Laser Power Stabilization beyond the Shot Noise Limit Using Squeezed Light

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High levels of laser power stability are necessary for high precision metrology applications. The classical limit for the achievable power stability is determined by the shot noise of the light used to generate a power control signal. Increasing the power of the detected light reduces the relative shot noise level and allows higher stabilities. However, sufficiently high power is not always available and the detection of high laser powers is challenging. Here, we demonstrate a nonclassical way to improve the achievable power stability without increasing the detected power. By the injection of a squeezed vacuum field of light we improve the classical laser power stability beyond its shot noise limit by $9.4^{+0.6}_{-0.6}$ dB at Fourier frequencies between 5 and 80 kHz. For only 90.6 μ A of detected photocurrent we achieve a relative laser power noise of $2.0^{+0.1}_{-0.1} \times 10^{-8}/\sqrt{\text{Hz}}$. This is the first demonstration of a squeezed light-enhanced laser power stabilization and its performance is equivalent to an almost tenfold increase of detected laser power in a classical scheme. The analysis reveals that the technique presented here has the potential to achieve stability levels of $4.2 \times 10^{-10}/\sqrt{\text{Hz}}$ with 58 mA photocurrent measured on a single photodetector.

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Laser power stabilization is essential for many optical and quantum-optical experiments. For example, among the most demanding applications are laser interferometric gravitational wave detectors [1–4] and experiments to reveal the standard quantum limit [5]. All of today's gravitational wave observations [6–12] are based on the sensing of differential arm-length changes of km-scale Michelson-type laser interferometers. One of the noise contributions that must be mitigated when aiming for an increase of the astrophysical reach of future-generation instruments is the fluctuating power of the lasers which, in the dc-readout configuration couple to the output of the interferometer [13] and can mask gravitational wave signals.

In general, laser power noise can be reduced by means of active controls loops for which a fraction of the laser field is detected with a photodetector and then compared to an ultrastable reference to generate an error signal. Noise related to this detection process gets imprinted on the output and limits the achievable noise suppression of the control loop. The fundamental classical limit is determined by the photon shot noise, which scales with the square root of the detected laser power. The relative power noise (RPN) is given by the residual fluctuations divided by average power and, consequently, scales with 1 over the square root of the detected power such that detecting higher powers improves the achievable noise performance.

Following this classical approach, a RPN of $2.7 \times 10^{-9}/\sqrt{\text{Hz}}$ was achieved by detecting a photocurrent of 58 mA with a single photodiode [14]. For a single-diode-

based detection scheme with a low-noise photodetector, a further increase of the detected power entails a high effort in handling the thermal load due to higher photocurrents. To circumvent these limitations and increase the optical power nonetheless, a total photocurrent of 200 mA was detected by splitting the optical power on an array of four photodiodes achieving a RPN of down to $1.8 \times 10^{-9} / \sqrt{\text{Hz}}$ [5,15]. Using a power detection method called optical ac coupling, in which the carrier power can be reduced while preserving the power fluctuation sidebands, a RPN of $7.0 \times 10^{-10} / \sqrt{\text{Hz}}$ was demonstrated at MHz frequencies with 3 mA detected photocurrent [16]. In contrast to gravitational wave detection, experimental situations might exist in which only a limited amount of light power can be spared for power stabilization purposes. In these situations the above-mentioned strategies are not applicable and a fundamental limit of classical power stabilization is reached.

The potential of using squeezed states to enhance measurement processes beyond their classical limit has been demonstrated in numerous experiments, e.g., Refs. [17–22]. To use squeezed vacuum states of light to improve a classical laser power stabilization beyond the shot noise limit was already proposed in 1987 [23]. The scheme relies on substituting the vacuum field, which couples into the detection, by a squeezed vacuum field. The squeezed field has a reduced (squeezed) noise in one quadrature and, following Heisenberg's uncertainty principle, accordingly increased (antisqueezed) noise in the orthogonal quadrature. As the quantum-mechanical description of laser power stabilization suggests [23–25],

only the detected amplitude noise affects the power stabilization scheme, such that by manipulating the vacuum field to be squeezed in the measured amplitude quadrature the power fluctuations of the output field can be reduced.

