Higher-Spin Gauge Theories and Bulk Locality

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We present a no-go result on consistent Noether interactions among higher-spin gauge fields on anti–de Sitter space-times. We show that there is a nonlocal obstruction at the classical level to consistent interacting field theory descriptions of massless higher-spin particles that are described in the free limit by the free Fronsdal action, under the assumption that such theories arise from the gauging of a global higher-spin symmetry. Our result suggests that the Fronsdal program for introducing interactions among higher-spin gauge fields cannot be completed without introducing new guiding principles, which could potentially lie beyond the framework of classical field theory.

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Introduction.—In this Letter, we consider the problem of constructing consistent interacting higher-spin (HS) gauge theories on anti–de Sitter (AdS) space-times. By now, the free propagation of massless HS particles is rather well understood, with an off-shell classical description given by the Fronsdal action [1]. But the question of whether massless HS particles can interact in a consistent manner is a highly nontrivial one. This question has been the subject of decades of intense efforts, which have over the years gathered increased attention owing (in part) to the tantalizing hypothesis that a HS symmetry may govern the high-energy regime of a UV-complete theory of gravity [2]. That this is a possibility within string theory was argued by Gross in the 1980s [3], based on the high-energy behavior of string amplitudes [4,5].

Among these efforts, in flat space there are various well-known no-go results [6–18] on the interactions of massless HS particles. Backgrounds of constant nonzero curvature appear to show more promise with, e.g., the possibility of a minimal coupling of HS gauge fields to gravity [16,19,20] and the existence of a well-defined HS algebra [2,21,22]. This promise was given further weight by the advent of conjectured holographic dualities between HS gauge theories on AdS and free conformal field theories [23–25] and Vasiliev's system [22,26], which proposes a field-theoretical framework to describe putative consistent (semi) classical interacting theories of HS gauge fields.

A time-honored approach to constructing interacting theories with a gauge symmetry at the classical level is the Noether procedure [27]. To this end, given the free theory, one attempts to systematically construct interactions as deformations of the free action or equations of motion through the requirement of gauge invariance. An important subtlety in using such approaches in a field theory setting is that of locality. This is particularly relevant for HS gauge theories: It is well known that a putative interacting theory of HS gauge fields would require us to allow nonlocal interactions that are unbounded in their number of derivatives at quartic order [28]. The crucial subtlety in the search for nontrivially interacting HS field theories is whether or not the functional class of such nonlocalities renders the theory equivalent to the free theory under field redefinitions. This point was first clarified by Barnich and Henneaux in Ref. [32], where this triviality of interactions was shown to arise as a consequence of placing no restriction on the functional class of nonlocalities. This was further refined in Refs. [14,17], where it was shown that a nontrivially interacting field theory is not possible with a functional class that allows contact interactions that are as nonlocal as the total exchange amplitude. We refer to such functional classes as nonlocal obstructions throughout [33].

A key open question is whether or not there exists a weaker notion of nonlocality for HS gauge theories which would lie somewhere between locality in the strict sense and a nonlocal obstruction as defined above. In this Letter, we present a no-go result [39] on the consistent interactions of HS gauge fields on AdS_{d+1} that are described in the free limit by the free Fronsdal action and which arise from the gauging of a global higher-spin symmetry. We find that it is not possible to have interacting classical field theory descriptions of massless HS particles on AdS backgrounds without a nonlocal obstruction. In particular, we clarify the degree of nonlocality that would be needed to construct consistent quartic couplings, which are shown to be as nonlocal as the four-point exchange amplitudes in the theory.

This result suggests that any classical interacting field theory description of massless HS particles on AdS would require new guiding principles to circumvent the triviality problem. We discuss some possibilities at the end of this Letter. Note that this would also provide motivation for revisiting the interaction problem for massless HS in flat space, for which there are analogous no-go results [14,15,17,18] due to the appearence of the same type of nonlocal obstruction.

In the final section, we make contact with recent observations [40] on the properties of correlators in the dual conformal field theory (CFT) picture under crossing.

