Thermalization and Heating Dynamics in Open Generic Many-Body Systems

Yuto Ashida,[1] Keiji Saito,[2] and Masahito Ueda[1,3]
[1]Department of Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan
[2]Department of Physics, Keio university, Hiyoshi 3-14-1, Kohoku-ku, Yokohama, Japan
[3]RIKEN Center for Emergent Matter Science (CEMS), Wako, Saitama 351-0198, Japan

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The last decade has witnessed remarkable progress in our understanding of thermalization in isolated quantum systems. Combining the eigenstate thermalization hypothesis with quantum measurement theory, we extend the framework of quantum thermalization to open many-body systems. A generic many-body system subject to continuous observation is shown to thermalize at a single trajectory level. We show that the nonunitary nature of quantum measurement causes several unique thermalization mechanisms that are unseen in isolated systems. We present numerical evidence for our findings by applying our theory to specific models that can be experimentally realized in atom-cavity systems and with quantum gas microscopy. Our theory provides a general method to determine an effective temperature of quantum many-body systems subject to the Lindblad master equation and thus should be applicable to noisy dynamics or dissipative systems coupled to nonthermal Markovian environments as well as continuously monitored systems. Our work provides yet another insight into why thermodynamics emerges so universally.

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Statistical mechanics offers a universal framework to describe thermodynamic properties of a system involving many degrees of freedom [1–8]. Systems described by statistical mechanics can be divided into three distinct classes: (i) systems in contact with large thermal baths, (ii) isolated systems, and (iii) systems coupled to nonthermal environments. Thermalization in the first class can be described by a phenomenological master equation in which the detailed balance condition ensures that the system always relaxes to the Gibbs ensemble with the temperature of the thermal bath [9–14]. The last decade has witnessed considerable progress in our understanding of thermalization in the second class [15–38], as promoted by quantum gas experiments [39–43]. In particular, the eigenstate thermalization hypothesis (ETH) [6,7,28–38] has emerged as a generic mechanism of thermalization under unitary dynamics of isolated quantum systems. The ETH has been numerically verified for a number of many-body Hamiltonians [28–38] with notable exceptions of integrable [44–50] or many-body localized systems [51,52].

In class (iii), coupling to a nonthermal environment violates the detailed balanced condition, as it permits arbitrary nonunitary processes such as continuous measurements [53–72] and engineered dissipation [73–89]. There, the bath temperature does not exist in general, and a number of fundamental questions arise. Does the system still thermalize and, if yes, in what sense? How are steady states under such situations related to the thermal equilibrium of the system Hamiltonian? These questions are directly relevant to recent experiments realizing various types of controlled dissipations and measurements [83–89] and to the foundations of open-system nonequilibrium statistical mechanics. The related questions were previously addressed in numerical studies of specific examples [53,65,90–92]. Yet, model-independent, general understanding of thermalization and dynamics in open many-body systems is still elusive.

The aim of this Letter is to extend the framework of quantum thermalization to many-body systems coupled to Markovian environments permitted by quantum measurements and controlled dissipations. We consider open-system dynamics under a continuous measurement process, i.e., weak and frequently repeated measurement, which can be realized by experimental setups of, e.g., atom-cavity systems [93–95] and quantum gas microscopy [96,97]. Combining the ETH and quantum measurement theory, we derive a matrix-vector product expression of the time-averaged density matrix, and show that a generic many-body system under continuous observation will thermalize at a single trajectory level. The obtained density matrix can also be used to determine an effective temperature of open many-body systems governed by the Lindblad master equation. Our results can thus be applied to dissipative many-body dynamics of a system coupled to a (not necessarily thermal) Markovian environment [74–90] or under noisy unitary operations [98–107]. We also present numerical evidence of these findings by applying our theory to specific models. Our results give yet another insight into why thermodynamics emerges so universally.

