Negative Thermal Magnetoresistivity as a Signature of a Chiral Anomaly in Weyl Superconductors

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We propose that the chiral anomaly of Weyl superconductors gives rise to negative thermal magnetoresistivity induced by emergent magnetic fields, which are generated by vortex textures of order parameters or lattice strain. We establish this scenario by combining the argument based on Berry curvatures and the quasiclassical theory of the Eilenberger equation with quantum corrections arising from inhomogeneous structures. It is found that the chiral anomaly contribution of the thermal conductivity exhibits characteristic temperature dependence, which can be a smoking-gun signature of this effect.

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Introduction.—In Weyl semimetals and Weyl superconductors, low-energy excitations behave as Weyl fermions characterized by nonzero Berry curvatures in momentum space, which stem from monopole charges at Weyl points [1–9]. This feature results in various intriguing electromagnetic responses associated with a chiral anomaly. For instance, in the case of Weyl semimetals, chiral anomaly gives rise to the anomalous Hall effect, chiral magnetic effect, and negative magnetoresistivity [10–17], some of which have already been experimentally verified in real materials [18–24]. For Weyl superconductors, however, chiral anomaly phenomena cannot be realized by simply applying electromagnetic fields, because Weyl-Bogoliubov quasiparticles do not carry definite charges. Instead, chiral anomaly in the superconducting state can be induced by emergent electromagnetic fields which are generated by spatially inhomogeneous textures of order parameters, or lattice strain [25–40].

In this Letter, we demonstrate that negative magnetoresistivity of longitudinal thermal currents induced by an emergent magnetic field can be a signature of chiral anomaly; i.e., thermal conductivity of Weyl quasiparticles increases as the emergent magnetic field parallel to the temperature gradient increases, even when pair-breaking effects due to magnetic fields are negligibly small. We examine two scenarios for realizing emergent magnetic fields. One is that induced by vortex textures in the mixed state, and the other one is a chiral magnetic field arising from lattice strain [26,39,40]. We establish the abovementioned result by combining the argument based on the semiclassical equation of motion with Berry curvatures characterizing Weyl fermions and microscopic analysis using the quasiclassical theory of the Keldysh Green function. Our finding is relevant to putative Weyl superconductors such as multilayer systems [9] and uranium-based systems, URu_2Si_2, UPt_3, UCoGe, $U_{1-x}Th_xBe_{13}$ [41–53].

Semiclassical argument for thermal transport with Berry curvature.—We, first, present a semiclassical argument for thermal transport. This approach is useful for qualitative understanding of chiral anomaly effects. We consider a paradigmatic model of Weyl superconductors which describes a three-dimensional chiral $p_x + ip_y$ pairing state of spinless fermions, though our basic idea can be generalized to any Weyl superconductors. The superconducting gap function for homogeneous cases is given by $\Delta_{\boldsymbol{k}} = \Delta(k_x - ik_y)/k_F$. In this system, low-energy excitations from point nodes of the superconducting gap at $\boldsymbol{k} = (0, 0, \pm k_F)$ behave as Weyl fermions. The model Hamiltonian for low-energy Weyl quasiparticles with the monopole charge $s = \pm 1$ in the case with spatial inhomogeneity is given by

$$\mathcal{H}_s(\boldsymbol{k}, \boldsymbol{r}) = s e^{\mu}_a V^a_b \tau^b (k_\mu - s k_{0\mu}), \tag{1}$$

where $V^a_b = \mathrm{diag}[(\Delta/k_F), (\Delta/k_F), v_F]$, with v_F the Fermi velocity and τ^a the Pauli matrix in the particle-hole space. Spatial inhomogeneity is described in terms of the vielbein e^μ_a. We use greek letter indices $\mu = 1, 2, 3$ as space indices for the laboratory frame, and roman letters $a = \bar{1}, \bar{2}, \bar{3}$ as indices for a local orthogonal frame. As mentioned above, the spatial inhomogeneity gives rise to an emergent magnetic field $\boldsymbol{\mathcal{B}} = \boldsymbol{T}^\mu k_\mu$ with the torsion field, $(\boldsymbol{T}^\mu)^\nu = \frac{1}{2}\epsilon^{\nu\lambda\rho} T^a_{\lambda\rho} e^\mu_a$, $T^a_{\mu\nu} = \partial_\mu e^a_\nu - \partial_\nu e^a_\mu$, where e^a_μ is the inverse of e^μ_a [26–29,54]. It is noted that $\boldsymbol{\mathcal{B}}$ plays the role of a chiral magnetic field, when $\boldsymbol{T}^z$ is nonzero, since the sign of k_z at the Weyl points of the model Eq. (1) corresponds to the chirality of Weyl fermions. There are several ways of realizing nonzero $\boldsymbol{\mathcal{B}}$ in superconductors. For instance, a vortex line texture parallel to the z axis, i.e., $\Delta = \Delta_0 e^{i\phi}$, generates the emergent magnetic field, $\boldsymbol{\mathcal{B}} = (0, 0, \mathcal{B}_z)$ with $\mathcal{B}_z = T^\mu_{12} k_\mu = (k_y \cos\phi - k_x \sin\phi)/r$, which does not depend on k_z, and is not a chiral magnetic field, but imitates a usual magnetic field. Also, lattice strain such as twist of a crystal structure with a rotation axis parallel to z direction gives rise to an emergent chiral magnetic field along the z axis. In the following, we consider magnetoresistivity of a thermal current for these two cases.

