Disorder-Driven Quantum Transition in Relativistic Semimetals: Functional Renormalization via the Porous Medium Equation

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In the presence of randomness, a relativistic semimetal undergoes a quantum transition towards a diffusive phase. A standard approach relates this transition to the $U(N)$ Gross-Neveu model in the limit of $N \to 0$. We show that the corresponding fixed point is infinitely unstable, demonstrating the necessity to include fluctuations beyond the usual Gaussian approximation. We develop a functional renormalization group method amenable to include these effects and show that the disorder distribution renormalizes following the so-called porous medium equation. We find that the transition is controlled by a nonanalytic fixed point drastically different from that of the $U(N)$ Gross-Neveu model. Our approach provides a unique mechanism of spontaneous generation of a finite density of states and also characterizes the scaling behavior of the broad distribution of fluctuations close to the transition. It can be applied to other problems where nonanalytic effects may play a role, such as the Anderson localization transition.

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Introduction.—The interplay between disorder and quantum fluctuations leads to unique phenomena, the most remarkable being the Anderson localization. After more than half a century of intensive efforts, it remains a topical subject of research with applications to various domains of physics ranging from condensed matter to cold atoms and light propagation [1]. Remarkably, a different type of disorder-driven quantum phase transition was discovered recently when considering waves with a quantum relativistic dispersion relation [2]. This transition happens between a pseudoballistic phase and a diffusive metal as a function of the disorder strength (or the energy). It is predicted to occur in particular in the recently discovered three-dimensional (3D) Weyl [3,4] and Dirac semimetals [5–7] in which, respectively, two and four electronic bands cross linearly at isolated points. However, we expect these phenomena to be relevant to other relativistic waves beyond condensed matter, such as ultracold atoms [8].

In spite of numerous efforts, the understanding of this transition remains elusive. In this Letter we show that the fluctuations of the randomness beyond the standard Gaussian approximation invalidate previous field-theoretic descriptions of this transition. A very similar mechanism occurs in the context of the Anderson transition: there, discrepancies between the results obtained using renormalization group (RG) and numerical simulations grow with the number of loops [9]. One may attribute them to the existence of infinitely many relevant operators of the associated field theory [10] which destabilize the fixed point (FP) usually considered to describe the transition [11–13]. We find that the same problem appears at the new semimetal-diffusive metal transition. We demonstrate how to overcome this obstacle by deriving and solving a functional renormalization group (FRG) for the whole (non-Gaussian) disorder distribution. To our knowledge, this solution constitutes the only example of an analytical description of a disorder-driven quantum phase transition controlled by non-Gaussian disorder fluctuations. Besides the present work we are aware of only one other example, namely, the classical 2D XY model with random phases, for which not only the flow equation for the probability distribution but also its solution was obtained by a mapping to the so-called Kolmogorov-Petrovskii-Piscounov equation [14,15]. Hence, we believe that our work sheds new light on the description of critical non-Gaussian disorder fluctuations in quantum systems beyond the disorder-driven semimetal-diffusive metal phase transition.

Here we focus on the transition between a pseudoballistic semimetal phase with a vanishing density of states (DOS) at the nodal point and a diffusive metal phase with a finite DOS at zero energy [16–23]. The field-theoretic description of this transition using both replica and SUSY approaches [24–27] implies that in the absence of scattering between different nodal points the transition is controlled by a perturbative in $\varepsilon = d - 2$ FP of the d-dimensional $U(N)$ Gross-Neveu (GN) model taken in the unusual limit of a vanishing number of fermion flavors $N \to 0$. As for the Anderson transition, numerical studies [28–32] demonstrate quantitative discrepancies with the predictions of the GN model [24–26] which grow with the order of approximation. We show that the GN FP is infinitely unstable in the limit of $N \to 0$ implying that the non-Gaussian fluctuations of the randomness are at the origin of the breakdown of the GN description. It is the purpose of the present Letter to resolve this problem by deriving the flow equation for the whole disorder distribution and solving it through a mapping to the well-known porous medium equation (PME) [33]. This reveals that the phase transition is governed by a nonanalytic FP which is crucially different from that of the GN model.