Noether procedure.—The Noether procedure is a systematic scheme to solve for interacting field theories as deformations of free actions governed by gauge and global symmetries: Given the latter, one postulates the existence of a fully nonlinear action and gauge symmetries, which are expanded on a given background in weak fields as

$$S = S^{(2)} + \sum_{n>2} S^{(n)}, \qquad \delta_\xi = \delta_\xi^{(0)} + \sum_{n>0} \delta_\xi^{(n)}. \quad (1)$$

The notation (n) signifies that the corresponding term is power n in the weak fields.

The requirement of gauge invariance translates into an infinite set of coupled equations:

$$\delta_\xi^{(n-2)} S^{(2)} + \left(\sum_{k=3}^{n-1} \delta_\xi^{(n-k)} S^{(k)} \right) + \delta_\xi^{(0)} S^{(n)} = 0, \quad (2)$$

$n = 2, 3, 4, \ldots,$ whose most general solution at a given order can be written as

$$S^{(n)} = S_h^{(n)} + S_p^{(n)}, \quad (3)$$

where $S_h^{(n)}$ solves the homogeneous equation

$$\delta_\xi^{(0)} S_h^{(n)} \approx 0, \quad (4)$$

where $\approx$ means on shell and $S_p^{(n)}$ is the particular solution to the original Eq. (2) which contains the information about the lower-order solutions.

A particular solution that has a nice physical interpretation is given by *minus* the exchange amplitudes generated by the lower-order couplings. For instance, at quartic order,

$$S_p^{(4)} = -\mathcal{A}^{(4)} \equiv -(\mathcal{A}^{\mathsf{s}} + \mathcal{A}^{\mathsf{t}} + \mathcal{A}^{\mathsf{u}}), \quad (5)$$

where $\mathcal{A}^{\mathsf{s}}$ is the four-point exchange diagram generated by the cubic couplings in the s channel, etc. That this solves the quartic ($n = 4$) consistency condition (2) can be seen extending the analysis from Refs. [14,41,42] to prove

$$\delta_\xi^{(0)}(-\mathcal{A}^{(4)}) \approx \delta_\xi^{(1)} S^{(3)}. \quad (6)$$

A further attractive feature of the above choice of a particular solution is that the corresponding homogeneous solution,

$$S_h^{(n)} = \mathcal{A}^{(n)} + S^{(n)}, \quad (7)$$

is then directly related to "scatteringlike" observables of the theory [43]. This link is especially significant for theories on AdS_{d+1}, where one can draw upon the dual interpretation of such observables as correlation functions of single-trace operators on the d-dimensional conformal boundary. We make this relationship more concrete in the following.

The homogeneous solution, global symmetries, and AdS/CFT.—Further constraints are placed on the homogeneous solution $S_h^{(n)}$ if the gauge symmetry (1) arises from the gauging of a global symmetry. This imposes

$$\delta_{\bar\xi}^{(1)} S_h^{(n)} \approx 0, \quad (8)$$

where the $\bar\xi$'s are the gauge parameters associated with the global symmetries [44].

For theories on AdS_{d+1} backgrounds, for boundary conditions on the bulk fields compatible with conformal symmetry, the form (7) of the homogeneous solution is the generating function of connected n-point correlation functions of single-trace operators in the dual CFT_d at large N [45,46]. In particular, the boundary value $\bar\varphi$ of the bulk field φ sources its dual single-trace operator O such that

$$\langle O_1 \cdots O_n \rangle_{\text{conn}} = (-1)^n \frac{\delta}{\delta \bar\varphi_n} \cdots \frac{\delta}{\delta \bar\varphi_1} S_h^{(n)}[\varphi_i|_{\partial \mathrm{AdS}} = \bar\varphi_i]. \quad (9)$$

The construction of consistent interactions on AdS can thus be mapped to the classification of consistent conformal correlators, which serve as the scattering observables of the bulk theory. In particular, in the dual CFT picture the global symmetry constraints (8) are equivalent to Ward identities

$$\delta_{\bar\xi}^{(1)} S_h^{(n)} \approx 0 \Leftrightarrow 0 = \sum_i \langle O_1 \cdots [Q_{\bar\xi}, O_i] \cdots O_n \rangle, \quad (10)$$

where $Q_{\bar\xi}$ is the charge associated with the Killing tensor $\bar\xi$.

