


Optical Ring Cavity Search for Axion Dark Matter

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We propose a novel experiment to search for axion dark matter that differentiates the phase velocities of the left- and right-handed polarized photons. Our optical cavity measures the difference of the resonant frequencies between two circular polarizations of the laser beam. The design of our cavity adopts a double-pass configuration to realize a null experiment and give a high common mode rejection of environmental disturbances. We estimate the potential sensitivity to the axion-photon coupling constant $g_{a\gamma}$ for the axion mass $m \lesssim 10^{-10}$ eV. In a low mass range $m \lesssim 10^{-15}$ eV, we can achieve $g_{a\gamma} \lesssim 3 \times 10^{-16}$ GeV⁻¹, which is beyond the current bound by several orders of magnitude.

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Introduction.—The axion is one of the most known particles in the extended theories beyond the standard model of particle physics. It was originally introduced by Peccei and Quinn to solve the strong CP problem in QCD [1]. Moreover, string theory and supergravity generically predict a plenitude of axionlike particles (ALPs), which can have a broad range of mass [2]. Especially, the axion with a small mass $m \ll 1$ eV is well motivated by cosmology, since it behaves as a nonrelativistic matter fluid in our Universe and is a good candidate for dark matter. Axion dark matter may weakly interact with known standard model particles so that we can explore axion dark matter through direct search experiments.

It has been known that if an axion is coupled to a photon, the axion-photon conversion under static magnetic fields takes place [3]. Making use of this conversion process, many different types of experiments have been considered, such as axion haloscopes [4], axion helioscopes [5,6], “light shining through a wall” experiments [7,8], laser interferometry [9,10], and magnetometers [11,12] (for more details, see recent reviews [13] and references therein). They put the constraints on the photon-axion coupling constant $g_{a\gamma}$ for a vast range of the ALPs’ mass. Astronomical observations can be also used to probe the axion-photon conversion. For the low mass range, the absence of γ -ray emission from SN1987A [14,15] and the spectral of cosmic rays from galaxy clusters [16–18] are used to put significant bounds on $g_{a\gamma}$. Furthermore, some observational results that can be attributed to the photon-axion conversion might imply its presence [19–21].

Here, we propose another way to find the coupling of photons to axion dark matter without using the axion-photon conversion. The dark matter axion whose field value oscillates around the minimum of its potential provides a small difference in the phase velocity between

the left- and the right-handed photon. The optical cavity is useful to detect such a small deviation of the phase velocity. The birefringence generally caused by ALPs [22] (and specifically by their oscillating background [23]) has been studied, and recently the authors in [10] suggested an experiment with a Michelson interferometer. On the other hand, ring cavity experiments have emerged to test the parity-odd Lorentz violation in the photon sector [24]. They have measured the variation of the resonant frequency depending on the direction of the light path. A similar technique can be applied for our purpose, because the resonant frequency of the cavity shifts depends on the polarization of photon, provided that the dark matter axion is coupled to a photon. The dark matter axion predicts the phase velocities of the left- and right-handed polarized photon shift with the opposite signs and the same magnitude. Therefore, such shifts of the resonant frequencies of the polarized laser in the optical cavity are the measurement target in our experiment. We estimate the reach of our cavity experiment and obtain the potential sensitivity.

This Letter is organized as follows. In the next section, we derive the difference in the phase velocity of polarized photons in the presence of axion dark matter and estimate its magnitude. Following that, we describe the experimental method to probe the axion-photon coupling with our designed cavity. We then show the potential sensitivity of our experiment and give a short discussion. Finally, we conclude the result of this Letter. In this Letter, we set the unit $\hbar = c = 1$.

Phase velocities of photons.—In this section, we present the equations of motion (EOM) for two circular-polarized photons coupled with the axion dark matter and estimate their phase velocities. We consider the axion-photon coupling term

$$\frac{g_{a\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu} = g_{a\gamma} \dot{a} A_i \epsilon_{ijk} \partial_j A_k + (\text{total derivative}), \quad (1)$$

where the dot denotes the time derivative, $a(t)$ is the axion field value, A_μ is the vector potential, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, and $\tilde{F}_{\mu\nu} \equiv \epsilon_{\mu\nu\rho\sigma} \partial_\rho A_\sigma / (2\sqrt{-g})$. Here, we choose the temporal gauge $A_0 = 0$ and the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$. Then the EOM for gauge fields reads

$$\ddot{A}_i - \nabla^2 A_i + g_{a\gamma} \dot{a} \epsilon_{ijk} \partial_j A_k = 0. \quad (2)$$

The present background axion field is written as

$$a(t) = a_0 \cos[mt + \delta_\tau(t)], \quad (3)$$

with its constant amplitude a_0 , its mass m , and a phase factor $\delta_\tau(t)$. In this experiment, we search for the axion dark matter with the mass $m \lesssim 10^{-10}$ eV, and the corresponding frequency f is given by

$$f = \frac{m}{2\pi} \simeq 2.4 \text{ Hz} \left(\frac{m}{10^{-14} \text{ eV}} \right). \quad (4)$$

The phase factor δ_τ can be assumed to be a constant value within the coherence timescale of dark matter τ .