Quantum many-body dynamics under measurement.—We consider a generic (nonintegrable) quantum many-body system subject to continuous observation. We assume

that the system is initially prepared in a thermal equilibrium state $\hat{\rho}_{\rm eq}$, which is characterized by the mean energy E_0, or, equivalently, by the corresponding temperature $T_0 = 1/\beta_0$. We set $\hbar = 1$ and $k_B = 1$ throughout this Letter. Following the standard theory of quantum measurement [108], we model a measurement process as repeated indirect measurements. We start from a separable state

$$\hat{\rho}_{\rm tot}(0) = \hat{\rho}_{\rm eq} \otimes \hat{P}_0, \tag{1}$$

where $\hat{P}_0$ is a projection operator on the reference state of the meter. The system interacts with a meter during a time interval δt via the total Hamiltonian $\hat{H}_{\rm tot} = \hat{H} + \hat{V}$, where $\hat{H}$ is the many-body Hamiltonian of the measured system and $\hat{V} = v\sum_{m=1}^{M} \hat{L}_m \otimes \hat{A}_m$ describes the system-meter interaction, where M is the number of possible interaction terms. We assume that $\hat{L}_m$ is either a single local operator or the sum of local operators on the system and conserves the total particle number. We also assume that $\hat{A}_m$ acts on the state of the meter such that $\hat{P}_l\hat{A}_m = \delta_{lm}\hat{A}_m$, where $\hat{P}_l$'s are projection operators satisfying $\sum_{l=0}^{M}\hat{P}_l = 1$. After each interaction, we perform a projection measurement $\{\hat{P}_l\}$ on the meter to read out an outcome $l = 0, 1, \ldots, M$. The meter is then reset to the reference state $\hat{P}_0$, which ensures that the dynamics is Markovian (see the Supplemental Material [109] for discussions on a non-Markovian case). For each measurement process, the meter exhibits either (i) a change in the state of the meter corresponding to outcome $m = 1, 2, \ldots, M$ or (ii) no change. Case (i) is referred to as a quantum jump process and accompanied by the following nonunitary mapping:

$$\mathcal{E}_m(\hat{\rho}) = {\rm Tr}_M[\hat{P}_m\hat{U}(\delta t)\hat{\rho}_{\rm tot}\hat{U}^\dagger(\delta t)\hat{P}_m] \simeq \gamma\delta t\hat{L}_m\hat{\rho}\hat{L}_m^\dagger, \tag{2}$$

where ${\rm Tr}_M[\cdot]$ denotes the trace over the meter, $\hat{U}(\tau) = e^{-i\hat{H}_{\rm tot}\tau}$, and $\gamma = v^2\delta t{\rm Tr}_A[\hat{P}_0\hat{A}_m^\dagger\hat{A}_m]$ [110]. In deriving the last expression in Eq. (2), we assume $\gamma\delta t \ll 1$. The case (ii) is referred to as a no-count process, leading to

$$\mathcal{E}_0(\hat{\rho}) \simeq (1 - i\hat{H}_{\rm eff}\delta t)\hat{\rho}(1 + i\hat{H}_{\rm eff}^\dagger\delta t), \tag{3}$$

where $\hat{H}_{\rm eff} = \hat{H} - i\hat{\Gamma}/2$ is an effective non-Hermitian Hamiltonian with $\hat{\Gamma} = \gamma\sum_m \hat{L}_m^\dagger\hat{L}_m$. Each outcome l is obtained with probability $p_l = {\rm Tr}[\mathcal{E}_l(\hat{\rho})]$. Taking the limit $\delta t \to 0$ while keeping $v^2\delta t$ finite, the system exhibits a nonunitary stochastic evolution, which is continuous in time and known as the quantum trajectory dynamics [108, 111–114]. Each realization of a trajectory is characterized by a sequence of measurement outcomes and given as

$$\hat{\varrho}_{\mathcal{M}}(t;\mathcal{T}) = \hat{\Pi}_{t;\mathcal{T}}^{\mathcal{M}}\hat{\rho}_{\rm eq}\hat{\Pi}_{t;\mathcal{T}}^{\dagger\mathcal{M}}, \tag{4}$$