By using the semiclassical equation of motion with Berry curvatures for Weyl quasiparticles [54], and the Boltzmann equation, we obtain the chiral anomaly contribution of the local thermal current $\boldsymbol{J}_H(\boldsymbol{r})$ up to leading terms in $\boldsymbol{\mathcal{B}}$,

$$\boldsymbol{J}_H(\boldsymbol{r}) = \sum_{s=\pm1}\sum_{\boldsymbol{k}}(\boldsymbol{v}_{ps}\cdot\boldsymbol{\Omega}_{\boldsymbol{k}\boldsymbol{k}s})^2\varepsilon_{\boldsymbol{k}s}^2\left(\frac{\partial f}{\partial\varepsilon_{\boldsymbol{k}s}}\right)\tau_{\boldsymbol{k}s} \times \left(\frac{\nabla T}{T}\cdot\boldsymbol{\mathcal{B}}\right)\boldsymbol{\mathcal{B}}, \tag{2}$$

where $\varepsilon_{\boldsymbol{k}s} = \sqrt{v^2(k_z - sk_{0z})^2 + \Delta^2(k_x^2+k_y^2)/k_F^2}$, $\boldsymbol{v}_{\boldsymbol{k}s} = \partial\varepsilon_{\boldsymbol{k}s}/\partial\boldsymbol{k}$, $\tau_{\boldsymbol{k}s}$ is the relaxation time, f is the Fermi distribution function, and $\boldsymbol{\Omega}_{\boldsymbol{k}\boldsymbol{k}s}$ is the Berry curvature generated by the monopole charge at the Weyl point, which characterizes the chiral anomaly contribution. Equation (2) evidences the negative thermal magnetoresistivity (NTMR) due to the emergent magnetic field $\boldsymbol{\mathcal{B}}$. It is noted that the chiral anomaly contribution of the thermal conductivity κ_A extracted from Eq. (2) exhibits singular temperature dependence. In the case of a constant relaxation time, we have

$$\kappa_A \propto 1/T, \tag{3}$$

for low T. If one takes into account the temperature dependence of $\tau_{\boldsymbol{k}s}$ more precisely, the low-temperature behavior becomes more singular. This behavior is due to the singularity of the Berry curvature in the vicinity of Weyl points, i.e., $\Omega_{\boldsymbol{k}\boldsymbol{k}s} \sim 1/|\delta\boldsymbol{k}|^2$ for the deviation from the Weyl points $|\delta\boldsymbol{k}| \to 0$. The characteristic T dependence of Eq. (3) can be utilized for discriminating the chiral anomaly contribution from usual contributions of thermal conductivity of nodal excitations, $\kappa_0 \propto T$ for $T \to 0$. However, we must be careful about the applicability of Eq. (2). The divergent behavior of Eq. (3) implies that it cannot be used in the low-temperature limit, for which adiabatic approximation postulated for the derivation of the Berry curvature formula fails. Thus, Eq. (3) is applicable only in the intermediate temperature region. To investigate thermal transport for the whole temperature region, we exploit alternative approaches based on the Keldysh formalism in the following.

