Disentangling Coalescing Neutron-Star–White-Dwarf Binaries for LISA

Thomas M. Tauris*

Argelander Institut für Astronomie, Auf dem Hügel 71, D-53121 Bonn, Germany,
Max-Planck-Institut für Radioastronomie, Auf dem Hügel 69, D-53121 Bonn, Germany,
and Department of Physics and Astronomy, Aarhus University, Ny Munkegade 120, 8000 Aarhus C, Denmark

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The prime candidate sources for the upcoming space-borne gravitational wave (GW) observatory LISA are the numerous Galactic tight binaries of white dwarfs (WDs) and neutron stars (NSs), many of which will coalesce and undergo mass transfer, leading to simultaneous emission of x rays and GWs. Here, detailed and coherent numerical stellar models are explored for the formation and evolution of these systems, including finite-temperature effects and complete calculations of mass transfer from a WD to a NS accretor. Evolutionary tracks of characteristic strain amplitude are computed, and the unique pattern of their evolution in the GW frequency–dynamical chirp mass parameter space enables a firm identification of the nature of the systems. Furthermore, it is demonstrated that a precise detection of the chirp allows determination of the NS mass to an accuracy of a few percent; with applications to constraining its equation of state, in particular for dual-line GW sources observed simultaneously at high and low frequencies.

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Introduction.—The recent detections of high-frequency gravitational waves (GWs) from mergers of black holes (BHs) and neutron stars (NSs) in distant galaxies [1,2] have excited the scientific community and marks the start of a new era of multimessenger astrophysics. Sources of continuous emission of low-frequency GWs, however, are numerous within the Milky Way [3]. These sources include mainly tight binaries of compact objects: white dwarfs (WDs), NSs, and BHs. As these compact objects orbit each other and produce ripples in the local space time, GWs are emitted which result in a gradual orbital decay over time. This causes a chirp of the emitted GW signal, which is an increase in frequency and amplitude, reaching a maximum when the two compact objects finally merge. A space-borne GW observatory (LISA [4]) is planned for launch in about a decade, with an aim to detect the chirp signals from such low-frequency GW sources. This opens up for the possibility to explore full multimessenger detections in both GWs and electromagnetic waves from such tight binaries in which stable mass transfer (leading to emission of x rays) is operating between the two compact objects, e.g., from a low-mass helium WD donor to a NS or WD accretor. More massive carbon-oxygen WD donors are not considered here as their mass transfer is dynamically unstable [5].

Vigorous studies are known in the literature on WD + WD evolution (e.g., Refs. [3,6–8]). However, thus far, attempts to model the chirp of the emitted GW signal are based on semianalytic modeling, with limited possibilities to resolve finite-temperature (entropy) effects of the WD and the stability of the mass-transfer process. Here, the aim is to expand beyond semianalytical results by using numerical modeling and investigate GW calculations of NS + WD systems for the first time. An advantage of applying state-of-the-art numerical calculations is that one is not restricted to applications of approximate zero-temperature mass-radius relations of the WD, which, therefore, results in more realistic ultracompact x-ray binary (UCXB) modeling [9]. This is particularly important for the low-mass helium WD donors studied here, since they can remain bloated on a Gyr timescale [10] until they settle on the WD cooling track. Finally, the ability to follow the coherent evolution of the same system through two consecutive mass-transfer stages leads to a self-consistent modeling of the WD donor.

Binary star modeling.— Using the numerical binary stellar evolution tool MESA [11], the complete evolution of NS binaries with a low-mass main sequence (MS) companion star is calculated until a double compact object is formed and beyond (see the Supplemental Material [12] for further details on the calculations). This includes two consecutive stages of mass transfer: (i) the low-mass x-ray binary (LMXB) stage [35] where the NS accretes matter from the MS donor star and (ii) the UCXB stage [5,36] where the NS accretes matter from the WD remnant of the former MS star. The computation of the UCXB stage, which had not been calculated numerically until recently [9], holds the key for tracking the observable properties of such systems in both GWs and electromagnetic waves.