Higher-spin gauge theories.—The free Fronsdal action [1] for spin-s gauge fields serves as a starting point to construct theories of interacting HS gauge fields on AdS_{d+1}. The free theory alone encodes the global symmetries that govern the spectrum, which can be extracted from the free theory Noether currents [16]. The HS symmetry algebra closing on totally symmetric gauge fields [2,21,22] is unique in generic dimensions [47]. This constrains the spectrum via the global symmetry requirement

$$\delta^{(1)}_{\bar{\xi}} S^{(2)} = 0. \tag{11}$$

The minimal HS-symmetric spectrum consists of a tower of even spin totally symmetric gauge fields φ_s, $s = 2, 4, 6, \ldots$ and a parity even scalar φ_0. This is the so-called type A HS theory [22,26], and it is the unique HS theory in generic dimensions consisting of only totally symmetric fields. This minimal HS-symmetric spectrum forms a closed subsector of any theory of totally symmetric HS gauge fields on AdS_{d+1}, which we can therefore restrict to throughout.

At the interacting level, global HS symmetry fixes all interactions and their couplings completely up to field redefinitions. This can be seen by analyzing the constraint (8) for HS Killing tensors $\bar{\xi}$, which forces the homogeneous solution (7) to take the unique form of correlators of a free scalar conformal theory on the d-dimensional boundary when evaluated on shell with AdS/CFT boundary conditions [47–49]. In particular, for fixed external legs of spins $s_1 - s_2 - \cdots - s_n$, we have

$$S^{(n)}_h = \langle \mathcal{J}_{s_1} \cdots \mathcal{J}_{s_n} \rangle_{\mathrm{conn}}, \tag{12}$$

where $\mathcal{J}_{s_i}$ is the spin s_i single-trace operator of twist $\tau = \Delta = d - 2$ on the d-dimensional boundary that is sourced by the bulk field φ_{s_i}. The explicit form of Eq. (12) was first given in Ref. [50] for a general d (see also Refs. [51–53]).

Combined with the particular solution (5), the complete solution to the Noether procedure with the minimal HS spectrum is thus dictated uniquely up to field redefinitions by global HS symmetry:

$$S^{(3)}_{s_1,s_2,s_3} = \langle \mathcal{J}_{s_1} \mathcal{J}_{s_2} \mathcal{J}_{s_3} \rangle_{\mathrm{conn}}, \tag{13a}$$

$$S^{(n)}_{s_1,s_2,\ldots,s_n} = \langle \mathcal{J}_{s_1} \cdots \mathcal{J}_{s_n} \rangle_{\mathrm{conn}} - \mathcal{A}^{(n)}_{s_1,s_2,\ldots,s_n}, \tag{13b}$$

with $n > 3$, where $\mathcal{A}^{(n)}_{s_1,s_2,\ldots,s_n}$ is the tree-level exchange amplitude with external legs of spins $s_1 - s_2 - \cdots - s_n$, generated by the lower-order couplings. Note that solution (13) does not assume the AdS/CFT duality since Eq. (12) holds purely as a consequence of global HS symmetry.

At cubic order, the couplings for any triplet of fixed spins are local (up to field redefinitions),

$$S^{(3)}_{s_1,s_2,s_3} = \int_{\mathrm{AdS}} \mathcal{V}_{s_1,s_2,s_3}, \tag{14}$$

with (schematically) [35,50]

$$\mathcal{V}_{s_1,s_2,s_3} = g_{s_1,s_2,s_3} \nabla^{s_3}\varphi_{s_1} \nabla^{s_1}\varphi_{s_2} \nabla^{s_2}\varphi_{s_3} + O(\Lambda), \tag{15a}$$

$$g_{s_1,s_2,s_3} = \frac{1}{\sqrt{N_{\mathrm{d.o.f.}}}} \frac{\pi^{[(d-3)/4]} 2^{[(3d-1+s_1+s_2+s_3)/2]}}{\Gamma(d + s_1 + s_2 + s_3 - 3)}. \tag{15b}$$

At cubic order, there is thus no apparent issue of locality, with the couplings (14) involving a finite number of derivatives (15a) with finite coupling constants (15b). However, the couplings at quartic and higher orders generically involve an arbitrary number of derivatives [33]. In the following section, we study the (non)locality of the quartic interactions (13b), which is possible with the explicit form of the homogeneous solution (12) with $n = 4$ and the four-point exchange diagrams generated by the local cubic couplings (15a).