Keldysh-Eilenberger approach for cases with vortex textures.—To confirm the prediction obtained above, and go beyond adiabatic approximation, which fails in the low-temperature region, we exploit the Keldysh formalism of the quasiclassical Eilenberger equation. We consider the 3D chiral $p_x + ip_y$ pairing model again, and, first, examine the case of an emergent magnetic field generated by vortex textures of the superconducting order parameter. The case of strain-induced chiral magnetic fields will be considered later. A merit of the scenario of a vortex-induced emergent magnetic field is that it can be easily realized for any type-II superconductors. Transport properties of systems with inhomogeneous textures are described in terms of the quasiclassical Green function $\check{g}(\hat{\boldsymbol{k}},\boldsymbol{r},\epsilon)$ with $\hat{\boldsymbol{k}}$ a unit vector parallel to the Fermi momentum [54,57–59]. Using the Keldysh Green function $\hat{g}^K$, we can express a thermal current as

$$\boldsymbol{J}_H(\boldsymbol{r}) = N_F\int_{-\infty}^{\infty}\frac{d\epsilon}{4\pi i}\int d\hat{\boldsymbol{k}}\epsilon\boldsymbol{v}_F\frac{1}{2}\mathrm{Tr}[\hat{g}^K(\hat{\boldsymbol{k}},\boldsymbol{r},\epsilon)], \tag{4}$$

where N_F is the density of states at the Fermi level, $\boldsymbol{v}_F$ is the Fermi velocity, and $\int d\hat{\boldsymbol{k}}\cdots$ is the normalized Fermi surface average. In this Letter, we consider the spherical Fermi surface with $\boldsymbol{v}_F = v_F\hat{\boldsymbol{k}}$.

Effects of emergent magnetic fields arising from spatial inhomogeneity can be incorporated via spatial gradient expansion of the Eilenberger equation, which gives higher-order quantum corrections to the quasiclassical approximation. Up to the first order in $1/(k_F\xi)$ with ξ the coherence length, the Eilenberger equation with quantum corrections is given by [54]

$$[(\epsilon + e\boldsymbol{v}_F\cdot\boldsymbol{A})\tau_3 - \check{h}, \check{g}] + i\boldsymbol{v}_F\cdot\boldsymbol{\nabla}_{\boldsymbol{r}}\check{g} = \frac{i}{2}\{\check{h}\cdot\check{g}\} - \frac{i}{2}\{\check{g}\cdot\check{h}\}, \tag{5}$$

where $\{\check{a}\cdot\check{b}\} = \boldsymbol{\nabla}_{\boldsymbol{r}}\check{a}\cdot\boldsymbol{\nabla}_{\boldsymbol{k}}\check{b} - \boldsymbol{\nabla}_{\boldsymbol{k}}\check{a}\cdot\boldsymbol{\nabla}_{\boldsymbol{r}}\check{b}$, $\boldsymbol{A}$ is a vector potential due to an external magnetic field, $\check{h} = \check{\Delta} + \check{\sigma}_{\mathrm{imp}}$ with $\check{\Delta}$ the gap function, and $\check{\sigma}_{\mathrm{imp}}$ the self-energy due to impurity scattering, which determines the relaxation time τ [54]. The nonzero right-hand side term of Eq. (5) describes leading quantum corrections. For simplicity, we assume that $\check{\sigma}_{\mathrm{imp}}$ does not depend on temperature T. In general, $\check{\sigma}_{\mathrm{imp}}$ should depend on T, because of the energy dependence of the density of states of Weyl quasiparticles and the T dependence of the gap function. However, this simplification is useful for the investigation of characteristic T dependence of thermal conductivity arising from chiral anomaly, which is predicted from the semiclassical analysis Eq. (3). Effects of an emergent magnetic field caused by vortex textures are included in the right-hand side of Eq. (5). We deal with this term in a perturbative way. We expand the Green function up to the second order in $1/(k_F\xi)$: $\check{g} = \check{g}_0 + \check{g}_1 + \check{g}_2$. The nonperturbative part $\check{g}_0$ can be easily calculated from the standard Eilenberger equation without quantum corrections, supplemented with the normalization condition, $\check{g}_0^2 = -\pi^2$ [60]. The correction terms $\check{g}_1$ and $\check{g}_2$ are obtained from an inhomogeneous Eilenberger equation with leading quantum corrections:

$$[(\epsilon + e\boldsymbol{v}_F\cdot\boldsymbol{A})\tau_3 - \check{h}, \check{g}_n] + i\boldsymbol{v}_F\cdot\boldsymbol{\nabla}_{\boldsymbol{r}}\check{g}_n = \frac{i}{2}\{\check{h}\cdot\check{g}_{n-1}\} - \frac{i}{2}\{\check{g}_{n-1}\cdot\check{h}\}. \quad (6)$$

The thermal conductivity $\kappa = J_H^z/(-\partial_z T)$ is obtained by substituting the solution of $\check{g} = \check{g}_0 + \check{g}_1 + \check{g}_2 + \cdots$ to Eq. (4). The temperature gradient along the vortex line is incorporated as the boundary condition of the Keldysh component at $z = \pm\infty$, $g_n^K(\infty) = -2\pi(g_n^R - g_n^A)\tanh[\epsilon/2T(\pm\infty)]$ [54], where $g_n^{R,A}$ are calculated in the absence of the temperature gradient.