The example shown in Fig. 1 is based on an initial binary with a $1.40M_{\odot}$ MS star orbiting a $1.30M_{\odot}$ NS with an orbital period of $P \simeq 3.0$ d. After orbital decay caused by magnetic braking and the subsequent LMXB phase, the system detaches with a $0.162M_{\odot}$ helium WD orbiting a $1.63M_{\odot}$ NS with an orbital period of 4.8 h. At this stage,

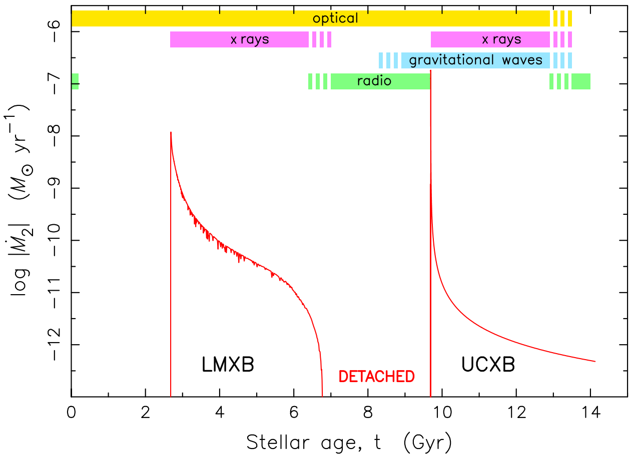

FIG. 1. Mass-transfer rate of the donor star as a function of stellar age. The initial MS star + NS binary has components of 1.40 and $1.30M_\odot$, respectively. The system evolves through two observable stages of mass transfer: a LMXB for 4 Gyr followed by a detached phase lasting about 3 Gyr where the system is detectable as a radio millisecond pulsar orbiting the helium WD remnant of the donor star until GW radiation brings the system into contact again producing an UCXB. The color bars indicate detectability in different regimes.

the system is observable as a binary radio millisecond pulsar (MSP, a recycled NS [37]). Over the next ~3 Gyr, the system spirals in further due to emission of low-frequency GWs with a constant chirp mass $\mathcal{M} = (M_{\rm NS}M_{\rm WD})^{3/5}/(M_{\rm NS}+M_{\rm WD})^{1/5} = 0.401M_\odot$, until the WD fills its Roche lobe (at $P = 24$ min and with a temperature of $T_{\rm eff} = 10580$ K) and initiates mass transfer (Roche-lobe overflow, RLO) to the NS, and the system becomes observable as an UCXB. It is anticipated that a large subpopulation of LISA sources [3,4] will indeed be such x-ray binaries, where mass is transferred from a low-mass WD to a NS. The sample system shown here is calculated until an age of ~14 Gyr, at which point the WD has become an $\sim 0.006M_\odot$ planetlike remnant orbiting a MSP—somewhat similar to a system like PSR J1719-1438 [38].

Dynamical chirp mass.—Figure 2 displays the calculated GW frequency $f_{\rm GW}$ as a function of stellar age (left panel) and the so-called *dynamical* chirp mass $\mathcal{M}_{\rm dyn}$ (right panel) before and after the onset of the UCXB stage. These tracks represent a unique fingerprint of GW frequency evolution (or a "song") for a given binary system. In the quadrupolar formalism, $f_{\rm GW}$ is simply twice the orbital frequency ($f = 1/P$), and the latter quantity $\mathcal{M}_{\rm dyn}$ depends on f and its time derivative $\dot{f}$.

For a detached binary system (i.e., without mass transfer between the two stellar components) where the only contribution to loss of orbital angular momentum is caused by GW radiation, the chirp mass is a constant quantity defined as [39]

$$\mathcal{M} \equiv \frac{(M_1 M_2)^{3/5}}{(M_1+M_2)^{1/5}}. \quad (1)$$

Given that the loss rate of orbital angular momentum caused by GW radiation for a circular binary can be expressed as [40]

$$\frac{\dot{J}_{\rm GWR}}{J_{\rm orb}} = -\frac{32G^3}{5c^5}\frac{M_1 M_2 M}{a^4}, \quad (2)$$

where $J_{\rm orb}$ is the orbital angular momentum, G is the constant of gravity, c is the speed of light in vacuum, $M = M_1 + M_2$ is the total mass of the system, and a is the orbital