where $\mathcal{M} = (m_1, \ldots, m_n)$ and $\mathcal{T} = (t_1, \ldots, t_n)$ denote the types and occurrence times of quantum jumps, and $\hat{\Pi}_{t;\mathcal{T}}^{\mathcal{M}} = \prod_{i=1}^{n}[\hat{\mathcal{U}}(\Delta t_i)\sqrt{\gamma}\hat{L}_{m_i}]\hat{\mathcal{U}}(t_1)$ with $\Delta t_i = t_{i+1} - t_i$, $t_{n+1} = t$, and $\hat{\mathcal{U}}(\tau) = e^{-i\hat{H}_{\rm eff}\tau}$.

Statistical ensemble.—We are interested in the thermalization process caused by the interplay between many-body dynamics and measurement backaction of continuous observation. We consider a situation in which the equilibration time in the measured many-body system is shorter than a typical waiting time between quantum jumps. We ensure this by taking the limit $\gamma \to 0$ while keeping γt finite. This guarantees that a finite number of quantum jumps typically occur during a given time interval $[0, t]$, but that the system has not yet reached a steady state (such as an infinite-temperature state) even in the long-time regime.

For closed many-body systems, it has been argued that the equilibration time can be estimated as the Boltzmann time $\hbar/k_{\rm B}T$ with T being the temperature of the system [115–117]. In the present context, these results imply the fast equilibration during no-jump process since the dominant contribution of $\hat{H}_{\rm eff}$ in the above limit is the (Hermitian) many-body Hamiltonian $\hat{H}$. Indeed, our numerical results presented below support this expectation, though its mathematically rigorous proof remains open.

When the waiting time exceeds the equilibration time, the exact state and its time-averaged density matrix become indistinguishable in terms of an expectation value of a physical observable. The reason is that the time-dependent elements of the density matrix make negligible contributions to an expectation value due to their rapid phase oscillations [19]. This emergent time-independent feature indicates that the memory of the occurrence times $\mathcal{T}$ will be lost and expectation values of physical observables can be studied by the time-averaged density matrix

$$\hat{\varrho}_{\mathcal{M}}(t) = \int_0^t dt_n \cdots \int_0^{t_2} dt_1 \hat{\varrho}_{\mathcal{M}}(t;\mathcal{T}). \tag{5}$$

To proceed with the calculation, we assume that for each eigenstate $|E_a\rangle$ of $\hat{H}$ the expectation values of arbitrary few-body observables coincide with those of the corresponding Gibbs ensemble. This condition is generally believed to hold when the system satisfies the ETH [36], as numerically supported for a number of different Hamiltonians [28–38]. The leading contribution to $\hat{\varrho}_{\mathcal{M}}$ can be given upon the normalization as

$$\hat{\rho}_{\mathcal{M}} = \frac{\Lambda_{\mathcal{M}}[\hat{\rho}_{\rm eq}]}{Z(\mathcal{M})}, \tag{6}$$

where we define $\Lambda_{\mathcal{M}} = \prod_{i=1}^{n}(\Lambda\circ\mathcal{L}_{m_i}\circ\Lambda)$ with $\mathcal{L}_m[\hat{O}] = \hat{L}_m\hat{O}\hat{L}_m^\dagger$, $\Lambda[\hat{O}] = \sum_a \hat{P}_a\hat{O}\hat{P}_a$ and $\hat{P}_a = |E_a\rangle\langle E_a|$, and $Z(\mathcal{M})$ is a normalization constant. While the non-Hermiticity in $\hat{H}_{\rm eff}$ can slightly modify the energy distribution, its contribution can be neglected in the

thermodynamic limit [109]. This follows from strong suppression of fluctuations in the decay rate $\hat{\Gamma}$ among eigenstates that are close in energy [see, e.g., the top panel in Fig. 1(c)]. This suppression can be understood from the ETH [6,36], which predicts the exponential decay of the fluctuations of the matrix elements in the energy basis in the thermodynamic limit.