We, first, consider the case of a single vortex with vorticity m, i.e., $\Delta(\boldsymbol{r}) = \Delta_0(T)[\tanh(r/\xi)]^{|m|}e^{im\phi}$ with $r = \sqrt{x^2+y^2}$. In this case, we can neglect the vector potential $\boldsymbol{A}$ in Eq. (6). Solving Eq. (6) numerically for $\check{g}_1$ and $\check{g}_2$, we found that the contribution from $\check{g}_1$ to the thermal current is negligible. The leading quantum correction associated with the vortex-induced emergent magnetic field arises from $\check{g}_2$. The calculated results of this quantum correction term of the thermal conductivity κ_2 for vorticity $m = 1, 2, 3$ are shown in Fig. 1(a), where κ_2 is spatially averaged over the core region within $r \le 5\xi$. In this calculation, the BCS-type temperature dependence of the gap function is assumed, the energy unit is scaled by $2\pi T_c$, and the parameters are set as $v_F = 20$, $k_F = 1$, $\xi = 20$, $\Delta_0(0) = 1.765T_c$, and $1/\tau = 0.002$. It is noted that κ_2 increases as the vorticity increases. Since the emergent magnetic field is proportional to the vorticity, this behavior implies negative magnetoresistivity of thermal currents. Furthermore, the T dependence of κ_2 remarkably exhibits an upturn increase in the intermediate temperature region, which is indeed in agreement with the prediction from the semiclassical analysis, Eq. (3). However, in contrast to the semiclassical result, which fails in the low-temperature limit, the T dependence turns to decreasing behaviors in the low-temperature region, which is consistent with the third law of thermodynamics. Thus, it is concluded that the negative magnetoresistivity of thermal currents is a signature of the chiral anomaly of Weyl quasiparticles. We, here, comment on the T dependence of the normal self-energy neglected in our calculations. If one takes into account the T dependence due to the energy dependence of the density of states, the increase of the thermal conductivity is more magnified in the intermediate T region, because of the longer relaxation time. Thus, the detection of the chiral anomaly effect becomes more feasible.

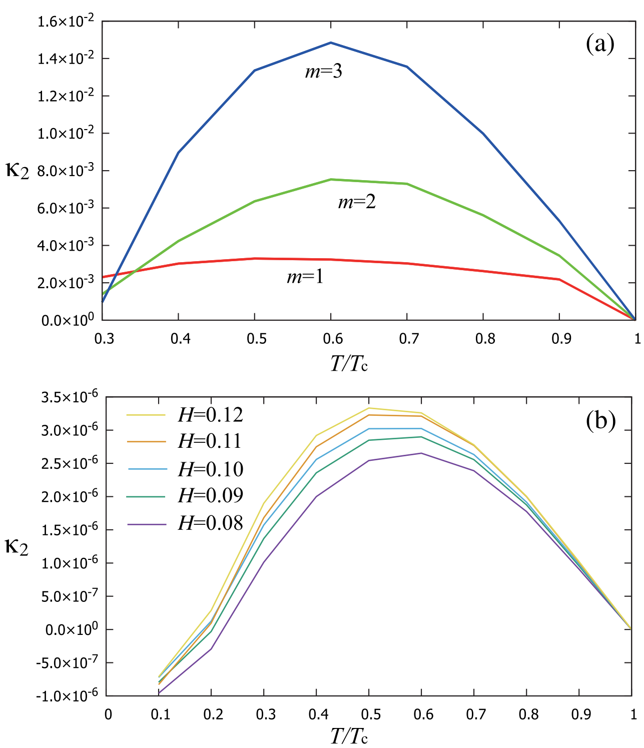

FIG. 1. (a) κ_2 versus T in the case of single vortex with vorticity $m = 1, 2, 3$. (b) κ_2 versus T in the case of a vortex lattice for $H = 0.08, 0.09, 0.10, 0.11, 0.12$ from bottom to top.