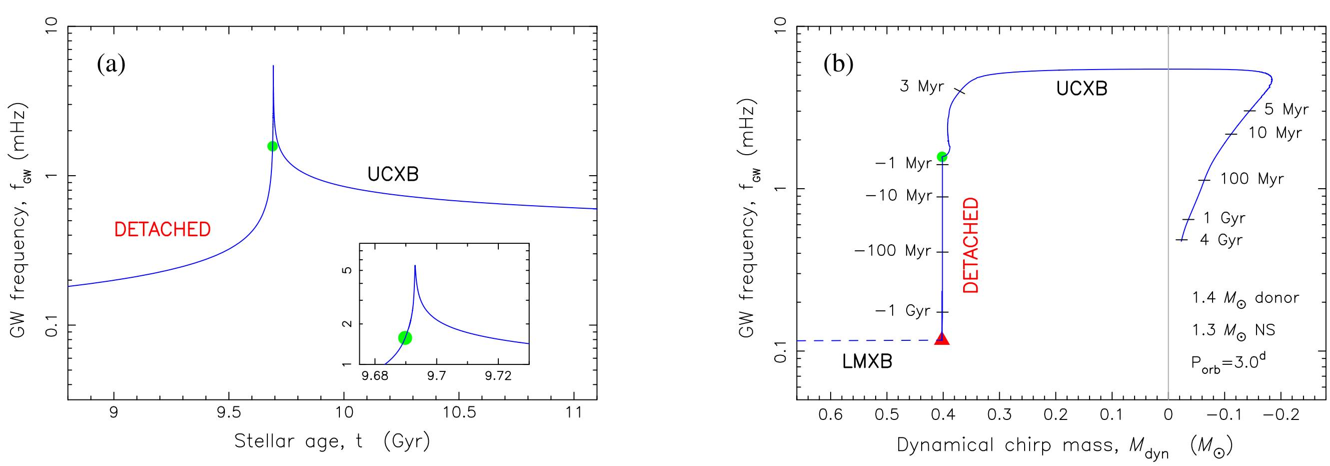

FIG. 2. Calculated GW spectrum evolution during the mass transfer from a $0.162M_\odot$ WD to a $1.63M_\odot$ NS in the UCXB of Fig. 1. (a) Emitted GW frequency vs stellar age. The inset shows an enlargement near the peak frequency. (b) Emitted GW frequency vs dynamical chirp mass. The evolution is from left to right in a clockwise direction. The ages along the evolutionary track are relative to the onset of the UCXB phase (green solid circle) at $t = 9.690$ Gyr. The dynamical chirp mass becomes negative when the orbit is widening as a result of mass transfer.

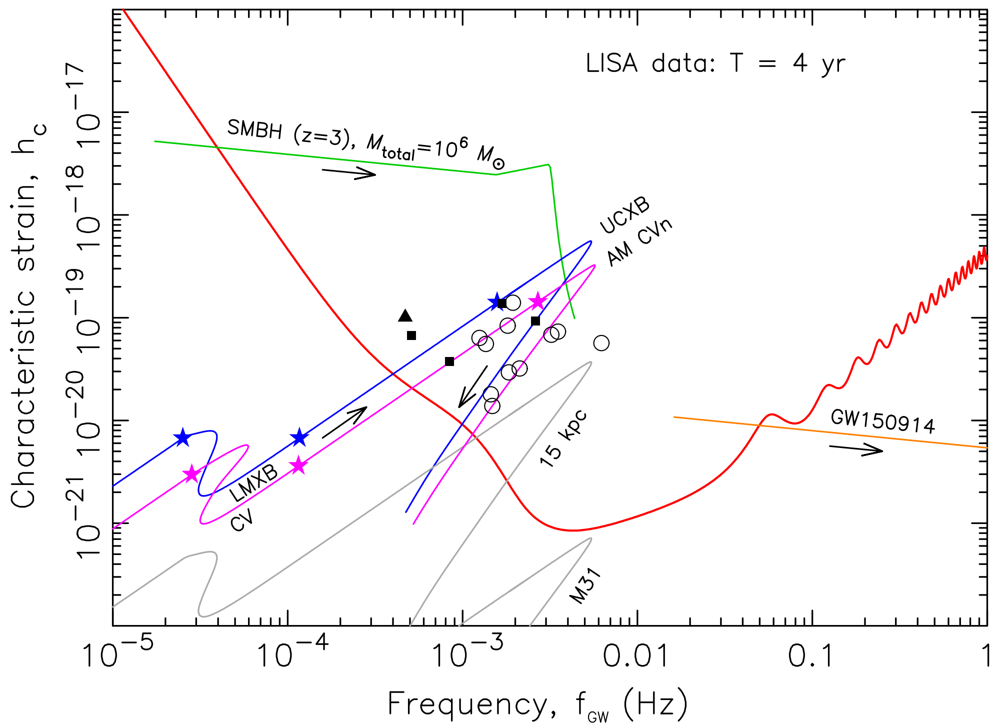

FIG. 3. Characteristic strain amplitude vs GW frequency for LISA. Evolutionary tracks for the UCXB system (blue) shown in Figs. 1 and 2, and the example AM CVn2 system (magenta; see Supplemental Material [12]) at a distance of $d_L = 1$ kpc. The stars represent (with increasing f_{GW}) onset LMXB/CV stage, termination LMXB/CV stage, and onset UCXB/AM CVn stage. The LISA sensitivity curve [41] (red line) is based on four years of observations. The grey curves are for the UCXB at $d_L = 15$ kpc and $d_L = 780$ kpc (M31), respectively. Comparison tracks are shown for a SMBH merger (green) and GW150914 (orange). Data from LISA verification sources [42] include detached double WD binaries (solid squares), AM CVn systems (open circles), and a hot subdwarf binary (solid triangle).