Here we report, for the first time, on the experimental realization of such a squeezed light enhanced laser power stabilization scheme. With a detected light power of only 106μ W (photocurrent of 90.6 μ A) we measured a squeezed light enhanced RPN of $2.0^{+0.1}_{-0.1}\times 10^{-8}/\sqrt{\text{Hz}}$ at Fourier frequencies between 5 and 80 kHz. This corresponds to a $9.4_{-0.6}^{+0.6}$ dB reduction beyond the shot noise limited relative power noise of $6.0^{+0.1}_{-0.1} \times 10^{-8}/\sqrt{\text{Hz}}$. Here, 10 dB is equivalent to a tenfold increase of detected light power in a classical scheme and corresponds to an improvement of RPN by $\sqrt{10}$, which allows for a significant performance enhancement of high precision measurements, that are either limited by the available laser power or the detectable photocurrent. Our analysis reveals that the significant squeezing level in conjunction with a laser stabilization scheme, which not only suppressed the laser noise down to shot noise level but even beyond, has the potential to achieve stability levels of $4.2 \times 10^{-10} / \sqrt{\text{Hz}}$ with 58 mA detected on a single photodetector.

The field of a laser can be described by time-dependant annihilation operators $\hat{A}(t)$. These operators can be split into a constant field component α and a time varying noise component $\delta \hat{A}(t)$, called noise annihilation operators

$$\hat{A}(t) = \alpha + \delta \hat{A}(t). \tag{1}$$

A laser power stabilization can be described by these operators defining the noise levels within the system as depicted in Fig. 1. Here, $\delta \hat{A}_{IN}$ is the initial laser noise, $\delta \hat{A}_{MOD}$ is the signal after the amplitude modulator, $\delta \hat{A}_{IL}$ is the signal in front of the in-loop photodetector (PD_{IL}), $\delta \hat{A}_{OUT}$ is the downstream output noise of the laser power stabilization and $\delta \hat{A}_{BS}$ is the noise coupling through the second input port of the beam splitter. The relation between these fields can be described by

$$\delta \hat{A}_{\rm MOD} = \delta \hat{A}_{\rm IN} + \delta \hat{A}_{\rm IL} H, \qquad (2)$$

$$\delta \hat{A}_{\rm IL} = t \delta \hat{A}_{\rm MOD} + r \delta \hat{A}_{\rm BS},\tag{3}$$

$$\delta \hat{A}_{\rm OUT} = -r\delta \hat{A}_{\rm MOD} + t\delta \hat{A}_{\rm BS},\tag{4}$$

where *r* and *t* are the beam splitter's amplitude reflectivity and transmissivity, respectively. Here, optical loss is neglected and we assume $r^2 + t^2 = 1$. The complex electronic control loop feedback gain is denoted as *H*. For a steady state, these equations can be combined to yield



FIG. 1. Laser power stabilization described by means of frequency dependent noise annihilation operators. The laser power noise $\delta \hat{A}_{IN}$ is modulated by an amplitude modulator yielding $\delta \hat{A}_{MOD}$. A fraction of this field is transmitted through a beam splitter and interferes with noise $\delta \hat{A}_{BS}$ entering through the second input port of the beam splitter. Both fields constitute the in-loop field $\delta \hat{A}_{IL}$ which is detected by a photodetector (PD_{IL}), gets frequency dependently amplified by an electronic controller, and is fed back to the amplitude actuator. The bright light field reflected by the beam splitter is the stabilized output field with noise $\delta \hat{A}_{OUT}$. For a nonclassical laser power stabilization scheme the vacuum fluctuations coupling into the system ($\delta \hat{A}_{BS}$) are substituted by an amplitude quadrature squeezed vacuum field leading to an output field $\delta \hat{A}_{OUT}$ with reduced noise.

$$\delta \hat{A}_{\text{MOD}} = \frac{\delta \hat{A}_{\text{IN}} + rH\delta \hat{A}_{\text{BS}}}{1 - tH},$$
(5)

$$\delta \hat{A}_{\text{OUT}} = \frac{-r\delta \hat{A}_{\text{IN}} - r^2 H \delta \hat{A}_{\text{BS}}}{1 - tH} + t\delta \hat{A}_{\text{BS}}.$$
 (6)

For frequencies with high loop gain H

$$\lim_{H \to \infty} \delta \hat{A}_{\text{OUT}} = \frac{r^2}{t} \delta \hat{A}_{\text{BS}} + t \delta \hat{A}_{\text{BS}} = \frac{1}{t} \delta \hat{A}_{\text{BS}}, \qquad (7)$$

such that the initial noise is fully suppressed and only the noise from the second beam splitter input port couples to the output field. This result needs to be converted into an experimentally accessible quantity. The photocurrent measured by a photodetector is proportional to the number of photons, which is given by the number operator $\hat{N} = \hat{A}^{\dagger} \hat{A}$. Using the linearization given by Eq. (1) and neglecting the higher order term it can be rewritten as

$$\hat{N}(t) \approx \alpha^2 + \alpha \delta \hat{X}^+(t),$$
 (8)

where $\delta \hat{X}^+(t) = \delta \hat{A}(t) + \delta \hat{A}^{\dagger}(t)$ is the amplitude quadrature operator. The average photon number on the detector is described by α^2 .