We, next, performed the calculation for the case of a vortex lattice. For simplicity, a square lattice structure of vortices is assumed [54,61]. The calculated results of κ_2 are shown in Fig. 1(b), which is the spatially averaged value over the unit cell. The qualitative characteristic features are similar to the results for the case with a single vortex. The thermal conductivity increases as a function of a magnetic field, and the T dependence qualitatively coincides with the Berry phase formula Eq. (3) in the intermediate T region, signifying the chiral anomaly effect. We also calculated the spatial distribution of thermal currents, and found that thermal currents are mainly carried by bulk quasiparticles, rather than bound states in vortex cores, confirming that the increase of κ_2 is due to the chiral anomaly of Weyl quasiparticles. It is noted that the NTMR in this scenario is free from the issue of current jetting, which disturbs the detection of negative magnetoresistivity as a signature of the chiral anomaly in the case of Weyl semimetals [62]. The current jetting is caused by inhomogeneity of current distribution due to the strong Landau quantization. Since the wave function in the vortex state is the Bloch function, the current jetting is absent in this case. We stress that the characteristic temperature dependence found in Fig. 1 cannot be realized for any non-Weyl (non-Dirac) superconductors, as revealed by numerous previous studies on thermal transport in the vortex state [63–75]. Thus, the NTMR with the characteristic temperature dependence is a unique feature of Weyl (Dirac) superconductors.

Although the above results establish the NTMR as a signature of chiral anomaly, the chiral anomaly contribution shown in Fig. 1(b), which corresponds to the case of high magnetic fields, is about 0.1% of the total contribution. The calculation for low fields is not attainable because of numerical costs. It is known that for small magnetic fields close to a lower critical field and for $\boldsymbol{J}_H \| \boldsymbol{H}$, the field dependence of the thermal conductivity due to usual pair breaking is quite small. Thus, in this case, the experimental detection of the chiral anomaly contribution is still feasible by measuring the field-dependent part of the thermal conductivity. A more promising approach for the detection of the chiral anomaly effect is to utilize an emergent chiral magnetic field induced by lattice strain. We consider this scenario in the following.

Case of strain-induced chiral magnetic fields.—We, now, explore the case that lattice strain induces a chiral magnetic field $\boldsymbol{\mathcal{B}}_C$ in the 3D chiral $p_x + ip_y$-wave spinless superconductor. To simplify the analysis, we introduce the strain-induced chiral vector potential by hand in the mode, though the realization of the strain-induced magnetic field requires multiorbital degrees of freedom [26,37]. Since a chiral magnetic field causes neither the Meissner effect nor the vortex state; the pair-breaking effect due to the chiral magnetic field is remarkably weak [54]. In fact, for the parameters used in our calculations, the superconducting state survives against a chiral magnetic $e\mathcal{B}_C \sim <0.03$, and thus, we can expect enormous NTMR due to a large value of $\mathcal{B}_C$. The chiral magnetic field in superconductors gives rise to a pseudo-Lorentz force, which is obtained from the right-hand side of Eq. (5) [76]. For simplicity, we assume a uniform chiral magnetic field parallel to the z axis, $\boldsymbol{\mathcal{B}}_C = (0, 0, \mathcal{B}_C)$. Then, we end up with the Eilenberger equation:

$$[\epsilon\tau_3 - \check{h}, \check{g}] + i\boldsymbol{v}_F \cdot \nabla_{\boldsymbol{r}}\check{g} + ie\boldsymbol{v}_F \times \boldsymbol{\mathcal{B}}_C \cdot \frac{\partial}{\partial \boldsymbol{k}_\|}\check{g} = 0. \quad (7)$$

The last term of Eq. (7) is the pseudo-Lorentz force term. Since this equation is homogeneous, we need an additional normalization condition for $\check{g}$ to solve it, i.e., $\check{g}^2 = -\pi^2$. To derive an approximate analytic solution of Eq. (7), we expand $\check{g}$ in terms of $1/(\xi k_F)$ and $\boldsymbol{\mathcal{B}}_C$ up to the second order. An explicit expression for quantum corrections of $\check{g}$ due to $\boldsymbol{\mathcal{B}}_C$ is given in Supplemental Material [54]. Although the superconducting state is robust against large values of $\mathcal{B}_C$, one cannot neglect the Landau quantization of quasiparticles for a sufficiently strong chiral magnetic field, which cannot be treated within the quasiclassical approximation. Thus, the temperature range in which our method is valid is limited to $T > T_L \equiv \sqrt{2e\mathcal{B}_C}\Delta/k_F$, for which the Landau levels are smeared by a temperature broadening effect. We calculate a thermal current from Eq. (4) up to linear order in ∇T [58,74]. Numerical results of the thermal conductivity $\kappa = \kappa_0 + \kappa_2$ with κ_0 the nonperturbed zero-field part and κ_2 the field-dependent quantum correction are shown in Fig. 2. In this calculation, the BCS-type T dependence of the gap function and the same parameters as those in the case with vortex-induced magnetic fields are used.