Locality.—For simplicity, we restrict to the quartic self-interaction of the scalar φ_0 [Eq. (13b) with $n = 4$ and $s_i = 0$ [54]]:

$$S^{(4)}_{0,0,0,0} = \langle \mathcal{J}_0 \mathcal{J}_0 \mathcal{J}_0 \mathcal{J}_0 \rangle_{\mathrm{conn}} - \mathcal{A}^{(4)}_{0,0,0,0}, \tag{16}$$

where the homogeneous solution reads explicitly

$$\begin{aligned} &\langle \mathcal{J}_0(y_1) \mathcal{J}_0(y_2) \mathcal{J}_0(y_3) \mathcal{J}_0(y_4) \rangle_{\mathrm{conn}} \\ &= \frac{1}{c} \frac{1}{(y^2_{12} y^2_{34})^{d-2}} \\ &\quad \times \left[u^{[(d/2)-1]} + \left(\frac{u}{v}\right)^{[(d/2)-1]} + u^{[(d/2)-1]} \left(\frac{u}{v}\right)^{[(d/2)-1]} \right], \end{aligned} \tag{17}$$

in terms of cross ratios $u = (y^2_{12} y^2_{34} / y^2_{13} y^2_{24})$ and $v = (y^2_{41} y^2_{23} / y^2_{13} y^2_{24})$, and c is proportional to the central charge of the boundary theory. The exchange amplitude is given by

$$\mathcal{A}^{(4)}_{0,0,0,0} = \mathcal{A}^{\mathsf{s}}_{0,0,0,0} + \mathcal{A}^{\mathsf{t}}_{0,0,0,0} + \mathcal{A}^{\mathsf{u}}_{0,0,0,0}, \tag{18}$$

where, e.g.,

$$\mathcal{A}^{\mathsf{s}}_{0,0,0,0} = \sum_{s \in 2\mathbb{N}} \mathcal{A}^{\mathsf{s}}_{0,0|s|0,0}, \tag{19}$$

with each spin-s exchange $\mathcal{A}^{\mathsf{s}}_{0,0|s|0,0}$ is generated by the local 0-0-s cubic coupling (14).

To study the locality of Eq. (16), we first need to perform the sum (19) over the exchanged spin. For this, it is useful to decompose into conformal blocks. For the spin-s exchange, we have [55–57]

$$\mathcal{A}^{\mathsf{s}}_{0,0|s|0,0} = \mathsf{c}^2_{\mathcal{J}_0 \mathcal{J}_0 \mathcal{J}_s} W^{\mathsf{s}}_{0,0|s|0,0} + \text{local contact interactions}, \tag{20}$$

where $W^{\mathsf{s}}_{0,0|s|0,0}$ is the conformal block encoding the contributions in the s channel from the exchanged spin-s single-particle state [58]. In the CFT picture, this is the contribution induced by the dual single-trace primary operator $\mathcal{J}_s$, with operator product expansion coefficient

$\mathsf{c}_{\mathcal{J}_0\mathcal{J}_0\mathcal{J}_s}$. Given Eq. (20), in the view of studying the locality of Eq. (16), it is useful to recall the following standard assumptions of field theory when considering the sum over spin in (19): $\mathbb{A}1$. Infinite summations over derivatives do not generate additional single-particle exchanges, in any channel [59]. $\mathbb{A}2$. Summations over spin do not generate additional single-particle exchanges, in any channel.

While $\mathbb{A}1$ and $\mathbb{A}2$ are not necessary from an S-matrix perspective, in field theory they provide necessary conditions for single-particle exchanges to arise only from cubic graphs at quartic order. In other words, in a field theory setting, in forgoing $\mathbb{A}1$ and $\mathbb{A}2$, one would encounter nonlocal obstructions anyway as defined in the Introduction.

In the following, we are going argue by contradiction to show that $\mathbb{A}1$ and $\mathbb{A}2$ do not hold in any interacting HS gauge field theories on AdS. In other words, we start by assuming that there is no nonlocal obstruction and then uncover that this is not the case.

