Marginally Self-Averaging One-Dimensional Localization in Bilayer Graphene

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The combination of a field-tunable band gap, topological edge states, and valleys in the band structure makes insulating bilayer graphene a unique localized system, where the scaling laws of dimensionless conductance g remain largely unexplored. Here we show that the relative fluctuations in $\ln g$ with the varying chemical potential, in strongly insulating bilayer graphene (BLG), decay nearly logarithmically for a channel length up to $L/\xi \approx 20$, where ξ is the localization length. This "marginal" self-averaging, and the corresponding dependence of $\langle \ln g \rangle$ on L, suggests that transport in strongly gapped BLG occurs along strictly one-dimensional channels, where $\xi \approx 0.5 \pm 0.1$ μm was found to be much longer than that expected from the bulk band gap. Our experiment reveals a nontrivial localization mechanism in gapped BLG, governed by transport along robust edge modes.

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The nature of subgap electrical transport in bilayer graphene (BLG) at large transverse electric fields (D) has led to considerable debate [1–6]. D lifts the interlayer symmetry, opening a band gap in the quasiparticle energy spectrum. At large D, the charge carriers in BLG are strongly localized when the Fermi level is tuned close to the charge neutrality point (CNP) [1]. Moreover, gapped BLG behaves as a "marginal topological insulator," where the finite Berry phase and field-induced inversion symmetry breaking lead to topologically protected one-dimensional conduction modes along specific edge and stacking boundary configurations [7–9]. While this has been experimentally verified through observation of the valley Hall effect [10,11] and ballistic 1D channels along artificial [12] and natural [13] stacking boundaries, the topological properties also raise doubts on the current understanding of the localized state transport in gapped BLG at low temperature ($T < 50$ K). Although initial results were analyzed in terms of two-dimensional Mott-type variable range hopping (VRH) associated with localized states in the bulk [1–4,14], recent supercurrent interferometry experiments [5,6] suggest strong edge-mode transport in short gapped BLG transistors. While this seems consistent with the apparent saturation of g at large D reported recently [6], the dimensionality of localized state transport in generic gapped BLG remains uncertain so far.

In BLG subjected to large D at low T, the localization at the edge (due to short-range lattice defects, chemical adsorbates, etc.) and that in the gapped bulk are hard to distinguish because of limited experimental temperature range for VRH. Here, we have followed a new route based on evaluating the full conductance statistics in the insulating regime and we specifically study its self-averaging properties. A macroscopic variable X in a disordered system of linear dimension L is spatially ergodic, or self-averaging, when the relative fluctuations $R_X = \langle (\Delta X)^2 \rangle / \langle X \rangle^2 \to 0$ as $L \to \infty$, where $\langle \cdots \rangle$ represents averaging over different realizations of disorder. For strongly localized noninteracting carriers, the electrical conductance g (in units of e^2/h) does not self-average, but the logarithm of g does [15–18], in a manner that is uniquely sensitive to the dimensionality and the scaling properties of Anderson localization for $L \gg \xi$, the localization length [19–24]. In two and three dimensions, the ensemble fluctuations in $\ln g$ are strongly self-averaging with $R_{\ln g} \sim L^{-d}$ ($d = 2, 3$) [20,24], whereas in 1D disordered systems at finite T, $\ln g$ is only marginally self-averaging because $R_{\ln g}$ decays logarithmically with L [19–21]. This purely 1D effect, which so far remains experimentally elusive, to the best of our knowledge, is predicted to occur because the conductance of the system is determined primarily by the most resistive, but unavoidable, hop at the percolation threshold [22]. To determine $R_{\ln g}$, we directly obtain the mean and the variance of $\ln g$ by measuring the full conductance probability distribution function (PDF) in the localized state for many dual-gated BLG devices with varying channel lengths. We find that for small electric fields (typically $|D| \lesssim 0.5$ V/nm), the relative fluctuations in $\ln g$ with the Fermi level close to the CNP decay with L as $\sim 1/L^2$, but the decay becomes nearly logarithmic at larger D—a characteristic of strictly 1D localized transport.