Model.—We start from the imaginary time action of relativistic fermions moving in a d-dimensional space in the presence of an external potential $V(r)$

$$S = i\int d^d x d\tau \bar{\psi}(x,\tau)[\partial_\tau - i\gamma_j\partial_j + V(x)]\psi(x,\tau), \quad (1)$$

where $\bar{\psi}$ and ψ are independent Grassmann fields and τ is the imaginary time. γ_j are elements of a Clifford algebra satisfying the anticommutation relations: $\gamma_j\gamma_k + \gamma_k\gamma_j = 2\delta_{jk}\mathbb{I}$ $(j,k = 1,\ldots,d)$, which reduce to the Pauli matrices $\gamma_j = \sigma_j$ in $d = 3$. The disorder potential is assumed to be uncorrelated in space, and thus, its distribution can be described by a local characteristic function $W(\Theta)$ defined as $\overline{\exp(-i\int d^d x V(x)\Theta(x))} = \exp[-\int d^d x W(\Theta(x))]$. Here the overbar stands for averaging over disorder configurations. To perform averaging directly in the action Eq. (1) we use the replica trick. Since the fermions are noninteracting it is convenient to switch in the action Eq. (1) from the imaginary time to the Matsubara frequency and write down the bare replicated action at fixed energy ω as [34]

$$S = \int d^d x \sum_{\alpha=1}^{N} \bar{\psi}_\alpha(x)(\gamma_j\partial_j + \omega)\psi_\alpha(x) + W(\Theta(x)), \quad (2)$$

where $\Theta(x) = \sum_{\alpha=1}^{N}\bar{\psi}_\alpha(x)\psi_\alpha(x)$ is the local density of fermions.

Renormalization.—To derive the FRG flow equations we use the effective average action formalism developed by Wetterich [35] together with $\varepsilon = d - 2$ expansion. Introducing the infrared cutoff in the form of mass m we obtain the flow equation for the characteristic function [36]

$$-m\partial_m W(\Theta) = 2m^\varepsilon(\Theta W'(\Theta)W''(\Theta) - \tilde{N}W'(\Theta)^2), \quad (3)$$

where $\tilde{N} = (N/2)\mathrm{tr}\mathbb{I}$, Θ is the expectation value of $\Theta(x)$, and m goes from m_0 to 0. A counterpart of Eq. (3) derived in a fixed dimension d can be found in Ref. [36]. The renormalized Green's function corresponding to action Eq. (2) is $G_{\alpha\beta}(k) = \delta_{\alpha\beta}/[\gamma_j k_j - i\omega - iW'(0)]$. For physically relevant disorder distributions the bare characteristic function $W(\Theta)$ is analytic and satisfies $W'(0) = 0$. Hence, the bare DOS given by $\rho(\omega) = -1/\pi \mathrm{Im}\int_k G_{\alpha\alpha}(k,\omega)$ vanishes at zero energy [51]. However, as we will see later the renormalized characteristic function can develop a cusp at the origin, and thus, generate a nonvanishing DOS at zero energy.

To demonstrate how one can recover the FP of the $U(N)$ GN model we rewrite the FRG equation [Eq. (3)] in dimensionless form by substituting $W(\Theta) = m^{2+\varepsilon}w(\theta)$ and $\Theta = m^{1+\varepsilon}\theta$. This gives

$$-m\partial_m w(\theta) = (2+\varepsilon)w(\theta) - (1+\varepsilon)\theta w'(\theta) + 2(\theta w'(\theta)w''(\theta) - \tilde{N}w'(\theta)^2). \quad (4)$$