We decompose A_i into two helicity modes with wave number \mathbf{k} as

$$A_i(t, \mathbf{x}) = \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^3} A_k^\lambda(t) e_i^\lambda(\hat{\mathbf{k}}) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (5)$$

where $e_i^\lambda(\hat{\mathbf{k}}) = e_i^{\lambda*}(-\hat{\mathbf{k}})$ is the circular polarization vector that obeys $e_i^\lambda(\hat{\mathbf{k}}) e_i^{\lambda'}(\hat{\mathbf{k}}) = \delta^{\lambda\lambda'}$, and $\epsilon_{ijm} k_j e_m^\pm(\hat{\mathbf{k}}) = \pm k e_i^\pm(\hat{\mathbf{k}})$. Then, one finds EOM for the two polarization modes as

$$\ddot{A}_k^\pm + \omega_\pm^2 A_k^\pm = 0, \quad (6)$$

with

$$\omega_\pm^2 \equiv k^2 \left(1 \pm \frac{g_{a\gamma} a_0 m}{k} \sin(mt + \delta_\tau) \right). \quad (7)$$

From (7), we obtain their phase velocities as

$$c_\pm \equiv \frac{\omega_\pm}{k} = \left(1 \pm \frac{g_{a\gamma} a_0 m}{k} \sin(mt + \delta_\tau) \right)^{1/2} \quad (8)$$

and define their difference as $\delta c \equiv |c_+ - c_-|$. The tiny coupling $g_{a\gamma}$ allows us to approximate δc by

$$\delta c \simeq \frac{g_{a\gamma} a_0 m}{k} \sin(mt + \delta_\tau) \equiv \delta c_0 \sin(mt + \delta_\tau). \quad (9)$$

Assuming the laser light with the wavelength $\lambda = 2\pi/k = 1550$ nm, we can estimate

$$\delta c_0 \simeq 3 \times 10^{-24} \left(\frac{g_{a\gamma}}{10^{-12} \text{ GeV}^{-1}} \right), \quad (10)$$

where we used the present energy density of the axion dark matter $\rho_a = m^2 a_0^2 / 2 \simeq 0.3 \text{ GeV/cm}^3$.

Search for axion dark matter using optical ring cavity.— In this section, we describe our experiment to detect δc caused by the axion dark matter. The setup of our experiment is schematically illustrated in Fig. 1. First, a laser beam that is circularly polarized by a 1/4 wave plate enters our bow-tie cavity. For illustrative purposes, let us assume the incident beam has the left-handed polarization. The incident beam to the cavity is partially reflected by the input mirror and goes to the photodetector A, while the other part enters the cavity. Since the reflection off of a mirror flips the circular polarization of photon, the beam changes its polarization each time it is reflected by a mirror. It should be noted that the beam that enters the cavity from the left has the right-handed polarization most of the time, because the bow-tie optical path is stretched in the longitudinal direction. It eventually goes to either the photodetector A or the mirror on the far right. The beam that is reflected from the mirror on the far right is partially reflected into the photodetector B or reenters the cavity. Then, it has the left-handed polarization most of the time, while traveling inside the cavity in the opposite direction. Finally, some part of the beam goes into the photodetector B.

From each photodetector, we can obtain the signal that is proportional to the frequency difference between the laser

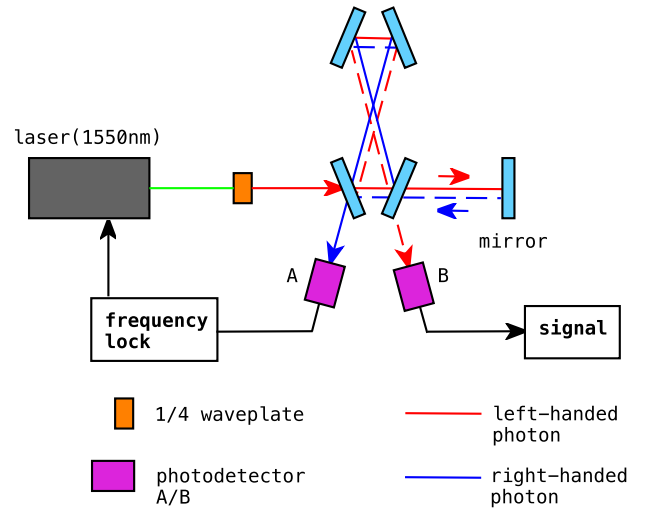


FIG. 1. The layout of our double-pass bow-tie cavity. The left-handed beam (solid line) is injected to the resonant cavity, while the transmit beam reflected by the mirror on the far right goes to the cavity as the right-handed beam (dashed line). The photodetector A is used to lock the laser frequency at the resonant frequency for the injected beam from the left, and the photodetector B monitors the modulation of the resonant frequency difference of two optical paths from the beam coming from the right.