The dual-gated BLG channels were created with mechanical exfoliation, followed by either one (top) or both sides covered with hexagonal boron nitride (hBN). Both surface and edge-contacting methods were adopted [25], and the top gate length defines the channel length L.

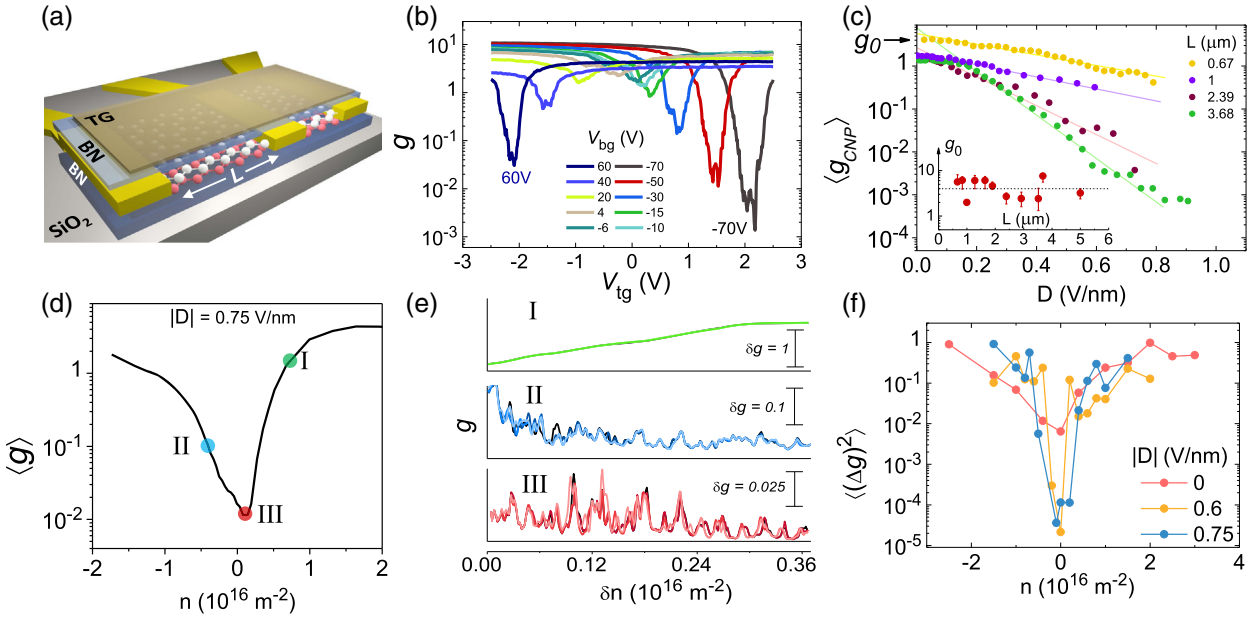

FIG. 1. Device characteristics and conductance fluctuations. (a) Device schematic. (b) Transfer characteristics ($g - V_{tg}$) for a few V_{bg} at 275 mK. (c) Mean conductance $\langle g_{CNP} \rangle$ at the charge neutrality point (CNP) as a function of D for several devices with different channel lengths. Inset shows extrapolated values of $\langle g_{CNP} \rangle$ to $D = 0$ V/nm. (d) Carrier density (n) dependence of $\langle g \rangle$ for $|D| = 0.75$ V/nm. (e) Conductance fluctuations as a function of small variations in n induced by the top gate at three different points marked in (d). (f) Variance of conductance fluctuations $\langle (\Delta g)^2 \rangle$ as a function of n for several D.