The $U(N)$ GN model corresponds to the model Eq. (2) with $W(\Theta)$ being a quadratic function, so that the FP of the GN model can be easily identified with $w^*(\theta) = \varepsilon\theta^2/[8(1-\tilde{N})]$. If we restrict $w(\theta)$ to a quadratic function its amplitude remains the only unstable direction. To check the full stability we linearize the flow [Eq. (4)] around this FP. The derivatives of the characteristic function $w^{(n)}(0)$ are coupled to the operators Θ^n corresponding to the fermion density moments. Using Eq. (4) we can calculate their scaling dimensions $[w^{(n)}(0)] = 2 + \varepsilon - n(1+\varepsilon) + \varepsilon n(2\tilde{N} - n)/(2\tilde{N} - 2)$ which are in agreement with diagrammatic [26] and conformal field theory [52] results. The coupling is relevant if its scaling dimension is positive. Hence, in the limit of $N \to 0$, which describes the disordered relativistic semimetal, infinitely many relevant operators corresponding to higher order cumulants of the disorder distribution are identified signaling the relevance of rare configurations of disorder at the transition [53].

Zero N limit and porous medium equation.—Since in the limit of $N \to 0$ the GN FP becomes unstable in infinitely many directions, it cannot control a continuous transition. Nevertheless, it is premature to conclude that the transition is smeared out or first order. A direct numerical integration of the rescaled flow equation, Eq. (4), however, failed to find any physical FP different from the GN one. As we will see below, this can be explained by the fact that the FP we are looking for has nonanalytical behavior at the origin in addition to the absence of boundary conditions at infinity. Notice, however, that if the large θ asymptotics of the FP $w^*(\theta)$ was known then the whole FP could be computed by numerical integration of Eq. (4). Fortunately, introducing the "time" $t = (m_0^\varepsilon - m^\varepsilon)/\varepsilon$, the "coordinate" $r = \sqrt{2\Theta}$ and the "density profile" $u = W'(\Theta)$ we can rewrite the unrescaled flow equation [Eq. (3)] in the form of a 2D nonlinear diffusion equation

$$2\partial_t u(r,t) = \frac{1}{r}\partial_r r\partial_r u^2(r,t) = \Delta u^2(r,t) \quad (5)$$

with the superimposed radial symmetry. Since m changes from m_0 to 0 one has to stop the evolution of the density profile $u(r,t)$ at the maximal observation time $T_0 = m_0^\varepsilon/\varepsilon$. Equation (5) is the 2D PME which has been intensively studied by mathematicians for several decades [33]. Because of the presence of degeneracy points (regions where $u = 0$ and thus vanishing diffusion constant) the

PME exhibits remarkable nonlinear phenomena. They include finite velocity propagation of fronts separating the regions with zero and nonzero u [54], waiting times before the front starts to move [55], and self-focusing solutions describing shrinking of holes in the support of u [56] with postfocusing accumulation of diffusing particles [57]. Following the route paved by these mathematical studies we look for a backward self-similar solution (BSS) to Eq. (5) which has the form

$$u(r,t) = (T-t)^{2\delta-1}F(\zeta), \qquad \zeta = \frac{r}{(T-t)^{\delta}}. \tag{6}$$

The self-similar solutions to the PME play a special role since they lead to a universal large time behavior. It is straightforward to identify the BSS Eq. (6) with a FP solution $w(\theta)$ to the rescaled FRG equation [Eq. (4)] setting $\delta = (1+\varepsilon)/(2\varepsilon)$, $T = T_0$ and $F(\zeta) = \varepsilon w'(\zeta^2/(2\varepsilon^2))$. Then the rescaled FRG equation [Eq. (4)] becomes

$$\partial_\tau F(\zeta) = F(\zeta)\left(F''(\zeta) + \frac{1}{\zeta}F'(\zeta)\right) + F'(\zeta)^2 - \delta\zeta F'(\zeta) + (2\delta-1)F(\zeta), \tag{7}$$