Usually, the photocurrent is measured in frequency domain and is expressed as a single-sided power spectrum [26]

$$PSD(\Omega) = \frac{2Var[\hat{N}(\Omega)]}{B} = \frac{2\alpha^2 V^+(\Omega)}{B}.$$
 (9)

Here, Ω indicates the Fourier frequency, *B* is the measurement bandwidth, and $V^+(\Omega) = \text{Var}(\delta \hat{X}^+)(\Omega)$ is the amplitude quadrature variance normalized to a vacuum state in frequency space. The RPN in units $1/\sqrt{\text{Hz}}$ is expressed as the square root of the normalized PSD for a bandwidth of 1 Hz

$$\operatorname{RPN}(\Omega) = \sqrt{\frac{\operatorname{PSD}(\Omega)}{\alpha^4}} = \sqrt{\frac{2V^+(\Omega)}{\alpha^2}}.$$
 (10)

Finally, to determine the RPN at the output of the laser power stabilization, the variance $V^+(\Omega)$ of the output field given by Eq. (7) needs to be determined. Using the amplitude quadrature operator $\delta \hat{X}^+(t)$, taking the Fourier transform and calculating the variance yields $V_{\rm OUT}^+ = (1/t^2)V_{\rm BS}^+$. Plugging this into Eq. (10) and substituting the average photon number α^2 by the optical power $P_{\rm OUT}$ at the output divided by the energy of a photon hc/λ results in

$$\operatorname{RPN}_{\operatorname{OUT}} = \sqrt{\frac{2hc}{\lambda}} \frac{V_{\mathrm{BS}}^+}{t^2 P_{\mathrm{OUT}}} = \sqrt{\frac{2hc}{\lambda}} \frac{V_{\mathrm{BS}}^+}{t^2 P_{\mathrm{IL}}}, \quad (11)$$

where *h* is the Planck constant, *c* the speed of light, λ the wavelength, and P_{IL} corresponds to the in-loop detected power.

Thus, for a high control loop gain H and a given $P_{\rm IL}$ (e.g., the maximal light power the photodetector can handle), the output noise level only depends on the noise coupling into the open beam splitter port. This yields the classical stabilization limit when vacuum fluctuations $(V_{\rm BS}^+ = 1)$ are coupling into this port. If, however, these vacuum fluctuations are replaced by a squeezed vacuum field $(V_{\rm BS}^+ < 1)$ the RPN improves accordingly, surpassing the classical limit. The purity of squeezed states of light is affected by decoherence effects such as residual quadrature phase noise and optical loss. In our experiment we assumed the phase noise contribution to be negligible. The optical loss can be modeled by mixing the (pure) squeezed state with vacuum fluctuations corresponding to

$$V_{\rm loss}^{+} = (1 - l)V_{\rm pure}^{+} + lV_{\rm vac}^{+}, \tag{12}$$

where *l* is the optical loss and $V_{\text{vac}}^+ = 1$.

The schematic of the experimental setup is depicted in Fig. 2. The main laser source was a continuous-wave single-mode monolithic Nd:YAG laser in a nonplanar ring oscillator configuration producing 500 mW output at a wavelength of 1064 nm. A direct modulation of the internal pump diode current allowed for a fast amplitude modulation of the output field. About 300 mW of the laser beam were sent towards the squeezed light source, which



FIG. 2. Schematic of the experimental setup. The performance of a classic laser power stabilization scheme was improved by the injection of squeezed vacuum states at the 99/1 pick-off beam splitter. The squeezed light source consists of a second harmonic generator (SHG) providing the pump field for a doubly resonant optical parametric amplifier (OPA). Beam splitter (BS);, polarizing beam splitter (PBS), dichroic beam splitter (DBS). For the sake of clarity further optical elements have been omitted. An out-of-loop photodetector (PD_{OOL}) was placed in the 10 mW bright laser field for an independent characterization of the nonclassical light enhanced noise reduction. A shutter acts as a beam dump for the squeezed vacuum and thereby allowing us to switch between a classical and the nonclassical laser power stabilization scheme.