frequency and the cavity resonant frequency, using, for example, the Pound-Drever-Hall method [25]. Using this error signal taken by photodetector *A*, the incident laser frequency is stabilized to the resonance of the (almost) right-handed polarized beam. We can also obtain the second error signal from photodetector *B*, which is proportional to the resonant frequency difference between (almost) the left- and right-handed beams. Without the phase velocity modulation δc given by the axion dark matter, the resonant frequency would not depend on the circular polarizations. Therefore, our setup works as a null experiment sensitive to the axion-photon coupling.

The bow-tie configuration of our optical cavity cancels the Sagnac effect from, for example, the spin of Earth [26]. Most of the environmental noises are also canceled due to the double-pass configuration [27], because the second error signal observes only the difference in the resonant frequency between the two counterpropagating optical paths in the cavity, and their common fluctuations become irrelevant. The difference of the resonant frequencies between the two optical paths is given by

$$\frac{\delta\nu}{\nu} = \frac{\delta c}{c} = 3 \times 10^{-24} \left(\frac{g_{a\gamma}}{10^{-12} \text{ GeV}^{-1}} \right) \sin(mt + \delta_\tau), \quad (11)$$

and hence the second error signal is expected to be oscillating. This oscillatory behavior is advantageous for the signal extraction.

Sensitivity to the axion-photon coupling.—In this section, we calculate the sensitivity of our experiment to the axion-photon coupling constant. By virtue of the double-pass configuration, most of the noises from the environmental disturbance are, in principle, canceled out by the common mode rejection. Then, the primary source of noise is the quantum shot noise. The one-sided spectrum of the shot noise of an optical ring cavity is written as [28]

$$\sqrt{S_{\text{shot}}} = \sqrt{\frac{\lambda}{4\pi P} \left(\frac{1}{t_r^2} + \omega^2 \right)}, \quad (12)$$

where λ is the laser wavelength, P is the input power, and ω is the angular frequency, which is the axion mass m in our case. Note that the quantum radiation pressure noise is canceled out by our double-pass configuration. Averaged round-trip time t_r is

$$t_r = \frac{L\mathcal{F}}{\pi}, \quad (13)$$

where L is the cavity round-trip length and \mathcal{F} is the finesse.

If our measurement is limited by the shot noise, the signal-to-noise ratio (SNR) improves with the measurement time T as

$$\text{SNR} = \frac{\sqrt{T}}{2\sqrt{S_{\text{shot}}}} \frac{\delta c_0}{c}, \quad (14)$$

as long as the axion oscillation is coherent for $T \lesssim \tau$. The axion dark matter can be regarded to show a coherent oscillation, and δ_τ in (11) is constant within the coherent timescale $\tau = 2\pi/(mv^2)$. Since the local velocity of dark matter v is around 10^{-3} , τ is roughly estimated as

$$\tau \sim 1 \text{ yr} \left(\frac{10^{-16} \text{ eV}}{m} \right). \quad (15)$$

When the measurement time becomes longer than this coherence time $T > \tau$, the phase δ_τ is not constant anymore and δ_τ behaves as a random variable staying constant for each period of τ . As a consequence, the growth of the SNR with the measurement time changes as [29]

$$\text{SNR} = \frac{(T\tau)^{1/4}}{2\sqrt{S_{\text{shot}}}} \frac{\delta c_0}{c}. \quad (16)$$

Therefore, the sensitivity to $\delta c_0/c$ is limited by

$$\frac{\delta c_0}{c} \lesssim \begin{cases} \frac{2}{\sqrt{T}} \sqrt{S_{\text{shot}}} & (T \lesssim \tau) \\ \frac{2}{(T\tau)^{1/4}} \sqrt{S_{\text{shot}}} & (T \gtrsim \tau) \end{cases}. \quad (17)$$

It can be translated into the sensitivity to $g_{a\gamma}$ as

$$g_{a\gamma} \lesssim \begin{cases} 10^{12} \sqrt{\frac{S_{\text{shot}}}{T}} [1/\text{GeV}] & (T \lesssim \tau) \\ 10^{12} \sqrt{\frac{S_{\text{shot}}}{(T\tau)^{1/2}}} [1/\text{GeV}] & (T \gtrsim \tau) \end{cases}. \quad (18)$$