A representative schematic of an edge-contacted device is shown in Fig. 1(a), and more details can be found in Ref. [26]. We have studied the conductance statistics of 20 BLG channels with L ranging from ~0.7 to 19.5 μm in different BLG flakes with channel widths ranging from ~1.0 to 3.1 μm. Although measurements were carried out down to ~20 mK, the conductance becomes nearly insensitive to T below $\lesssim$1 K in all devices in the localized regime [Fig. 3(c)]. Figure 1(b) illustrates the typical transfer characteristics of gated BLG, obtained at $T = 275$ mK in device Dev3 with $L = 1.28$ μm, where the two-probe conductance g is shown as a function of the top gate voltage (V_{tg}) at fixed backgate voltages (V_{bg}).

Figure 1(b) also shows that the conductance at CNP, $g_{CNP} \ll 1$ for large values of $|V_{bg}|$, which implies strong localization of carriers at the center of the band gap as $|D|$ increases. Between $|D| = 0$ and $\lesssim$0.4 V/nm, when localization in the bulk is weak, the variation in g_{CNP} with D is device dependent. However, for $|D| > 0.5$ V/nm, g_{CNP} decreases nearly exponentially in almost all our devices irrespective of L [Fig. 1(c)]. The absence of saturation in g_{CNP} at large D confirms that there are no accidental stacking or grain boundaries that shunt the source and drain leads [13], and that $L > \xi$ in all devices. To avoid gate leakage, the maximum $|D|$ was limited to ≈0.9–1.3 V/nm, which, for longer devices, led to g_{CNP} as low as ~10^{-3}–10^{-4}.

The key feature of the transfer characteristics in Fig. 1(b) is the strong relative fluctuations in g in the vicinity of the CNP, which become more apparent as $|D|$ increases. The fluctuations are remarkably reproducible as the Fermi level is varied, as illustrated in Fig. 1(e) with device Dev6 ($L = 2.92$ μm) at $T = 0.03$ K and $|D| = 0.75$ V/nm. The panels of Fig. 1(e) represent three separate regimes from metallic (large n) to strongly localized (CNP, $n \approx 0 \times 10^{16}$ m^{-2}) transport, where four traces of g within the window $\delta n = 0.36 \times 10^{16}$ m^{-2} demonstrate the reproducibility, and confirm negligible contribution from time-dependent noise [25,29]. Within δn, which is similar to the spatial variation in carrier density typically present in BLG [30], the fluctuations reflect traversing across the microscopic realizations at a fixed point in the phase space [19,20,22].

To quantify, we have first calculated the variance $\langle (\Delta g)^2 \rangle$ within δn (consisting of ≈400 points or realizations) and shown it as a function of n in Fig. 1(f) for three values of D. At large n (typically $|n| > 1 \times 10^{16}$ m^{-2}), the onset of a quasimetallic or weakly localized regime is characterized by $g \gtrsim 1$, where $\langle (\Delta g)^2 \rangle$ saturates to ≈0.1–1, irrespective of D. This is universal conductance fluctuations due to quantum interference of multiply backscattered electron waves [31,32]. Since the Fermi level lies within the conduction or valence bands, this is a bulk phenomenon, expected for diffusive 2D disordered systems when the phase coherence length is similar to L. As $|n|$ decreases, $\langle (\Delta g)^2 \rangle$ decreases, exhibiting a minimum around the CNP. The reduction in $\langle (\Delta g)^2 \rangle$ at CNP is weak for $D = 0$ V/nm, but is nearly 5 orders of magnitude for $|D| \geq 0.6$ V/nm. The fluctuations in the localized regime are largely immune

to contact effects [25], where the channel conductance is at least an order of magnitude lower than that of the contacts.

Fluctuations in g with gate voltage in the strongly localized regime may be caused by inhomogeneous charge distribution or single-particle localized states in the bulk of the BLG [33] through, e.g., local charging of electron-hole puddles [34] or multiple transmission resonances [35]. However, direct charging or resonance effects are expected to decay rapidly for $L \gtrsim O[100\ \text{nm}]$, the typical scale of inhomogeneity. Such fluctuations may also arise in disordered quasi-1D or (short) 2D systems due to the extreme sensitivity of the critical resistance-determining hop to the local chemical potential [36], as shown for short conducting channels in silicon and semiconductor heterostructures [22,37,38]. This mechanism manifests in the PDF of $\ln g$ that approaches a Gaussian for large L [17,20,21,23], and the characteristic L dependence of the mean and variance of $\ln g$ depend sensitively on the dimensionality of the system [19–21].