includes a second harmonic generator (SHG) and an optical parametric amplifier (OPA) as the main building blocks. Both, the SHG and the OPA were set up in a linear cavity design consisting of a nonlinear crystal and an additional coupling mirror. The SHG utilized a MgO:LiNbO₃ nonlinear crystal to produce a frequency doubled output light field at a wavelength of 532 nm. This pump field was injected into a doubly resonant OPA based on a periodically poled KTP (PPKTP) crystal. A more detailed description of the SHG and OPA as well as the electronic control scheme can be found in Ref. [27]. We injected 9.5 mW of the generated green pump field into the OPA, thereby operating the OPA below its oscillation threshold. The squeezed vacuum states of light at 1064 nm, generated by the OPA via a parametric down-conversion process, were separated from the pump light by a dichroic beam splitter (DBS).

The remaining intense infrared laser field in transmission of the first beam splitter was attenuated to approximately 10 mW with a half-wave plate and a polarization beam splitter. Another beam splitter with a power reflectivity of R = 99% (T = 1%) was used to split light between the inloop detection and the intense output light path. Both fields were measured by independent photodetectors. The in-loop photodetector (PD_{IL}) measured the laser power fluctuations by detecting 106 μ W. For this measurement a highsensitivity and high-speed photodetector was designed and built based on a custom made positive intrinsic negative (*p-i-n*) InGaAs photodiode with a quantum efficiency of about 99%. The signal measured by PD_{IL} carried the noise information of the main laser field and served as the sensor for the laser power stabilization control loop. This signal was fed into homemade servoelectronics (Controller) which provided a frequency dependent negative feedback to the lasers pump diode current at frequencies between 10 Hz and 1 MHz thereby stabilizing the laser output power at 1064 nm.

The out-of-loop photodetector (PD_{OOL}) was used to sense the intense output light field (10 mW). This allowed for control loop independent measurements of the relative laser power noise. The light power independent linearity of both photodetectors was validated in separate measurements.

In the case of a classic laser power stabilization (shutter closed in Fig. 2) the control loop stabilized the power noise of the free running laser to a relative power noise level of $6.0^{+0.1}_{-0.1}\times 10^{-8}/\sqrt{Hz}$ in the frequency range from 5 to 100 kHz. This corresponds to the shot noise limited relative power noise for 106 μ W detected in-loop, which can be determined using $\sqrt{2eI}R_{\rm TI}/U_{\rm dc}$ with a photocurrent $I = 90.6 \ \mu A$, deduced from the photodetectors transimpedance resistor $R_{\rm TI} = 33.1 \ \rm k\Omega$ and the measured output voltage $U_{dc} = 3$ V. The power noise measured on PD_{OOL} and normalized to the shot noise of the photocurrent of PD_{IL} with the controller feedback engaged is plotted as trace (a) in Fig. 3. The average value over the frequency interval from 10 to 80 kHz serves as the 0-dB shot noise reference. The shot noise reference normalized power noise measured with $\ensuremath{\text{PD}_{\text{IL}}}\xspace$ is shown in trace (b). This curve corresponds to the free-running laser power noise suppressed by the electronic control loop gain. It shows the in-loop shot noise suppression and hence, this measurement is an indicator for the achievable out-of-loop performance if an infinite amount of squeezing could be detected on the in-loop detector and electronic noise contributions were negligible.

The in-loop electronic dark noise is shown in trace (c). It was measured independently and was more than 15 dB below the shot noise reference at Fourier-frequencies above 10 kHz. In general the detector noise that gets imprinted on the laser output consists of the shot noise and the electronic dark noise contribution. Because of the high dark noise clearance our measurement was dominated by photon shot noise only and other noise contributions were negligible. The dark noise of the out-of-loop photodetector (not shown) was more than 30 dB below the shot noise.

