Cooling a Bose Gas by Three-Body Losses

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We report the demonstration of cooling by three-body losses in a Bose gas. We use a harmonically confined one-dimensional (1D) Bose gas in the quasicondensate regime and, as the atom number decreases under the effect of three-body losses, the temperature T drops up to a factor of 4. The ratio $k_BT/(mc^2)$ stays close to 0.64, where m is the atomic mass and c the speed of sound in the trap center. The dimensionless 1D interaction parameter γ, evaluated at the trap center, spans more than 2 orders of magnitudes over the different sets of data. We present a theoretical analysis for a homogeneous 1D gas in the quasicondensate regime, which predicts that the ratio $k_BT/(mc^2)$ converges towards 0.6 under the effect of three-body losses. More sophisticated theoretical predictions that take into account the longitudinal harmonic confinement and transverse effects are in agreement within 30% with experimental data.

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The identification and understanding of cooling processes, both on the theoretical and the experimental side, is crucial to the development of cold atom physics [1,2]. It can help to elaborate strategies to enter new regimes and it can also improve the control over state preparation in experiments where cold atoms are used as quantum simulators of many body systems. Ultracold atom gases are metastable systems, their ground state being a solid phase. They are thus plagued with intrinsic recombination processes, that in practice limit their lifetime. Such processes are mainly three-body collisions during which a strongly bound dimer is formed. It amounts to three-body losses because the dimer is typically no longer trapped and the remaining atom escapes because of its large kinetic energy. These losses are known to produce an undesired heating in cold gases. In the case of a thermal gas, since they occur predominantly in the regions of high atomic density, where the potential energy is low, these losses increase the energy per remaining particle, leading to an antievaporation process [3]. In Bose-Einstein condensates (BEC) confined in deep traps, it was predicted that three-body collisions produce a heating of the BEC through secondary collisions with high energy excitations formed by the loss process [4]. This Letter constitutes a breakthrough since we identify a cooling associated with three-body losses in a cold Bose gas.

A similar counterintuitive cooling was recently investigated in [5–8] where the effect of one-body losses in quasicondensate regime [9] is considered. Although one-body losses are also central for evaporative cooling, here the losses are energy independent and the cooling originates from a very different physics. Those works were recently extended [11] to any j-body loss process, for Bose gases in the BEC or quasicondensate regime, in any dimension d, and for homogeneous gases as well as gases confined in a smooth potential. These studies focus on the effect of losses on low energy excitations in the gas, the phononic modes, which correspond to density waves propagating in the condensate. On the one hand, the energy in these modes is reduced by losses since the amplitude of density modulations is decreased, removing interaction energy from the mode. On the other hand, the discrete nature of the loss process comes with accompanying shot noise which induces density fluctuations, increasing the energy per mode. It has been shown that the competition between these processes leads to a stationary value of the ratio $k_BT/(mc^2)$ where m is the atom mass and c the speed of sound. This value, of the order of one, depends on j, d, and on the confining potential [11]. For three-body losses in a 1D quasicondensates ($j=3$, $d=1$) confined in a harmonic potential one expects $k_BT/(mc_p^2)$ to converge to 0.70 [11], where c_p is evaluated at the peak density. In contrast to evaporative cooling, this loss-induced cooling does not rely on a thermalization mechanism in the gas [12].

In this Letter, we show experimentally that three-body losses induce a cooling and we identify the stationary value of $k_BT/(mc_p^2)$ associated with the three-body process. More precisely, investigating the time evolution of a 1D quasicondensate, we observe a decrease of the temperature as the atom number decreases under the effect of three-body losses. Moreover, on the whole observed time interval, the ratio $k_BT/(mc_p^2)$ stays about constant, at a value close to 0.64, which indicates that the stationary value of $k_BT/(mc_p^2)$ imposed by the loss process is reached. We took several data sets for different parameters. In terms of the 1D dimensionless parameter γ [13] characterizing the strength of the interactions [14], our data span more than 2 orders of magnitude. We compare the experimental data with numerical calculations based on the results of [11], which take into account the harmonic longitudinal confinement of the gas and the swelling of the transverse wave function under the effect of interactions. The experimental results are close to those predictions. In order to present the underlying physics, we derive in this Letter the evolution of the temperature under three-body losses, in the more simple case of a homogeneous purely 1D quasicondensate.