Figures 2(a)–2(d) show the PDF of the fluctuations in g observed within the δn window around the CNP for different values of D and L. Figures 2(a) and 2(b) present data from device Dev10 with $L = 3.68\ \mu$m at different D. At $D = 0$ V/nm [Fig. 2(a)], the device is quasimetallic ($g_{\text{CNP}} \simeq 2$), and the PDF of g_{CNP} is close to a Gaussian, symmetric around $\langle g_{\text{CNP}} \rangle$ (dashed line). However, at large $|D|$ of 0.75 V/nm [Fig. 2(b)], the device is strongly localized at the CNP with modal $g_{\text{CNP}} \sim 0.01$, and the PDF in g_{CNP} is strongly asymmetric around the peak. Instead, as shown in the inset of Fig. 2(b), the PDF of $\ln g_{\text{CNP}}$ is symmetric around $\langle \ln g_{\text{CNP}} \rangle$, and corresponds closely to a Gaussian distribution (solid line). The log-normal PDF in g_{CNP} is observed for all but one L at large D (typically $|D| \gtrsim 0.5$ V/nm, see Ref. [26] for details), as illustrated with two other devices in Figs. 2(c) and 2(d). Occasionally, the distribution can exhibit weak asymmetry due to the blocking effect or "optimal shorts or punctures" in long and short channels, respectively [21,22] (Fig. S3 of Supplemental Material [26]).

Log-normal conductance PDF in strongly localized systems, when $L \gg \xi$, has been analytically shown in 1D systems [17,18], whereas only numerically in higher dimensions [24,39]. The Gaussian fits to the PDFs allow a direct evaluation of $\langle \ln g_{\text{CNP}} \rangle$ and $\text{var}(\ln g_{\text{CNP}})$ with varying D and L, shown in Figs. 2(e) and 2(f), respectively. Quantitatively, the mean conductance of a 1D disordered system can be expressed as [20,21]

$$\langle \ln g \rangle \approx -\left(\frac{T_0}{T}\right)^{1/2} f(L/\xi, T_0/T), \tag{1}$$

where $T_0 = 1/k_B \xi N$, and N is the 1D density of states at the Fermi level. The functional form of f depends on the details of the hopping mechanism. Serota, Kalia, and Lee (SKL) [20] considered that selective links which are the