For a nonclassical light enhanced laser power stabilization (shutter open in Fig. 2) we substituted the unsqueezed vacuum fluctuations at the open beam splitter port of the laser power stabilization by squeezed vacuum states. The squeezed vacuum field was carefully mode matched to the bright laser beam at the beam splitter and their relative phase was manually adjusted by a piezo actuated mirror, indicated as the phase shifter in Fig. 2. This allowed us to



FIG. 3. Noise budget of the squeezing enhanced laser power stabilization. Trace (a): Measured out-of-loop laser noise normalized to the shot noise level of the 90.6 uA in-loop detected power. This is the classical stabilization reference. Trace (b): Inloop measurement of the free-running laser power noise suppressed by the electronic loop gain. Trace (c): Electronic dark noise of the in-loop photodetector. Trace (d): Nonclassical laser power noise reduction. Between 5-80 kHz an average of $9.4^{+0.6}_{-0.6}$ dB below the shot noise was achieved. Trace (e): Simulated squeezing enhancement. Trace (f): Uncorrelated sum of limiting noise sources (b),(c),(e). All traces were normalized to the average of the shot noise reference. All measurements were performed with a fast Fourier transform (FFT) analyzer (model SR785) at Fourier frequencies from 0 to 100 kHz with 200 FFT lines, a sweep time of 1.95 ms, and ten averages. The peak around 40-50 kHz in trace (b) is due to an in-loop measurement related artifact. Further tests showed that it does not appear in the out-of-loop detected laser power noise.

control the squeezing phase for detecting minimal amplitude quadrature noise of the combined beam after the 99/1 beam splitter. The corresponding measurement of the nonclassically enhanced power stabilization is show as trace (d) in Fig. 3. We measured a reduction of laser power noise on PD_{OOL} by an average of $9.4^{+0.6}_{-0.6}$ dB beyond the shot noise reference at Fourier frequencies between 5 and 80 kHz. The squeezing enhanced power stabilization yields a relative power noise level of $2.0^{+0.1}_{-0.1} \times 10^{-8}/\sqrt{\text{Hz}}$ compared to $6.0^{+0.1}_{-0.1} \times 10^{-8}/\sqrt{\text{Hz}}$ for the classical case.

Squeezed states of light are highly susceptible to optical loss, which mixes in contributions of unsqueezed vacuum noise and is therefore a figure of merit for the application of squeezing. In our experiment various factors contributed to the loss budget. The loss due to the escape efficiency of the OPA was determined to be $0.95^{+0.45}_{-0.40}\%$ in Ref. [27]. The interference contrast between the intense laser field and the squeezed vacuum field was measured to be $98.5^{+0.1}_{-0.1}\%$ corresponding to loss of $3.0^{+0.2}_{-0.2}\%$. In a separate experiment we characterized the reflectivity and transmissivity of the 99/1 beam splitter and determined an optical loss for the squeezed state of $1.0^{+0.2}_{-0.2}\%$. The quantum efficiency of

the photodiode of the in-loop PD was assumed to be approximately $99.5^{+0.5}_{-0.5}\%$ [27]. Optical loss due to nonperfect antireflection coatings of the lenses and residual transmission through high-reflectivity mirrors were estimated to be smaller than 0.2%. In total, these contributions yield an overall optical loss $l = 5.6^{+0.9}_{-0.9}\%$.

The squeezing level for a detection efficiency $\eta = 1 - l$ can be computed as $X_{dB} = 10\log_{10}\{1 - \eta[4\sqrt{x}/(1 + \sqrt{x})^2]\}$, where *x* is equal to the pump power injected into the OPA divided by the threshold power [17,28]. We measured the pump power to be $9.5^{+0.2}_{-0.2}$ mW, which corresponds to x = 0.5 [27]. With the detection efficiency $\eta = 94.4^{+0.9}_{-0.9}$ % this leads to a predicted squeezing level of $10.8^{+0.4}_{-0.5}$ dB. The mean value is illustrated as trace (e) in Fig. 3. In contrast to the classical experiment, at this lowered noise level the electronic dark noise, trace (c), as well as the residual in-loop noise, trace (b), contribute significantly to the final stabilized power noise. By taking the uncorrelated sum of these noise sources and the simulated squeezing level the expectable nonclassical improvement was calculated which is illustrated in trace (f). Our measurement, as shown in trace (d), is in very good agreement with the theoretical expectation.

The performance of the nonclassical laser power stabilization could be improved by further reduction of optical loss for the squeezed field. A higher OPA pump power would increase the injected squeezing level; however, in our experiment the corresponding increase of the antisqueezing level led to instabilities in the control loop. Moreover, a further reduction of the electronic noise contributions would improve the measurable squeezing enhancement. As shown in Fig. 3 at frequencies below 50 kHz the dark noise has the main impact while at higher frequencies the bandwidth and gain of the control loop are currently limiting.