In the matrix representation, the ensemble (6) has a simple factorized form:

$$\hat{\rho}_{\mathcal{M}} \propto \sum_a [\mathcal{V}_{m_n} \cdots \mathcal{V}_{m_1} p_{\rm eq}]_a \hat{P}_a, \tag{7}$$

where we introduce the vector $(p_{\rm eq})_a = \langle E_a|\hat{\rho}_{\rm eq}|E_a\rangle$ and the matrix $(\mathcal{V}_m)_{ab} = |\langle E_a|\hat{L}_m|E_b\rangle|^2$. It follows from the cluster decomposition property [47,118,119] of thermal eigenstates for local operators $\hat{O}_{x,y}$ that

$$\lim_{|x-y|\to\infty} {\rm Tr}[\hat{O}_x\hat{O}_y\hat{P}_a] - {\rm Tr}[\hat{O}_x\hat{P}_a]{\rm Tr}[\hat{O}_y\hat{P}_a] = 0. \tag{8}$$

Then we can show [109] that the standard deviation of energy in the ensemble (7) is subextensive and thus its energy distribution is strongly peaked around the mean value $\bar{E}_{\mathcal{M}}$. We introduce an effective temperature $\beta_{\rm eff}^{\mathcal{M}}$ from the condition $\bar{E}_{\mathcal{M}} = {\rm Tr}[\hat{H}\hat{\rho}_{\beta_{\rm eff}^{\mathcal{M}}}]$ with $\hat{\rho}_\beta = e^{-\beta\hat{H}}/Z_\beta$ being the Gibbs ensemble. The ETH then guarantees that, if we focus on a few-body observable $\hat{O}$, $\hat{\rho}_{\mathcal{M}}$ is indistinguishable from the Gibbs ensemble:

$${\rm Tr}[\hat{O}\hat{\rho}_{\mathcal{M}}] \simeq {\rm Tr}[\hat{O}\hat{\rho}_{\beta_{\rm eff}^{\mathcal{M}}}]. \tag{9}$$

Here and henceforth we understand $\simeq$ to be the equality in the thermodynamic limit. Thus, a generic quantum system under a measurement process thermalizes by itself at a single-trajectory level.

The derivation of the matrix-vector product ensemble (MVPE) in Eq. (7) is one of the main results in this Letter. In open many-body dynamics it is highly nontrivial to precisely estimate an effective temperature of the system. One usually has to design *ad hoc* techniques for each individual problem. In contrast, the MVPE provides a general and efficient way to determine an effective temperature under physically plausible assumptions as demonstrated later. If the ETH holds, any physical quantity can be calculated from the Gibbs ensemble at an extracted temperature.

Before examining concrete examples, we discuss some general properties of thermalization under quantum measurement in comparison with thermalization in isolated systems. First, since $\hat{H}$ has no local conserved quantities, it satisfies $[\hat{H}, \hat{L}_m] \neq 0$ and thus the matrix $\mathcal{V}_m$ should change the energy distribution. It is this noncommutativity between the Hamiltonian and measurement operators that leads to heating or cooling under measurement. Second, it is worthwhile to mention similarities and differences between Eq. (7) and the density matrix of isolated systems under slow time-dependent operations [36] or a sudden quench [28,120–122]. Both density matrices are diagonal in the energy basis and coefficients are represented in the matrix-vector product form. The unitarity inevitably leads to the doubly stochastic condition of the transition matrix $\sum_a(\mathcal{V})_{ab} = \sum_b(\mathcal{V})_{ab} = 1$, which causes the energy of the system to increase or stay constant [26,27,123]. However, in the nonunitary evolution considered here, $\mathcal{V}$ cannot be interpreted as the transition matrix and the doubly stochastic condition is generally violated. This is why it is possible to cool down the system if one uses artificial (typically non-Hermitian) measurement operators $\hat{L}_m$ [74,75].