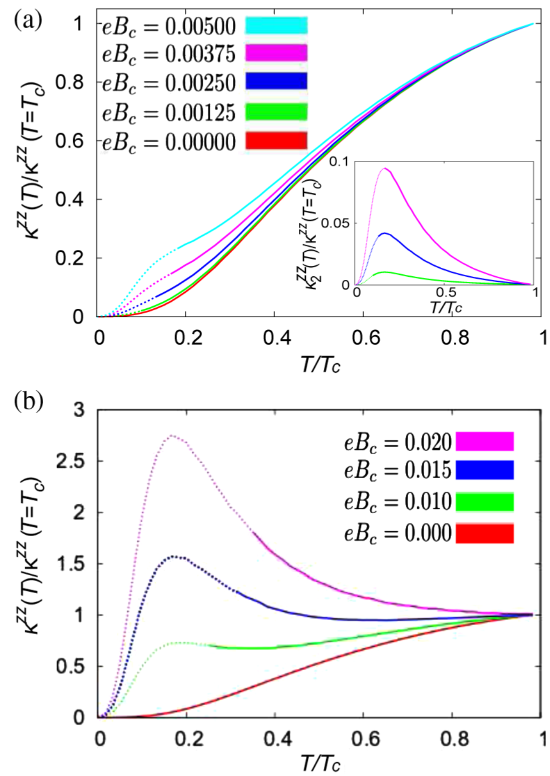

FIG. 2. (a) κ versus T for $e\mathcal{B}_C = 0.0, 0.00125, 0.0025, 0.00375, 0.005$ from bottom to top. For the temperature region $T < T_L$, in which the quasiclassical approximation fails, the results are shown as dotted lines. Inset: κ_2 versus T for $e\mathcal{B}_C = 0.00125, 0.0025, 0.00375, 0.005$ from bottom to top. (b) κ versus T for $e\mathcal{B}_C = 0.0, 0.01, 0.015, 0.02$ from bottom to top. The results for $T < T_L$ are shown as dotted lines.

As seen in Fig. 2, the thermal conductivity increases, as $\mathcal{B}_C$ increases, signifying NTMR. Furthermore, for $e\mathcal{B}_C > \sim 0.01$, the quantum correction part dominates, and hence, the total thermal conductivity exhibits a remarkable increase, as temperature is lowered in the intermediate temperature region, which is a characteristic feature of chiral anomaly contributions. The positions of the peaks of κ for different values of $\mathcal{B}_C$ shown in Fig. 2(b) are roughly $T_c \times \Delta/E_F$, and thus independent of $\mathcal{B}_C$. It is noted that the prominent increase of the thermal conductivity appears even for temperatures much above T_L for sufficiently large $e\mathcal{B}_C$, implying that the increasing behavior of the thermal conductivity is not an artifact of the quasiclassical approximation. For putative Weyl superconductors of uranium-based systems with lattice constants 4–9 Å, $\mathcal{B}_C \approx 2$–5 T can be realized by torsional distortion around the c axis by 2π per ~ 1 μm. On the other hand, for a lattice constant ~ 4 Å, $e\mathcal{B}_C = 0.00125$ in Fig. 2 corresponds to $\mathcal{B}_C \sim 5$ T. In such cases, the magnitude of the chiral anomaly part of the thermal conductivity is more than 10% of the total thermal conductivity, and thus, it is feasible to detect the characteristic T dependence of κ_2 experimentally by extracting the $\mathcal{B}_C$-dependent part of the thermal conductivity. We also note that the current jetting issue [62] can be avoided in this case, because the results in Fig. 2 show that the characteristic signature of chiral anomaly, i.e., the upturn increase of the thermal conductivity in the intermediate temperature region, appears even for sufficiently small chiral magnetic fields which do not cause the inhomogeneous current distribution due to the strong Landau quantization.

Conclusion.—We have investigated thermal transport in Weyl superconductors with emergent (chiral) magnetic fields. It is established that NTMR as a signature of the chiral anomaly of Weyl quasiparticles can be realized, and its experimental detection is feasible.

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