In conclusion, we have demonstrated the first nonclassical light enhanced laser power stabilization by substituting the vacuum fluctuations sensed in a classical laser power stabilization scheme by squeezed vacuum states. We demonstrated a $9.4^{+0.6}_{-0.6}$ dB improvement beyond the classical shot noise limit at Fourier frequencies between 5 and 80 kHz. The observed noise reduction is in good agreement with our theoretical prediction and corresponds to an almost tenfold increase in detected optical power required in a purely classical scheme. With the technique presented here it seems feasible to achieve 15 dB of nonclassical noise reduction, which would correspond to an almost 32-fold increase of directly detected laser power. Together with the detection of 58 mA optical power [14] our scheme would allow for a RPN of $4.2 \times 10^{-10} / \sqrt{\text{Hz}}$, surpassing the best laser power stabilization demonstrated so far. The squeezing enhancement could also be utilized to improve the performance of the array detection scheme [16]. Here, injecting 15 dB of squeezing would yield an equivalent detected power of 6.4 W and would allow for a RPN of $2.3 \times 10^{-10} / \sqrt{\text{Hz}}$.

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- J. Aasi *et al.* (LIGO Scientific Collaboration), Classical Quantum Gravity **32**, 074001 (2015).
- [2] F. Acernese *et al.* (Virgo Collaboration), Classical Quantum Gravity **32**, 024001 (2015).
- [3] K. L. Dooley *et al.*, Classical Quantum Gravity **33**, 075009 (2016).
- [4] B. P. Abbott *et al.* (KAGRA, LIGO Scientific, and Virgo Collaborations), Living Rev. Relativity 19, 1 (2016).
- [5] J. Junker, P. Oppermann, and B. Willke, Opt. Lett. 42, 755 (2017).
- [6] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Phys. Rev. Lett. **116**, 061102 (2016).
- [7] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Phys. Rev. Lett. **116**, 241103 (2016).
- [8] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Phys. Rev. Lett. **118**, 221101 (2017).
- [9] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Astrophys. J. Lett. 851, L35 (2017).
- [10] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Phys. Rev. Lett. **119**, 141101 (2017).
- [11] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Phys. Rev. Lett. **119**, 161101 (2017).
- [12] B. P. Abbott et al., Astrophys. J. Lett. 848, L12 (2017).
- [13] S. Hild *et al.*, Classical Quantum Gravity **26**, 055012 (2009).
- [14] F. Seifert, Ph.D. thesis, Leibniz Universität Hannover, 2010.
- [15] P. Kwee, B. Willke, and K. Danzmann, Opt. Lett. 34, 2912 (2009).
- [16] P. Kwee, B. Willke, and K. Danzmann, Opt. Lett. 33, 1509 (2008).
- [17] E. S. Polzik, J. Carri, and H. J. Kimble, Appl. Phys. B 55, 279 (1992).
- [18] M. A. Taylor, J. Janousek, V. Daria, J. Knittel, B. Hage, H.-A. Bachor, and W. P. Bowen, Nat. Photonics 7, 229 (2013).
- [19] S. Steinlechner, J. Bauchrowitz, M. Meinders, H. Müller-Ebhardt, K. Danzmann, and R. Schnabel, Nat. Photonics 7, 626 (2013).
- [20] T. Gehring, V. Händchen, J. Duhme, F. Furrer, T. Franz, C. Pacher, R. Werner, and R. Schnabel, Nat. Commun. 6, 8795 (2015).
- [21] J. Abadie *et al.* (The LIGO Scientific Collaboration), Nat. Phys. 7, 962 (2011).
- [22] J. Aasi et al., Nat. Photonics 7, 613 (2013).
- [23] C. M. Caves, Opt. Lett. 12, 971 (1987).
- [24] M. S. Taubman, H. Wiseman, D. E. McClelland, and H.-A. Bachor, J. Opt. Soc. Am. B 12, 1792 (1995).
- [25] B. C. Buchler, E. H. Huntington, C. C. Harb, and T. C. Ralph, Phys. Rev. A 57, 1286 (1998).
- [26] B.C. Buchler, Ph.D. thesis, The Australian National University, 2001.
- [27] H. Vahlbruch, M. Mehmet, K. Danzmann, and R. Schnabel, Phys. Rev. Lett. **117**, 110801 (2016).
- [28] T. Aoki, G. Takahashi, and A. Furusawa, Opt. Express 14, 6930 (2006).