Numerical simulations.—To demonstrate our general approach, we consider a Hamiltonian $\hat{H} = \hat{K} + \hat{U}$ of hard-core bosons on an open one-dimensional lattice with nearest- and next-nearest-neighbor hopping and an interaction: $\hat{K} = -\sum_l (t_h \hat{b}_l^\dagger \hat{b}_{l+1} + t_h' \hat{b}_l^\dagger \hat{b}_{l+2} + {\rm H.c.})$ and $\hat{U} = \sum_l (U\hat{n}_l\hat{n}_{l+1} + U'\hat{n}_l\hat{n}_{l+2})$, where $\hat{b}_l$ ($\hat{b}_l^\dagger$) is the annihilation (creation) operator of a hard-core boson on site l and $\hat{n}_l = \hat{b}_l^\dagger\hat{b}_l$. This model is, in general, nonintegrable and has been numerically confirmed to satisfy the ETH [29,33,36,38]. We set the system size and the total number

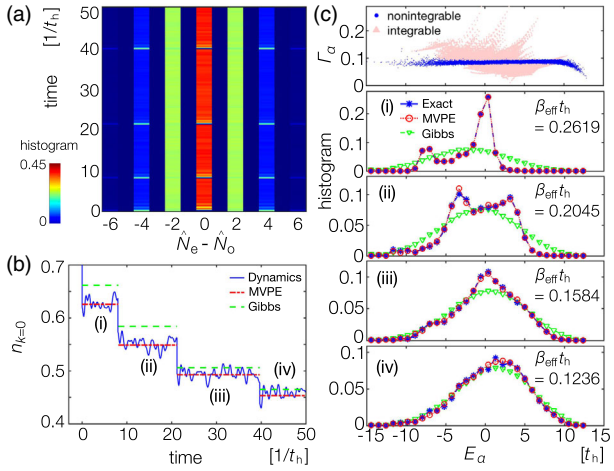

FIG. 1. (a) Time evolution of the distribution with the jump operator $\hat{L} = \sum_l (-1)^l \hat{n}_l$, which gives the difference $\hat{N}_e - \hat{N}_o$ of particle numbers at even and odd sites. Every time a quantum jump occurs, the distribution peaks at $\hat{L} = \pm 4$ after which it rapidly relaxes toward an equilibrium profile due to the noncommutativity of $\hat{L}$ with the system Hamiltonian $\hat{H}$. (b) The corresponding dynamics of $\langle \hat{n}_{k=0} \rangle$. Superimposed are the prediction from the MVPE conditioned on a sequence of quantum jumps that have occurred by time t (red dashed lines) and that from the Gibbs ensemble $\hat{\rho}_{\beta_{\rm eff}^{\mathcal{M}}}$ (green dashed lines). (c) Diagonal values of the detection rate $\hat{\Gamma}$ (top panel) and the energy distributions after each jump (the other panels). We set $\gamma = 0.02$ and $t_h = U = t_h' = U' = 1$ except for the integrable case in (c) where we use $t_h = U = 1$ and $t_h' = U' = 0$.

of particles to be $L_s = 18$ and $N = 6$. As a measurement process, we consider $\hat{L} = \sum_l (-1)^l \hat{n}_l$, which can be implemented by monitoring photons leaking out of a cavity coupled to a certain collective mode of atoms [65].

To test the validity of the MVPE (7) for describing open many-body dynamics, it suffices to use an energy eigenstate as the initial state. Results for a general initial distribution $p_{\rm eq}$ can be obtained simply as a linear sum of the results for individual eigenstates. To be specific, we start from an eigenstate $|E_0\rangle$ corresponding to the initial temperature $T_0 = 3t_h$. Without loss of generality, we choose the first detection time of a quantum jump as $t = 0$.

