $\delta_A = \Delta - E_A(0)$, follows from Eq. (15) as

$$\delta_A = 2\sin^4(\eta_F)\Delta^3[\tilde\alpha_1 + \mathrm{Re}(\gamma)]^2 \sim \frac{\Delta^3}{T_K^2}. \qquad (18)$$

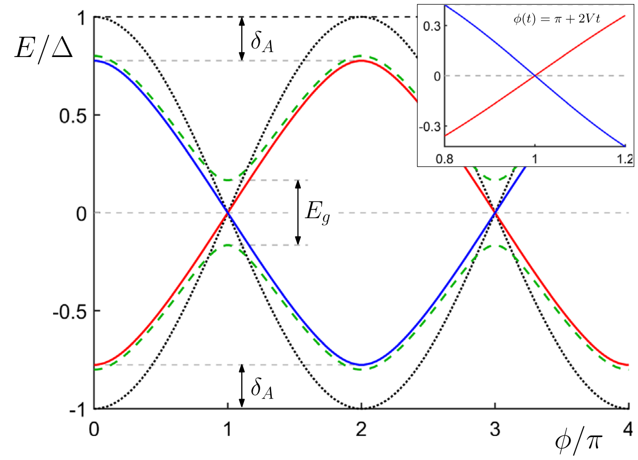

FIG. 2. ABS spectrum vs phase ϕ. Main panel: Black dotted curves show the particle-hole symmetric limit with $T_K/\Delta \to \infty$ [8]. Blue and red solid curves depict 4π periodic Andreev levels for $T_K/\Delta = 5$, $\lambda = 0$, and $\alpha_1 = \gamma = 1/T_K$. Green dashed curves illustrate the gap E_g formed away from particle-hole symmetry ($\lambda = 0.2$), leading to 2π periodicity. Inset: Asymmetry of 4π-periodic adiabatic Andreev levels near the crossing at $\phi = \pi$ with voltage $V = 0.33\Delta$ and $\mathrm{Im}(\gamma) = 3/T_K$.

While ABS detachment already arises from elastic scattering [12], AR and Hartree corrections can strongly renormalize δ_A. Since the Kondo resonance floats with the Fermi level and the ABS spectrum is detached from the continuum, the 4π periodic CPR in the Kondo limit should be observable for $T \ll \delta_A$.

ABS spectrum for small voltage V.—What will happen to the 4π periodic Andreev spectrum in the Kondo limit when a small bias voltage V is applied? For $V \ll \delta_A \ll \Delta$, adiabatic Andreev levels still represent good dynamical variables. Since the ABSs are removed from continuum states by a spectral gap, the retarded and advanced sectors of the Keldysh action decouple [49,50]. To investigate whether the 4π periodicity survives in the nonequilibrium case, we consider the phase dynamics, $\phi(t) = \pi + 2Vt$, at times where $\phi(t) \approx \pi(\mathrm{mod}\,2\pi)$, corresponding to zero-energy crossings. The retarded sector can equivalently be described [50] by the real-time action

$$S = \int dt\Phi^\dagger(t)[i\partial_t - E_A(t)(\sigma_z - \xi(t))]\Phi(t), \qquad (19)$$

where $\Phi = (c_+, c_-)^T$ contains the amplitudes for upper and lower ($\nu = +/-$) Andreev branches, the Pauli matrix σ_z acts in Andreev level space, and

$$E_A(t) \simeq \frac{1 - \mathrm{Re}(\gamma)\Delta}{1 + \alpha_1\Delta}\Delta\sin(Vt). \qquad (20)$$

In the near-adiabatic regime, the ABS degeneracy at each crossing is not lifted by the voltage, and the Andreev spectrum remains 4π periodic. We find $\xi(t) = \frac{1}{2}\mathrm{Im}(\gamma)V\cos(Vt)$ in

Eq. (19), where $\mathrm{Im}(\gamma) \neq 0$ requires that particle-hole symmetry has been broken, e.g., by the voltage. The only effect of $\xi(t)$ then consists of an asymmetric distortion of the $\nu = +/-$ Andreev levels, cf. Eqs. (19), (20), and inset of Fig. 2,

$$\nu E_A \to \nu(1 - \nu\xi)E_A. \tag{21}$$

Since $\xi \to -\xi$ at each subsequent crossing ($\phi \to \phi + 2\pi$), Eq. (21) implies an asymmetric absorption and/or emission spectrum near the ABS crossings. Importantly, this feature allows one to experimentally distinguish the predicted 4π Josephson effect from its topological counterpart in Majorana junctions [37,38,40] as well as from other proposed realizations [46–48]. The real and imaginary parts of γ can be measured via the detachment gap δ_A [Eq. (18)] in the equilibrium Andreev spectrum and via the low-voltage asymmetry ξ, see Eq. (21), respectively.

Conclusions.—In this work, we have presented a Fermi liquid approach to the Kondo problem in a conventional s-wave BCS superconductor with $T_K \gg \Delta$. While we have illustrated the theory for an Anderson dot between two superconducting leads in the (near) equilibrium regime, the Fermi liquid description also allows us to tackle many other setups featuring an interplay of Kondo physics with superconductivity.

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