In neglecting the local contact terms in Eq. (20), the sum over spin (19) is given by the HS (or twist) block [60,61]:

$$\mathcal{H}^{\mathsf{s}}_{(0,0|\tau|0,0)} = \sum_{s=0}^{\infty} \mathsf{c}^2_{\mathcal{J}_0\mathcal{J}_0\mathcal{J}_s} W^{\mathsf{s}}_{0,0|s|0,0} \tag{21}$$

$$= \frac{1}{c}\frac{1}{(y_{12}^2 y_{34}^2)^{d-2}}\left[u^{[(d/2)-1]} + \left(\frac{u}{v}\right)^{[(d/2)-1]}\right]. \tag{22}$$

In other words, we have

$$\mathcal{A}^{(4)}_{0,0,0,0} = \mathcal{H}^{\mathsf{s}}_{(0,0|\tau|0,0)} + \mathcal{H}^{\mathsf{t}}_{(0,0|\tau|0,0)} + \mathcal{H}^{\mathsf{u}}_{(0,0|\tau|0,0)} + \cdots, \tag{23}$$

where the ellipsis denotes terms which, under the field theory assumptions $\mathbb{A}$, encode only contact contributions—i.e., no single-particle exchanges—in any channel.

In analogy to conformal blocks, HS blocks represent the contribution to a four-point function from an entire HS multiplet (in a given channel). Accordingly the homogeneous solution (17), which is invariant under global HS symmetry (8), can be expressed purely in terms of HS blocks (21) as

$$\langle \mathcal{J}_0\mathcal{J}_0\mathcal{J}_0\mathcal{J}_0\rangle_{\rm conn} = \frac{1}{2}[\mathcal{H}^{\mathsf{s}}_{(0,0|\tau|0,0)} + \mathcal{H}^{\mathsf{t}}_{(0,0|\tau|0,0)} + \mathcal{H}^{\mathsf{u}}_{(0,0|\tau|0,0)}], \tag{24}$$

which can be verified explicitly from Eqs. (17) and (21).

Combined with the particular solution (23), we find that the nonlocal part of the quartic self-interaction (16) is proportional to the total exchange amplitude (18):

$$S^{(4)}_{0,0,0,0} = \langle \mathcal{J}_0\mathcal{J}_0\mathcal{J}_0\mathcal{J}_0\rangle_{\rm conn} - \mathcal{A}^{(4)}_{0,0,0,0} = -\frac{1}{2}\mathcal{A}^{(4)}_{0,0,0,0} + \cdots, \tag{25}$$

thus uncovering a nonlocal obstruction at quartic order.

The role of crossing symmetry.—We note that the nonlocal obstruction (25) can be traced back to the behavior of the tower of single-trace operators $\mathcal{J}_s$ under crossing. To see this, it is instructive to consider terms in the correlator (24) that are independent solutions to the crossing equation, and their microscopic interpretation.

For a four-point function of scalar operators O of dimension Δ,

$$\langle OOOO\rangle = \frac{G(u,v)}{(y_{12}^2 y_{34}^2)^{\Delta}}, \tag{26}$$

crossing symmetry is the requirement

$$f(u,v) = v^{\Delta}G(u,v) = u^{\Delta}G(v,u). \tag{27}$$

Equation (27) is straightforward to solve in free theories [62–64], with a general solution of the form [40]

$$f(u,v) = \sum_{i,j} c_{ij} u^{(\tau_i/2)} v^{(\tau_j/2)}, \qquad c_{ij} = c_{ji}, \tag{28}$$

which sums over the twists τ_i of the operators in the theory. The general solution (28) exhibits that, in free theories, pairs of twist trajectories are mapped onto each other under crossing,

$$\text{twist } \tau_1 \overset{\text{crossing}}{\leftrightarrow} \text{twist } \tau_2. \tag{29}$$

For the correlation function (24), the following two functions are the independent solutions to crossing (27):

$$f_{d-2}(u,v) = \frac{1}{c} u^{[(d/2)-1]} v^{[(d/2)-1]}, \tag{30a}$$

$$f_{2(d-2)}(u,v) = \frac{1}{c}(u^{[(d/2)-1]} v^{d-2} + v^{[(d/2)-1]} u^{d-2}). \tag{30b}$$