Figure 1 shows a typical realization of the quantum dynamics under measurement. Figure 1(a) plots the time evolution of the distribution of $\hat{L} = \sum_l (-1)^l \hat{n}_l$, which is the difference of particle numbers at even and odd sites. Each detection creates a catlike postmeasurement state having large weights on $\hat{L} = \pm 4$. It then quickly decays into a thermal state since $\hat{H}$ does not commute with $\hat{L}$. In Fig. 1(b), the corresponding dynamics of $\langle \hat{n}_{k=0} \rangle$, which is the average occupation number at zero momentum, is compared with the predictions from the MVPE $\hat{\rho}_{\mathcal{M}}$ and the Gibbs ensemble $\hat{\rho}_{\beta_{\rm eff}^{\mathcal{M}}}$. We find an excellent agreement between the exact values and the MVPE. Note that the MVPE is time independent by definition; the plotted values correspond to the MVPE conditioned on a sequence of quantum jumps that have occurred by time t. Figure 1(c) shows the diagonal values Γ_a of the detection rate (top panel) and energy distributions after each jump (the other panels). The latter shows a rapid collapse of the energy distribution into that of the Gibbs ensemble after only a few jumps. The similar results are also found in numerical simulations for a local density measurement $\hat{L}_i = \hat{n}_i$ [109], which is directly relevant to quantum gas microscopy.

Figure 2 shows relative deviations of the MVPE predictions from time-averaged values of $\hat{n}_{k=0}$ with varying system-meter coupling γ. Finite-size scaling analyses indicate that the relative deviations become exponentially small with increasing the system size for small γ [see, e.g., Fig. 2(b)], while they no longer converge for larger values (typically $\gamma \gtrsim 0.08 t_h$) in which the minimally destructive limit breaks down. A relatively slow convergence of the Gibbs ensemble predictions in Fig. 2(b) can be attributed to broad energy distributions and large fluctuations in diagonal elements of $\hat{n}_{k=0}$ in finite-size systems (see Ref. [91] for a similar observation).

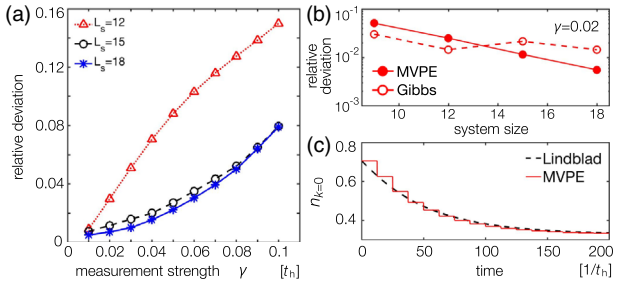

FIG. 2. (a) Relative deviations of MVPE predictions from time-averaged values of $\hat{n}_{k=0}$ plotted against measurement strength γ for different system sizes L_s. (b) Finite-size scaling analyses of the relative deviations of MVPE (solid circles) and the corresponding Gibbs ensemble (open circles) from time-averaged values of $\langle \hat{n}_{k=0} \rangle$. (c) Comparison between the MVPE predictions (red solid lines) and the Lindblad dynamics (black dashed curve) for $\hat{n}_{k=0}$ with $L_s = 18$. We use $t_h = U = t_h' = U' = 1$ and set $\gamma = 0.02$ in (b) and (c).

Application to many-body Lindblad dynamics.—Having established the validity of the MVPE, we now discuss its application to the Lindblad dynamics. The quantum trajectory dynamics offers a numerical method to solve the Lindblad master equation [14,92]:

$$\frac{d\hat{\rho}}{dt} = \mathcal{L}[\hat{\rho}] = -i(\hat{H}_{\rm eff}\hat{\rho} - \hat{\rho}\hat{H}_{\rm eff}^{\dagger}) + \gamma \sum_m \hat{L}_m \hat{\rho} \hat{L}_m^{\dagger}, \qquad (10)$$

where $\hat{\rho}(t) = \sum_{\mathcal{M}} \hat{\varrho}_{\mathcal{M}}(t)$ is the density matrix averaged over all trajectories. Equation (10) can describe the temporal evolution of a system weakly coupled to its environment [14] or a system under noisy unitary operations [98–101,107]. Yet, especially for a many-body system, it is often very demanding to take the ensemble average due to a vast number of possible trajectories.