The experiment uses an atom-chip setup [15] where ^{87}Rb atoms are magnetically confined using current-carrying microwires. An elongated atomic cloud is prepared using radio frequency forced evaporative cooling in a trap of transverse frequency $\omega_\perp$. Depending on the data set, $\omega_\perp/(2\pi)$ varies between 1.5 and 9.2 kHz and the atomic peak linear densities n vary between 22 and 257 μm^{-1}. The temperature fulfills $k_B T < \hbar\omega_\perp$ and the gas mostly behaves as a 1D Bose gas [17]. It, moreover, lies in the quasicondensate regime [18], characterized by weak correlations between atoms, as in Bose-Einstein condensates [19], and in particular small density fluctuations [20]. As long as the atoms are in the ground state of the transverse potential, interactions between atoms are well described by a 1D effective coupling constant $g = 2\hbar\omega_\perp a$, where $a = 5.3$ nm is the 3D scattering length [21], and the chemical potential is given by $\mu = gn$. This is valid only as long as $\mu \ll \hbar\omega_\perp$, which requires $na \ll 1$. In the presented data na takes values as large as 1.3 and the broadening of the transverse wave function due to interactions has to be taken into account for quantitative analysis. In particular, the equation of state becomes $\mu = \hbar\omega_\perp(\sqrt{1+4na} - 1)$ [23]. The quasicondensates are confined in the longitudinal direction with a harmonic potential $V(z)$ of trapping frequency $\omega_z/(2\pi) = 8.5$ Hz, weak enough so that the longitudinal profile $n_0(z)$ is well described by the local density approximation (LDA), with a local chemical potential $\mu(z) = \mu_p - V(z)$, where μ_p is the peak chemical potential. It extends over $2R$ where the Thomas-Fermi radius R fulfills $V(R) = \mu_p$. Once the quasicondensate is prepared, we increase the frequency of the radio-frequency field, by several kHz, a value sufficient so that it no longer induces losses. We then investigate the evolution during the waiting time t. Five different data set are investigated, differing in the value of the transverse confinement and the initial temperature and peak density.

Using absorption images we record the density profile of the gas, from which we extract the peak density n_p. Figure 1 shows the evolution of n_p with the waiting time t for the different data sets. The observed nonexponential decrease of n_p is neither due to one-body losses (whose rate is smaller than about 0.14 s^{-1} in our experiment), nor to inelastic two-body collisions, negligible for spin polarized ^{87}Rb [24,25]. Its origin is three-body recombinations, as justified by calculations presented below. In a three-body recombination, a molecule (a dimer) is formed and its

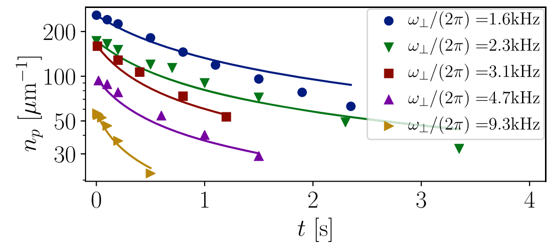

FIG. 1. Peak density, in log scale, versus the waiting time t, for the five different data sets. Solid lines are *ab initio* calculations of the effect of three-body losses, for initial peak densities equal to that of the experimental data.

binding energy is released in the form of kinetic energy of the molecule and the remaining atom. They both leave the trap since their energy is typically much larger than the trap depth, limited by the radio-frequency field. Thus, the effect of the three-body process is to decrease the gas density according to $d\rho/dt = -\rho^3 g^{(3)}(0)\kappa$, where ρ is the three dimensional atomic density, $g^{(3)}(0)$ is the normalized three-body correlation function at zero distance, and $\kappa = (1.8 \pm 0.5) \times 10^{-41}$ m^6/s is the three-body loss rate for ^{87}Rb [24]. In a quasicondensate, correlations between atoms are small and $g^{(3)}(0) \simeq 1$ [26]. Moreover, integrating $d\rho/dt$ over the transverse shape of the cloud, we obtain a one-dimensional rate of density decrease $dn_0(t)/dt = -Kn_0(t)^3$, where $K = (\kappa/n_0^3)\iint dxdy\rho(x,y)^3$. Taking into account the transverse broadening of the wave function using the Gaussian ansatz results of [27], we obtain $K = K^0/(1+2n_0 a)$, where $K^0 = \kappa m^2\omega_\perp^2/(3\pi^2\hbar^2)$ [28]. Finally, the rate of variation of the total atom number N is