where we have defined $\tau = -\ln(T-t)$ such that $\varepsilon\partial_\tau = -m\partial_m$. For a BSS $F(\zeta)$ the rhs of Eq. (7) identically vanishes. The GN FP corresponds to the BSS with $F(\zeta) = \zeta^2/8$. One may get the impression that rewriting the FRG equation, Eq. (4), in the form of Eq. (7) is just a beautiful mathematical trick which connects two *a priori* unrelated problems. However, there is much more to it than that. Indeed, while the BSS Eq. (6) translates into a FP at $T = T_0$, as we will see below, its dependance on T also provides an explicit expression for the flow of the whole disorder distribution along a single unstable direction. Moreover, the nontrivial BSS Eq. (6) can be captured by the phase-plane formalism [33] which is a powerful tool for analysis of the PME (5). To that end we define the phase variables Z and Y as $F(\zeta) = -\zeta^2 Z(\zeta)$ and $Y(\zeta) = -[2Z(\zeta) + \zeta Z'(\zeta)]$. They satisfy autonomous first order differential equations [36] whose solution for $\delta = 1$, i.e., $d = 3$, is shown in Fig. 1. In the phase plane (Z,Y) the BSS is represented by an integral curve which connects the singular point (0,0) controlling the large ζ behavior and a limiting cycle around the singular point $(-\frac{1}{8},\frac{1}{4})$ corresponding to the GN FP (see inset of Fig. 1). Although the function $Z(\zeta)$ is infinitely oscillating at the origin, the corresponding profile function $F(\zeta)$ is surprisingly monotonic as one can see in Fig. 2. It grows as $F(\zeta) \sim \zeta^{2-1/\delta}$ for large ζ and is strongly nonanalytic at $\zeta = 0$. This explains why the nontrivial FP can be easily overlooked when solving numerically the FRG equation, Eq. (4). The new nonanalytic FP exists only for $\delta > \delta_c \approx 0.856\,326\,5$, i.e., only below the critical dimension $d_c \approx 3.4$, and thus controls the transition in $d = 3$.

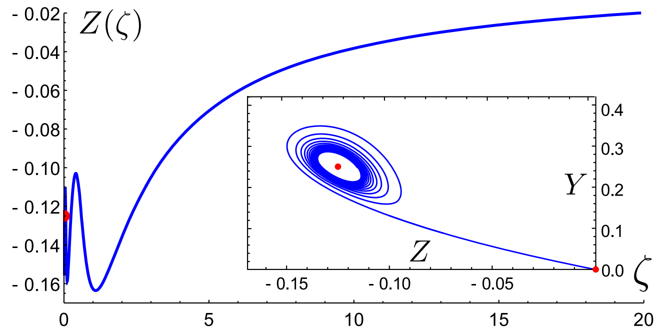

FIG. 1. Fixed point solution to the functional renormalization group equation for $d = 3$ expressed as a backward self-similar solution $Z(\zeta)$ to the porous medium equation, Eq. (5), with $\delta = 1$. The inset shows the integral curve representing the BSS in the phase plane (Z,Y), which clarifies the nature of nonanalyticity of the FP.

Stability analysis.—To study the stability of the new nonanalytic FP we add to the BSS Eq. (6) a time dependent perturbation $F(\zeta;\tau) = F(\zeta) + \phi(\zeta)e^{\lambda\tau}$, where λ is the stability eigenvalue and ϕ is the corresponding eigenfunction. Substituting it into Eq. (7) and linearizing around the BSS we arrive at

$$-Z\ddot{\phi} + [2Y-\delta]\dot{\phi} + [2\delta - 1 - \lambda + 2Y + \dot{Y}]\phi = 0, \tag{8}$$

where the dots stand for the logarithmic derivatives, $\dot{X} \equiv dX/d\ln\zeta$. In order to obtain the stability spectrum of the FRG FP one has to impose the boundary condition at $\zeta = 0$ using additional physical arguments [58]. Here we look for perturbations originating from higher order cumulants. Choosing $\phi_n(\zeta) = \zeta^n f_n(\zeta)$, $n = 2,4,6,\ldots$ such that the functions $f_n(\zeta)$ are bounded for $\zeta \to 0$ but