Figure 2 shows the sensitivity of our experiment to the axion-photon coupling constant for different configurations. Here, we set $\lambda = 1550 \text{ nm}$ and assume $T = 1 \text{ yr} = 3 \times 10^7 \text{ sec}$. With feasible parameters, we can achieve a sensitivity level $g_{a\gamma} \simeq 3 \times 10^{-13} \text{ GeV}^{-1}$ for $m \lesssim 10^{-16} \text{ eV}$, which is below the current constraints from axion helioscope experiments and SN1987A and *Chandra* x-ray observations. Moreover, with more optimistic parameters, our cavity can reach $g_{a\gamma} \simeq 3 \times 10^{-16} \text{ GeV}^{-1}$ for $m \lesssim 10^{-16} \text{ eV}$, which will be the best sensitivity among the proposed axion search experiments in this mass range.

Our optical cavity is made critically coupled so that most of the beam is transmitted, and reflected beam power impinging on the photodiode is minimized. This allows the shot noise limited detection at 100 W input power with current technology.

We note here that various technical noises at low frequency should be further investigated to determine the sensitivity for a lower mass range of the axion. Technical noises could be higher than the shot noise, especially at low frequencies, since technical noises are usually smaller at

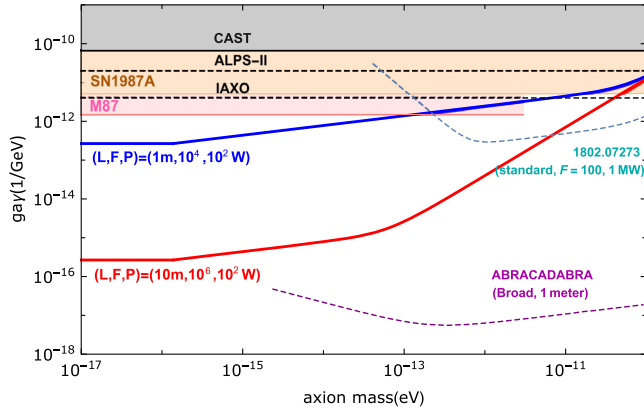


FIG. 2. The sensitivity curves for the axion-photon coupling constant $g_{a\gamma}$ with respect to the axion mass m . The solid blue (red) line shows the sensitivity of our experiment $(L, F, P) = (1(10) \text{ m}, 10^4(10^6), 10^2(10^2) \text{ W})$. The gray band represents the current limit from CAST [5]. The dashed black lines are the prospected limits of IAXO [6] and ALPS-II [7] missions. The dashed turquoise blue and purple lines show the proposed reaches of axion optical interferometer suggested in [10] and ABRA-CADABRA magnetometer [12]. The orange and pink bands denote the astrophysical constraints from the cosmic ray observations of SN1987A [15] and radio galaxy M87 [17].

higher frequencies, and shot noise degrades proportional to frequency above the cavity pole [see Eq. (12)]. One of the largest technical noises comes from mirror vibration. The vibration noise can be estimated with

$$\sqrt{S_{\text{vib}}} = \omega^2 A \gamma_{\text{CMRR}} \sqrt{S_{\text{seis}}}, \quad (19)$$

where A is the vibration sensitivity, γ_{CMRR} is the common-mode rejection ratio, and $\sqrt{S_{\text{seis}}}$ is the ground vibration spectrum. Assuming values from cutting-edge technologies, $A = 10^{-12}/(\text{m/s}^2)$, $\gamma_{\text{CMRR}} = 10^{-4}$, and $\sqrt{S_{\text{seis}}} = 10^{-9}/f^2 \text{ m}/\sqrt{\text{Hz}}$, we get $\sqrt{S_{\text{vib}}} = 4 \times 10^{-24}$. This is comparable to the shot noise level below the cavity pole in the optimistic cavity parameter case, and a reasonable vibration attenuation system would be necessary.

Furthermore, it would be interesting to consider an optical cavity that is sensitive to the axion-photon coupling at higher frequencies, because our cavity does not have much sensitivity at larger mass regions $m \gtrsim 10^{-10} \text{ eV}$. We leave these issues for future work.

Conclusion.—In this Letter, we proposed a novel experiment to probe the coupling of axion dark matter to photon with a resonant cavity. We considered the double-pass cavity, which aims to detect the difference in the resonant frequencies of the laser beam with the two circular polarizations. Because of the oscillation of the axion dark matter, the resonant frequencies are expected to periodically change in time and we can extract the signal from the irreducible noises. The sensitivity curve is, in principle, determined only by quantum shot noise by virtue of the

double-pass configuration, and hence we can achieve the great sensitivity level for the detection of the axion-photon coupling constant. In the concrete estimation of the sensitivity, we adopted two sets of parameters: a feasible one and an optimistic one. We have demonstrated that both of them can reach sensitivities beyond the current constraints by several orders of magnitude.

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