$$\frac{dN}{dt} = -\int_{-R}^{R} dz K(z) n_0(z)^3. \tag{1}$$

At any time, the measured profile is very close to an equilibrium profile, which indicates the loss rate is small enough to ensure adiabatic following of $n_0(z)$. Then N and $n_0(z)$ are completely determined by n_p and Eq. (1) can be transformed into a differential equation for n_p. We solve it numerically for the parameters of the experimental data, namely the frequency $\omega_\perp$ and the initial peak density, using the LDA to relate N and $n_0(z)$ to n_p. Calculations, shown in Fig. 1, are in good agreement with the experimental data, which confirms that losses are largely dominated by three-body losses. In contrast to [5], where losses are dominated by engineered large one-body losses, we rely here on intrinsic collisional properties of the gas.

The temperature of the gas is determined analyzing the large density ripples that appear after a time of flight t_f [5,29–32]. Interactions are effectively quickly turned off by the transverse expansion of the gas and the subsequent free evolution transforms longitudinal phase fluctuations into density fluctuations. Using an ensemble of images taken in

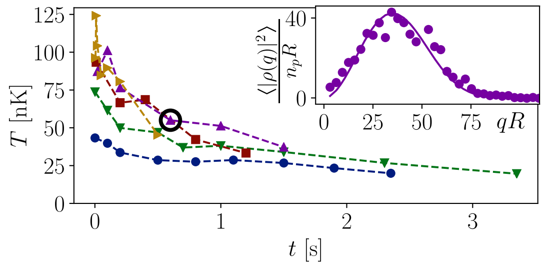

FIG. 2. Evolution of the temperature for the five data sets (same color code and symbols as in Fig. 1). Inset: density ripples power spectrum corresponding to the encircled point, with the fit in solid line yielding the temperature.

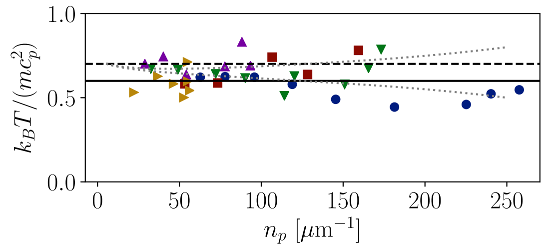

FIG. 3. Evolution of the ratio $k_BT/(mc_p^2)$, in the course of the three-body loss process, for the five data sets (same color code and symbols as in Fig. 1). The temperature decreases with c_p^2, which is approximately proportional to n_p. Solid (respectively, dashed) lines: asymptotic ratio for a 1D homogeneous (respectively, harmonically confined) gas. Dotted lines: numerical calculation, that takes into account the transverse swelling, for two different initial situations close to that of experimental data.

the same experimental condition, we extract the density ripple power spectrum

$$\langle|\rho(q)|^2\rangle = \left\langle\left|\int dz[n(z,t_f) - \langle n(z,t_f)\rangle]e^{iqz}\right|^2\right\rangle. \quad (2)$$

We choose t_f small enough so that the density ripples occurring near the position z are produced by atoms which were initially in a small portion of the cloud, located near z. We can thus use, within a LDA, the analytic predictions for homogeneous gases to compute the expected power spectrum of the trapped gas [32]. We take into account the finite resolution of the imaging system modeling its impulse response function by a Gaussian of rms width σ_{res}. For a given data set the density ripple power spectrum recorded at $t = 0$ is fitted with the temperature T and σ_{res}, the latter depending on the transverse width of the cloud and thus on $\omega_\perp$. We then fit $\langle|\rho_q|^2\rangle$ at larger values of t with T as a single parameter (see inset Fig. 2). The time evolution of T is shown in Fig. 2 for the five different data sets investigated in this Letter. The temperature decreases with t, which indicates a cooling mechanism associated with the three-body losses. Note that this thermometry probes phononic collective modes since the experimentally accessible wave vectors are much smaller than the inverse healing length $\xi^{-1} = \sqrt{mgn_0}/\hbar$.