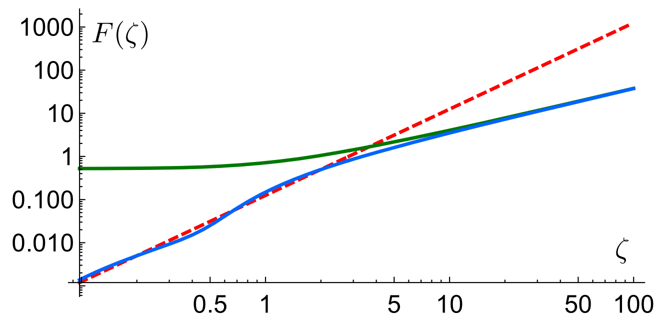

FIG. 2. Analytical continuation of the backward self-similar solution, Eq. (6), for $t < T$ to the forward self-similar solution [Eq. (10)] for $t > T$ which provides a nonanalytic mechanism for the DOS generation at the nodal point. The generated DOS is related to the nonzero $\tilde{F}(0)$. Blue solid line is the BSS function $F(\zeta)$ for $\delta = 1$, green solid line is the corresponding FSS $\tilde{F}(\tilde{\zeta})$ with $\tilde{F}(0) > 0$. Both solutions have the same asymptotic behavior at large ζ, which ensures the matching between $u(r,T^-)$ and $u(r,T^+)$. Red dashed line is the GN FP $F(\zeta) = \zeta^2/8$ shown for comparison.

not necessarily analytic, we render the spectrum discrete. Numerical solution of Eq. (8) shows that only the eigenvalue corresponding to $n = 2$ is positive, and thus, the nonanalytic FP we have found indeed is a critical FP describing the disorder driven transition [36].

Remarkably, the relevant eigenvalue and eigenfunction can be identified from general symmetry considerations. Let $u(r,t)$ be a BSS with profile $F(\zeta)$ and waiting time T. Owing to the time-translational invariance of the PME (5) we can shift $T \to T + \Delta T$ to obtain another BSS: $u(r,t) \to u(r,t) + \Delta T \partial_T u(r,t)$. From Eq. (6) we find that $\partial_T u(r,t) = (T-t)^{2\delta-1}\phi_2(\zeta)e^{\tau}$ where

$$\phi_2(\zeta) = (2\delta - 1)F(\zeta) - \delta\zeta F'(\zeta). \tag{9}$$

It is straightforward to see that Eq. (9) is the eigenfunction of Eq. (8) which corresponds to the only positive eigenvalue $\lambda_2 = 1$. Thus, $T_0 - T$ is the only relevant parameter which controls the transition. Taking into account the relation between τ and m we can find the correlation length exponent $\nu^{-1} = \varepsilon\lambda_2$. Although the nonanalytic FP can always be expressed as a BSS [Eq. (6)] with $T = T_0$, higher loop order corrections to the critical exponents are expected.

Postfocusing regime and DOS generation.—The inverse waiting time T^{-1} determined by the full bare disorder distribution [59] turns out to be a natural measure of the disorder strength. If the bare disorder is weak ($T > T_0$) the system is in the semimetal phase, while for strong disorder ($T < T_0$) it is in the diffusive phase. The disorder is critical for $T = T_0$. In order see how the DOS at the zero energy is generated by the FRG flow the BSS Eq. (6) of PME Eq. (5) corresponding to $T < T_0$ has to be continued analytically from $t < T$ to $t > T$. Recalling the asymptotic behavior $F(\zeta) \sim \zeta^{2-1/\delta}$ for $\zeta \to \infty$ we find by the continuity that $u(r, t = T) = c^* r^{(2\delta-1)/\delta}(1 + o(1))$ for $r \to 0$. In the postfocusing regime, i.e., for $t > T$, the fictional particles, whose nonlinear diffusion is described by the PME, Eq. (5), start to accumulate at the origin. This is described by a forward self-similar solution (FSS) [57]:

$$u(r,t) = (t-T)^{2\delta-1}\tilde{F}(\tilde{\zeta}), \qquad \tilde{\zeta} = \frac{r}{(t-T)^{\delta}}. \tag{10}$$