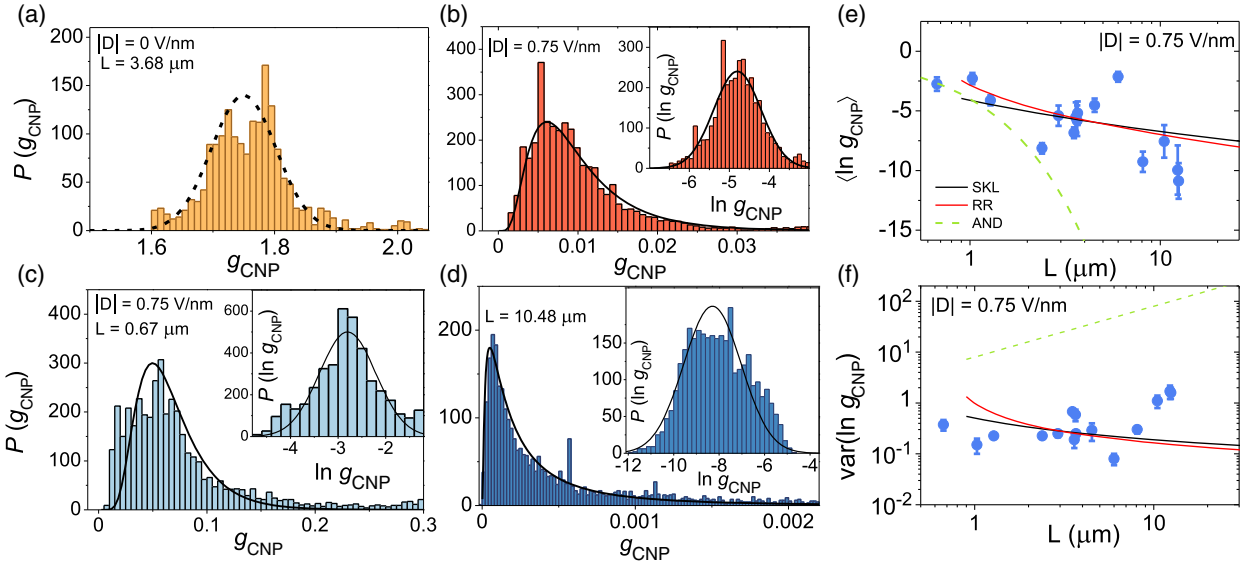

FIG. 2. Conductance distribution. Probability distribution function of conductance [$P(g_{\text{CNP}})$ and $P(\ln g_{\text{CNP}})$] around the CNP for different L and D, with a fixed length $L = 3.68\ \mu$m at (a) $D = 0$ V/nm and (b) $|D| = 0.75$ V/nm; (c),(d) for a fixed $|D| = 0.75$ V/nm at (c) $L = 0.67\ \mu$m and (d) $L = 10.48\ \mu$m. Insets in (b)–(d) show the corresponding probability distributions of $\ln g_{\text{CNP}}$. The dashed and solid lines represent normal and log-normal distributions, respectively. (e) Average logarithm of conductance around the CNP ($\langle \ln g_{\text{CNP}} \rangle$) as a function of L for $|D| = 0.75$ V/nm. The expected variations of $\langle \ln g_{\text{CNP}} \rangle$ from SKL [20] (black solid line), RR [21] (red solid line), and Anderson model [17] (green dashed line) are also indicated. The error bars indicate the standard deviation of $\langle \ln g_{\text{CNP}} \rangle$ over ensemble variations. (f) The variance of $\ln g_{\text{CNP}}$, $\text{var}(\ln g_{\text{CNP}})$, as a function of channel length L for $|D| = 0.75$ V/nm. The L dependence of variance expected from SKL, RR, and Anderson model, with $\xi \approx 0.5\ \mu$m, are shown. The error bars represent the uncertainty in log-normal fits.

weakest dominate the net conductance, and showed that $f \approx [\ln(2L/\xi)]^{1/2}$. On the other hand, Raikh and Ruzin (RR) [21] introduced the concept of an optimal break in the phase space of energy and position of the localized states and derived $f \approx \{\ln[(L/\xi)(T/T_0)^{1/2}\ln^{1/2}(L/\xi)]\}^{1/2}$. Moreover, the variance of conductance for a 1D disordered system is predicted to exhibit a logarithmic decrease $[\sim[\ln(2L/\xi)]^{-1}]$ [20,21]. Therefore, the relative fluctuations, obtained by dividing $\mathrm{var}(\ln g_{\mathrm{CNP}})$ with the corresponding $\langle \ln g_{\mathrm{CNP}} \rangle^2$, are expected to only marginally decay with L as $R_{\ln g} \sim [\ln(2L/\xi)]^{-2}$ in 1D.