Figure 3 shows the same data, with the temperature normalized to mc_p^2, where $c_p = \sqrt{n_p\partial_n\mu|_{n_p}/m}$ is the sound velocity at the center of the cloud, shown versus the peak density n_p. While n_p explores more than one order of magnitude, remarkably $k_BT/(mc_p^2)$ shows small dispersion and is close to its mean value 0.64, the standard deviation being 0.02 [33].

The absolute linear density is, however, not the most relevant quantity. A 1D gas at thermal equilibrium is characterized by the dimensionless quantities $\gamma = mg/(\hbar^2 n)$ and $t_{YY} = \hbar^2 k_BT/(mg^2)$ [18]. In particular, the quantum degeneracy condition corresponds to the line $\gamma^2 t_{YY} \simeq 1$. Moreover, the crossover between the ideal Bose gas regime and the quasicondensate regime occurs, within the region $\gamma \ll 1$, along the line $\gamma^{3/2}t_{YY} \simeq 1$. Finally, within the quasicondensate regime, the line $\gamma t_{YY} \simeq 1$ separates the high temperature regime, where the zero distance two- and three-body correlations functions $g^{(2)}(0)$ and $g^{(3)}(0)$ are dominated by thermal fluctuations and are larger than 1 from the low temperature regime, where $g^{(2)}(0)$ and $g^{(3)}(0)$ are dominated by quantum fluctuations and are smaller than 1 [34]. Here, we generalize these 1D parameters to quasi-1D gases introducing $\tilde{t}_{YY} = \hbar^2 k_BTn^2/(m^3c^4)$ and $\tilde{\gamma} = m^2c^2/(\hbar^2n^2)$. For a harmonically confined gas, we refer in the following to the values of $\tilde{t}_{YY}$ and $\tilde{\gamma}$ evaluated at the trap center. The evolution of the state of the gas during the three-body loss process is shown in Fig. 4 in the $(\tilde{t}_{YY}, \tilde{\gamma})$ space. All data collapse on the line $\tilde{\gamma}\tilde{t}_{YY} = k_BT/(mc_p^2) = 0.7$, with a maximum deviation of 36%, while $\tilde{t}_{YY}$ explore more than 2 orders of magnitude.

The physics at the origin of the observed behavior can be understood by considering the simple case of a pure 1D homogeneous quasicondensate. We give here a simplified analysis and refer the reader to [11] for a more complete study. At first, let us solely consider the effect of three-body losses, during a time interval dt, in a small cell of the gas of length Δ. The density is $n = n_0 + \delta n$, where n_0 is the mean density and $\delta n \ll n_0$ since we consider a quasicondensate.

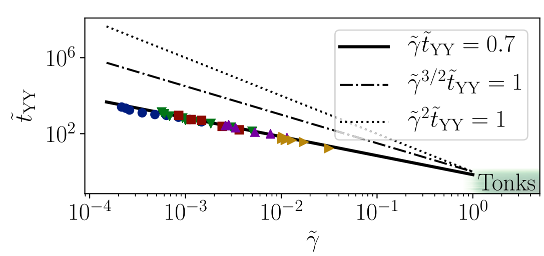

FIG. 4. The data collapse on the line $\tilde{\gamma}\tilde{t}_{YY} = 0.7$. The lower right corner corresponds to the strongly interacting Tonks-Girardeau regime. The data sets and color (symbols) codes are the same as in all other figures.

The density evolves according to $dn = -Kn^3 dt + d\eta$, where $d\eta$ is a random variable of vanishing mean value reflecting the stochastic nature of the loss process. During dt the loss process is close to Poissonian and $\langle d\eta^2 \rangle = 3Kn^3 dt/\Delta \simeq 3Kn_0^3 dt/\Delta$, where the factor 3 comes from the fact that each loss event amounts to the loss of 3 atoms. To first order in δn, the mean density evolves according to $dn_0 = -Kn_0^3 dt$, and the expansion of dn yields

$$d\delta n = -3Kn_0^2 \delta n dt + d\eta. \quad (3)$$

The two terms of the right-hand side correspond to the two competing effects of losses. The first term, a drift term, reduces the density fluctuations: it thus decreases the interaction energy, leading to a cooling. The second term, a stochastic term due to the discrete nature of the atom losses, increases the density fluctuations and thus induces a heating. Going to the continuous limit, one has $\langle d\eta(z) d\eta(z') \rangle = 3Kn_0^3 dt \delta(z - z')$.