The FSS Eq. (10) can be found using the same phase-plane formalism [36]. It implies that $\tilde{F}(0) = \text{const}$ and $\tilde{F}(\tilde{\zeta}) \sim \tilde{\zeta}^{2-1/\delta}$ for $\tilde{\zeta} \to \infty$ (see Fig. 2). Since $W'(0^+) = u(0,t) \neq 0$ in the postfocusing regime, the FSS Eq. (10) describes the diffusive phase of relativistic fermions and allows one to compute explicitly the DOS at zero energy. We find that close to the transition, i.e., for $T_0 - T \ll T_0$ the DOS at zero energy is given by $\overline{\rho(0)} \sim (T_0 - T)^{\beta}$ with the order parameter critical exponent $\beta = 2\delta - 1$. Assuming that the hyperscaling relation $\beta = (d - z)\nu$ is not broken we obtain the dynamic critical exponent as $z = 1 + \varepsilon + O(\varepsilon^2)$. Beside the averaged DOS the postfocusing regime of the FRG flow, Eq. (10) allows us to characterize the scaling behavior of the whole distribution of its fluctuations in the diffusive metal phase. We find that the scaling behavior of the nth cumulant of the DOS fluctuations at zero energy scales as $\overline{\rho^n(0)}^c \sim (T_0 - T)^{\beta - 2\delta(n-1)}$ close to the transition. This scaling signals that the corresponding distribution becomes very broad when one approaches the transition from the diffusive phase.

Conclusions and outlook.—We have developed a FRG approach to the semimetal-diffusive metal transition in disordered Weyl fermions. We have shown that the previously studied FP corresponding to the Gaussian distribution of disorder is unstable, demonstrating the relevance of rare disorder fluctuations at this transition. Indeed, the analysis of the flow equation derived in a fixed dimension d reveals the proliferation of infinite number of higher order cumulants in the running disorder distribution, even if starting from a pure Gaussian distribution [36]. In order to resolve this problem we have established a connection between the FRG equation and the celebrated PME, whose self-similar solution represents a nonanalytic FP describing the transition. Its analytical continuation to the postfocusing regime provides a unique mechanism of spontaneous generation of a finite DOS at zero energy [60]. Moreover, it shows that the distribution of fluctuations becomes very broad close to the transition. In particular, one expects that the critical wave functions exhibit multifractality at the transition [61], with a spectrum different from that at the GN FP [26,62].

It was argued that rare disorder configurations can give rise to a finite DOS in the semimetal phase [63]. Although more refined recent calculations [64] suggest that their contribution vanishes in the thermodynamic limit, a finite DOS was observed in numerical simulations [65]. While this can be due to a simple finite size correction, $\overline{\rho(0)} \sim L^{z-d}$, our FRG description provides another mechanism. Indeed, vanishing of the DOS stems from the existence of a finite waiting time in the FRG flow [Eq. (5)]. Once one introduces disorder correlations similar to that used in Ref. [63], the waiting time phenomenon is then superimposed with a slow creeplike motion, which generates a finite DOS. Nevertheless the universal critical properties of the underlying FP will dominate over the nonuniversal contributions depending on the UV cutoff.

Beyond the present transition, our approach can be applied to other problems where nonanalyticity in the renormalized disorder distribution may play a significant role, such as the Anderson localization transition. In that case, the relevance of infinitely many so-called high-gradient operators at the conventional FP of the corresponding nonlinear sigma model [11–13] raises the question of the existence of a new FP which indeed controls the transition. An argument against such an unconventional FP [12] can also be applied to the present semimetal–diffusive metal transition. Here, however, it is ruled out by nonanalyticity of the new FP, which hints at a similar scenario in the case of the Anderson localization transition [66].

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