Figure 3(a) shows the L dependence of the relative fluctuations $R_{\ln g}$ in the strongly localized regime ($|D| \gtrsim 0.5$ V/nm). Clearly, $R_{\ln g}$ remains nearly constant or decreases marginally even as L increases by more than an order of magnitude. This represents near absence of self-averaging in the localized BLG transport. While SKL and RR [20,21] generally capture the logarithmic decay in self-averaging with L [solid lines in Fig. 3(a)], we emphasize that the marginal self-averaging behavior is a model-independent phenomenon. This is expected in a randomly disordered purely 1D system because the most resistive link cannot be bypassed with an increasing system size. Intriguingly, the best fit to the data, both by SKL and RR, yields a similar estimate of $\xi \approx 0.5$ μm. This estimate exceeds the localization length $\sim\hbar/\sqrt{2m^*\Delta} \sim 3$–$10$ nm due to the bulk band gap (Δ) by nearly 2 orders, but is similar to the length scale of short-range edge defects and intervalley scattering [40], suggesting transport along the BLG edge [6]. The insensitivity of $R_{\ln g}$ to the magnitude of D for $|D| \gtrsim 0.5$ V/nm further suggests that, beyond a certain gap, the bulk of the BLG becomes largely inconsequential to the hopping transport. Importantly, the self-averaging properties expected in two dimensions are recovered at low $|D|$, where bulk transport contributes significantly, and manifest in $R_{\ln g} \sim 1/L^2$ [20,24] for $|D| \lesssim 0.5$ V/nm [inset of Fig. 3(a)].

The $\langle \ln g_{\mathrm{CNP}} \rangle^2$ and the $\mathrm{var}(\ln g_{\mathrm{CNP}})$, shown in Figs. 2(e) and 2(f), respectively, can also be roughly described by the SKL and RR models with the same $\xi \approx 0.5 \pm 0.1$ μm as used above but are inconsistent with the form expected from the $T = 0$ K Anderson localization model [17]. Importantly, both SKL and RR models suggest $\langle \ln g \rangle \sim -(T_0/T)^{1/2}$ in the leading order, which is indeed observed in our experiments in the range $T \gtrsim 1$ K as shown in Fig. 3(c).

Alternative edge-bound transport processes, in particular, those due to lateral confinement at the BLG boundary [5], cannot be completely ruled out. However, the inherent tendency to bypass around strong disorder renders a quasi-1D nature to these channels and it is unlikely that such a mechanism would lead to the suppression of self-averaging, which is a strictly 1D phenomenon. The self-averaging may also be absent in specific cases of fractal disorder landscape [41] or proximity to critical point [24], which are not likely in the BLG devices. Thus, in view of the theoretical [7] and recent experimental reports [5,6], a likely mechanism is hopping via low-energy electronic states of disordered edges in BLG [see Fig. 3(b)]. Notably, the observed prefactor $g_0 \sim 2$–6 in the exponential variation