For the case of a translationally invariant Hamiltonian $\hat{H}$ and a local operator $\hat{L}_m$, our approach suggests a simple way to overcome the above difficulty. In this case, the matrix $\mathcal{V}_m$ is independent of a spatial label m and thus the MVPE in Eq. (7) is characterized by the number n of quantum jumps alone: $\hat{\rho}_n \propto \sum_a [\mathcal{V}^n p_{\rm eq}]_a \hat{P}_a$. As the detection rate $\hat{\Gamma}$ of quantum jumps consists of few-body observables, the distribution of n is sharply peaked around the mean value $\bar{n}$ if the ETH holds. These observations lead to

$$\mathrm{Tr}[\hat{O} e^{\mathcal{L}t} \hat{\rho}_{\rm eq}] \simeq \mathrm{Tr}[\hat{O} \hat{\rho}_{\bar{n}_t}] \simeq \mathrm{Tr}[\hat{O} \hat{\rho}_{\beta_{\rm eff}^{\bar{n}_t}}], \qquad (11)$$

where $\bar{n}_t$ is the mean number of quantum jumps during $[0, t]$ that can be determined from the implicit relation $t \simeq \sum_{n=0}^{\bar{n}_t} 1/\Gamma_n$ with $\Gamma_n = \mathrm{Tr}[\hat{\Gamma}\hat{\rho}_n]$, and $\beta_{\rm eff}^{\bar{n}_t}$ is the corresponding effective temperature. Thus, expectation values of physical observables in the many-body Lindblad dynamics agree with those predicted from the typical MVPE or the Gibbs ensemble at an appropriate effective temperature. Since solving Eq. (10) requires the diagonalization of a $D^2 \times D^2$ Liouvillean with D being the dimension of the Hilbert space, our approach (11) allows a significant simplification of the problem. We have applied our approach to the Lindblad dynamics of the above lattice model and demonstrated the relation (11) aside from stepwise finite-size corrections (Fig. 2c).

Summary and discussions.—Combining the ideas of the ETH and quantum measurement theory, we find that a generic quantum many-body system under continuous

observation thermalizes at a single trajectory level. We have presented the matrix-vector product ensemble (7), which can quantitatively describe the dynamics and give an effective temperature of an open quantum many-body system. This can also be used to analyze a many-body Lindblad master equation and thus should have broad applicability to dissipative [74–89] or noisy Markovian systems [98–101,107], in addition to continuously monitored ones. These findings are supported by numerical simulations of nonintegrable systems under continuous measurement, which can be experimentally realized in atom-cavity systems or by quantum gas microscopy.

The present study opens several research directions. First, it is intriguing to elucidate thermalization at the trajectory level when the system Hamiltonian is integrable [44–50]. Under measurements, quantum jumps act as weak integrability-breaking perturbations and, if their effects are insignificant, we expect prethermalization [25,44], i.e., a phenomenon in which observables approach quasistationary values consistent with the generalized Gibbs ensemble [44]. Ultimate thermalization will happen when quantum jumps sufficiently mix the distribution, leading to the unbiased probability weights on nonthermal rare states admitted in the weak variant of the ETH [25,30,124,125]. We present our first attempt to outline this scenario in the Supplemental Material [109] and leave a detailed analysis as an interesting future problem. Another important system is a many-body localized system [51,52] where even the weak ETH can be violated [126]. Second, it remains an important problem to extend our analysis to non-Markovian open dynamics [127]. While an application of a non-Markovian trajectory approach [128] to a many-body system is challenging in general, our MVPE approach may still be useful if the support of a jump operator is restricted [109]. Third, it is interesting to explore possible connections between the predictions made in the random unitary circuit dynamics [101–106] and the nonintegrable open trajectory dynamics studied here. They share several intriguing similarities; they satisfy the locality, have no energy conservation, thus relaxing to the infinite-temperature state, and obey the Lindblad master equation upon the ensemble average (at least) in a certain case [101]. It is particularly interesting to test the predicted scrambling dynamics [104,105] or the Kardar-Parisi-Zhang universal behavior [103] in the present setup.

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