Solution (30b) originates from the exchange of operators of twist $\tau = d-2$, which are the single-trace operators $\mathcal{J}_s$ of the HS multiplet, together with the exchange of double-trace operators of twist $\tau = 2(d-2)$. These two twist trajectories thus map onto each other under crossing:

$$\text{Regge}_{\text{single trace}} \overset{\text{crossing}}{\leftrightarrow} \text{Regge}_{\text{double trace}}. \tag{31}$$

The mapping of single-trace contributions to double-trace contributions under crossing is quite generic of CFTs in $d > 2$ [40,65,66] (see also Refs. [67–69]) and is characteristic of CFTs with a local bulk dual [70].

On the other hand, solution (30a) is self-dual under crossing, with

$$\text{Regge}_{\text{single trace}} \overset{\text{crossing}}{\leftrightarrow} \text{Regge}_{\text{single trace}}, \tag{32}$$

which is typical of CFTs at or around a point of large twist degeneracy [40,71,72], such as the present case of theories with HS symmetry. It is this property that is responsible for the nonlocal obstruction (25). Indeed, it is straightforward to see that the absence of such a contribution would mean that one instead has

$$\langle OOOO\rangle_{\mathrm{conn}} = \mathcal{A}^{(4)} + \cdots, \tag{33}$$

precisely canceling the single-trace contributions in the particular solution (5), and it would thus avert the appearance of the nonlocal obstruction (25).

Discussion.—Let us briefly (and inexhaustibly) discuss the possibilities for interacting HS gauge theories that we feel deserve further understanding in light of the results presented in this Letter.

We stress that a common assumption which leads to nonlocal obstructions of the type of Eq. (25) is the requirement of nontrivial bulk interactions and observables. In particular, our conclusion does not rule out the possibility that HS theories, in both flat and AdS spaces [34], could be regarded as exotic topological theories with trivial bulk interactions and topological S-matrix–like observables. This possibility has already been discussed in Refs. [11,35,73,74].

From this perspective, let us emphasize that the nonlocal obstruction (25) should, of course, not be considered as an inconsistency of the boundary theory but rather as the statement that the bulk action reproducing the boundary CFT correlators at leading order in $1/N$ is at most an effective action, while the microscopic description leading to such an effective action in the bulk would lie outside of the so-called Fronsdal program [1]. The microscopic description should rather be that of an exotic topological (string) field theory (see, e.g., Ref. [75] for some ideas in this direction). From this topological viewpoint, cohomologically nontrivial interactions would then live on the boundary of AdS [76].

Let us also note that, from this perspective, a microscopic bulk definition of HS gauge theories may be possible in terms of properly defined topological string constructions [75,79]. In this setting, one may also attempt a second quantized string description which could provide the additional input to define proper nonlocal topological string field theory interactions. It is conceivable that the nontriviality of interactions could be restored by the requirement that the corresponding functional space of field redefinitions is globally defined on the underlying string Hilbert space [80]. We plan to closely investigate these options in the near future.

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gauge theories, by allowing nonlocalities of the same class as the exchange amplitudes, one can remove the offending terms at the root of the Weinberg theorem [34]. This is equivalent to considering distributional solutions to the Ward identities of the type $\delta(\Box)$ [35] in momentum space, which require one to allow for nonconvergent summations over spin and derivatives. In our language, the presence of a nonlocal obstruction in HS gauge theories would be equivalent to having such a distributional solution to the Noether procedure. Indeed, note that AdS HS amplitudes in Mellin space [36] have been observed to be given by distributions [37,38].

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[44] This constraint originates from the restriction of the $(n+1)$th order consistency condition (2) to global gauge parameters $\bar{\xi}$:

$$0 = \delta^{(1)}_{\bar{\xi}}(S^{(n)}_h + S^{(n)}_p) + \sum_{k=2}^{n-1} \delta^{(n+1-k)}_{\bar{\xi}} S^{(k)} \approx \delta^{(1)}_{\bar{\xi}} S^{(n)}_h. \qquad (34)$$

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