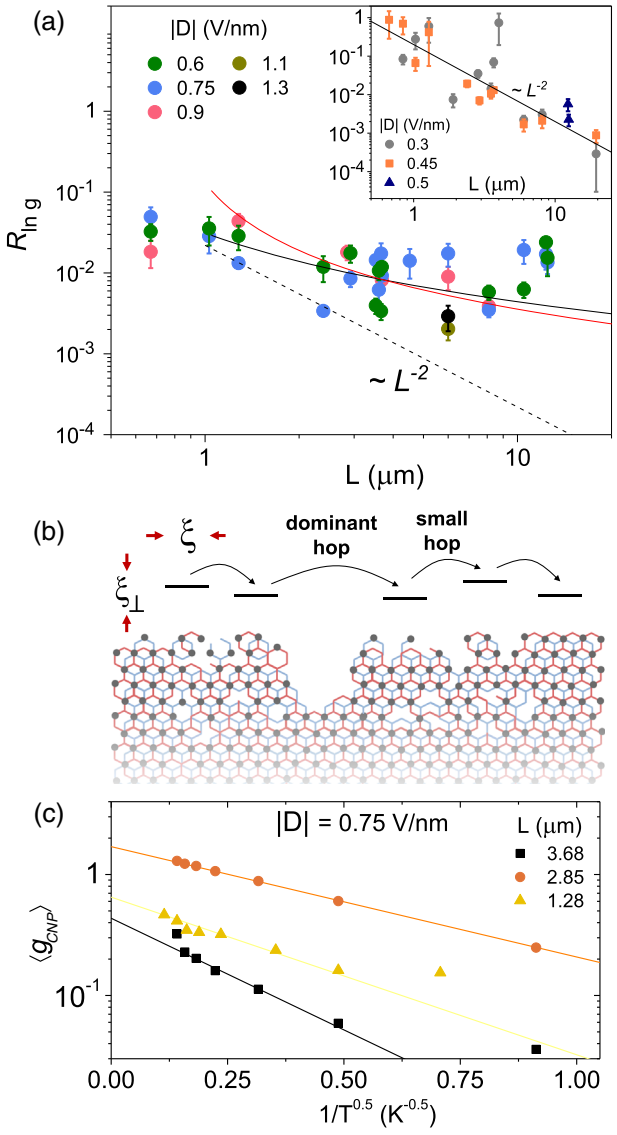

FIG. 3. Self-averaging of $\ln g_{\mathrm{CNP}}$ in gapped bilayer graphene. (a) Relative fluctuations $R_{\ln g} = \mathrm{var}(\ln g_{\mathrm{CNP}})/\langle \ln g_{\mathrm{CNP}} \rangle^2$ as a function of L for high $|D|$ (> 0.5 V/nm). The solid lines represent a logarithmic decay $R_{\ln g} \propto [\ln(2L/\xi)]^{-2}$ expected from the SKL and RR models with $\xi \approx 0.5$ μm. Inset shows $R_{\ln g}$ versus L for $|D| = 0.3$, 0.45, and 0.5 V/nm. Here, strong self-averaging is indicated by the decay in $R_{\ln g}$ as $R_{\ln g} \propto L^{-2}$ (solid line). The error bars have been computed as the net error from uncorrelated relative errors in $\langle \ln g_{\mathrm{CNP}} \rangle$ and $\mathrm{var}(\ln g_{\mathrm{CNP}})$. (b) Schematic depicting hopping transport along disordered 1D edge of the bilayer graphene with gapped bulk. (c) $\langle g_{\mathrm{CNP}} \rangle$ as a function of $T^{-0.5}$ for three devices at a fixed $|D| = 0.75$ V/nm, where the fits $\langle g \rangle \sim \exp -(T_0/T)^{1/2}$ indicate the validity of 1D hopping transport.

of $g_{\mathrm{CNP}} = g_0 \exp(-\alpha|D|)$ for $|D| \gtrsim 0.4$ V/nm [Fig. 1(c), inset] is similar to that expected in the localized edge transport [7], although the manner by which D modifies the tunneling probability of charge between two adjacent fragments needs further understanding (here, α is a device-dependent parameter).

Finally, we note an intriguing feature in the T dependence of conductance in the strongly localized phase ($|D| = 0.75$ V/nm) as shown in Fig. 3(c), where $\ln g_{\mathrm{CNP}}[\sim -(T_0/T)^{1/2}]$ extrapolates to a prefactor of the order of the conductance quantum ($\sim 0.5e^2/h$–$2e^2/h$) in three separate channels. While 1D localized channels may naturally exhibit this in the $T \to \infty$ limit [42], the universal prefactor in localized 2D electron systems has previously been attributed to electron-electron interaction-driven hopping transition [43,44]. In localized 1D systems [45], the effect of electron-electron interaction on the hopping mechanism remains poorly understood, compounded by the difficulty in distinguishing the Efros-Shklovskii mechanism [46] with $\ln g \sim -(T_0/T)^{1/2}$, from the Mott hopping law. Nonetheless, recent spectroscopy [47] and transport [48] experiments reveal strong on-site electron-electron interaction along the edges of graphene.

In summary, our experiment probes self-averaging of the logarithm of conductance in strongly localized bilayer graphene with full conductance statistics as a function of the device length and band gap. We observed a logarithmically slow marginal self-averaging, which is a strictly one-dimensional phenomenon, and may be connected to an interplay of topological states at the bilayer graphene edge and frozen disorder (edge lattice defects).

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