Let us now consider the intrinsic dynamics of the gas. Within the Bogoliubov approximation, valid in the quasicondensate regime, one identifies independent collective modes and, up to a constant term, the Hamiltonian of the gas writes $H = \sum_k H_k$, where

$$H_k = A_k \delta n_k^2 + B_k \theta_k^2 \quad (4)$$

is the Hamiltonian of the collective mode of wave vector k [8]. Here, the conjugate quadratures δn_k and θ_k are the Fourier components of δn and θ, $B_k = \hbar^2 k^2 n_0/(2m)$, and, as long as phononic modes are considered, $A_k = g/2$. At thermal equilibrium the energy is equally distributed between the quadratures so that $\langle H_k \rangle /2 = A_k \langle \delta n_k^2 \rangle = B_k \langle \theta_k^2 \rangle$. Let us compute the evolution of $\langle H_k \rangle$ under the effect of losses, assuming the loss rate is small compared to the mode frequency ω_k such that the equipartition holds for all times. First, the Hamiltonian parameter B_k changes according to $dB_k = -Kn_0^2 B_k dt$. Second, according to Eq. (3), the losses modify the distribution on the quadrature δn_k and we obtain $d\langle \delta n_k^2 \rangle /dt = -6Kn_0^2 \langle \delta n_k^2 \rangle + 3Kn_0^3$ [35]. Summing this two contributions leads to

$$\frac{d\langle H_k \rangle}{dt} = -\frac{7}{2} Kn_0^2 \langle H_k \rangle + \frac{3}{2} Kn_0^3 g. \quad (5)$$

From this equation, and using $dn_0/dt = -Kn_0^3$, we derive the evolution of the ratio $y = \langle H_k \rangle /(mc^2)$, where $c = \sqrt{gn_0/m}$ is the speed of sound. We find that y converges at long times towards the stationary value $y_\infty = 0.6$. Phononic modes typically have large occupation numbers for values of y of the order of or larger than 1 so that $\langle H_k \rangle \simeq k_B T$, where T is the mode temperature, and $y = k_B T/(mc^2)$.

In the presence of a harmonic longitudinal potential, calculations which assume that the loss rate is small enough to neglect nonadiabatic coupling between modes, predict a stationary value of the ratio $k_B T/(mc_p^2)$ equal to $y_\infty = 0.70(1)$ [11], a value close to experimental data. For a more precise comparison of data with theory, we compute the time evolution of y according to the formula derived in [11], that takes into account the transverse swelling of the wave function which occurs in our data at large na. The results, shown in Fig. 3 for two different initial situations, is close to experimental data. Even at the beginning of the observed time evolution, the ratio $k_B T/(mc_p^2)$ in our gases is close to its asymptotic value. Data are taken only for gases that were sufficiently cooled by evaporative cooling to be in the quasicondensate regime, where both our thermometry and the theoretical description of the effect of losses are applicable. It occurs that, in our experiment, when the gas enters the quasicondensate regime the ratio $k_B T/(mc_p^2)$ is already close to 0.7.

In conclusion, we showed in this Letter that, under a three-body losses process, the temperature of a quasicondensate in the quasi-1D regime decreases in time. The ratio $k_B T/(mc_p^2)$ stays close to the predicted stationary value, which results from the competition between the cooling effect of losses and the heating due to the stochastic nature of losses. This work raises many different questions. First, the cooling mechanism presented in this Letter is not restricted to 1D quasicondensates and it would be interesting to investigate it in other regimes and dimensions, in particular as one approaches the Tonks regime of 1D gases. Second, while results presented in this Letter concern only the phononic modes, it would be interesting to study the effect of losses on higher energy modes. They might reach higher temperatures than phononic modes, as predicted for one-body losses [7], and the stability of such a nonthermal situation might be particular to the case of 1D gases. Finally, it is interesting to compare the three-body losses cooling to the commonly used evaporative cooling mechanism, which occurs via the removal of atoms whose energy is larger than the trap depth. Its efficiency drops drastically for temperatures lower than mc_p^2/k_B: the relevant excitations are then phonons, which do not extend beyond the condensate, and are thus very difficult to "evaporate." Thus, obtaining, by means of evaporative cooling, temperatures lower than the asymptotic temperature imposed by three-body losses is not guaranteed.

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