separation between the stellar components, one can combine the above expression with Kepler's third law ($4\pi^2 f^2 = GM/a^3$, where $f = f_{GW}/2$ is the orbital frequency) and easily derive

$$\mathcal{M} = \frac{c^3}{G}\left(\frac{5}{96}\pi^{-8/3} f_{GW}^{-11/3} \dot{f}_{GW}\right)^{3/5}. \qquad (3)$$

In an x-ray binary system, however, the exchange of mass between the stellar components and mass lost from the system (as well as other effects giving rise to loss of J_{orb}) affect the orbital period evolution, and, hence, $\dot{f}_{GW}$ cannot be evaluated using Eq. (2). In particular, for binaries where RLO results in a widening of the binary system, one has $\dot{f} < 0$, which means that there is no real number solution to Eq. (3). Instead, I define the *dynamical* chirp mass $\mathcal{M}_{dyn}$ as

$$\mathcal{M}_{dyn} \equiv \begin{cases} \frac{c^3}{G}\left(\frac{5}{96}\pi^{-8/3}(2f)^{-11/3} 2\dot{f}\right)^{3/5} & \text{for } \dot{f} \geq 0, \\ -\frac{c^3}{G}\left(\frac{5}{96}\pi^{-8/3}(2f)^{-11/3} |2\dot{f}|\right)^{3/5} & \text{for } \dot{f} < 0, \end{cases} \qquad (4)$$

which will be negative for expanding orbits. That is, the orbital frequency and, hence, the GW frequency, from expanding orbits will decrease and give rise to a negative chirp [8].

The reason for the change in the sign of orbital frequency (i.e., switching from a decreasing to an increasing orbital period) and the shape of computed UCXBs tracks (Fig. 2) can be understood from the ongoing competition between GW radiation and orbital expansion caused by mass transfer and loss [9]. The peak at $f_{GW} \simeq 5.5$ mHz (corresponding to the minimum orbital period of $P_{orb} \simeq 6.1$ min) coincides with the maximum value of the mass-transfer rate $|\dot{M}_2| = 10^{-6.8}\ M_\odot \mathrm{yr}^{-1}$. As the onset of RLO in the UCXB phase leads to a very high mass-transfer rate (Fig. 1), an outward acceleration of the orbital size results from the small mass ratio between the two stars ($q \sim 0.1$), such that at some point the rate of orbital expansion dominates over that of orbital shrinking due to GW radiation.

An analogy to the numerical computations of the described UCXB model can be made to RLO in double WD systems (see Figs. S2–S5 in the Supplemental Material [12]), i.e., the so-called AM Canum Venaticorumn (CVn) binaries [6,7] which constitute a main population of LISA sources [3].

LISA observations.—Figure 3 shows the characteristic GW strain amplitude calculated from the above evolutionary tracks for sources located at different distances with respect to the Solar System. The LISA sensitivity curve [41] based on four years of observations is plotted for comparison (see Supplemental Material [12]). The resulting signal-to-noise ratio (SNR) is above 100 out to distances of about 1 kpc, and for such a source located in the Andromeda galaxy (M31) at a distance of 780 kpc, the peak characteristic strain is almost detectable. A comparison track calculated with MESA for a double WD system (AM CVn) with component masses of 0.160 and $0.706 M_\odot$ is included (see Supplemental Material [12]), as well as computed tracks for a supermassive BH (SMBH) merger with a total mass of $10^6 M_\odot$ at a redshift of $z = 3$ (green line) and the last four years of in-spiral for the first LIGO event [1], the double BH binary GW150914 (orange line), as it would appear in the LISA frequency band.

Derivations of individual component masses are not possible from LISA measurements alone, since higher order relativistic terms [43] to the quadrupole formula are needed (but not measurable) to break the degeneracy in component masses obtained from the observed chirp mass. To first post-Newtonian order (see Sec. 5.2.2 in Ref. [43]), the correction to the measured binary phase scales with $(v/c)^2$, where (v/c) is the ratio between the relative orbital velocity and the speed of light. As an example, consider the UCXB model shown in Fig. 2. Near the onset of the mass transfer from the WD to the NS (at which point the subsequent orbital evolution at latest deviates from pure GW radiation, here neglecting tidal effects), the GW frequency is $f_{\mathrm{GW}} = 1.63$ mHz and $v \simeq 1068\ \mathrm{km\,s^{-1}}$ and, thus, $(v/c)^2 \simeq 1.3 \times 10^{-5}$. Such a small deviation in phase is not measurable with LISA.

A new method to determine NS masses.—A tight correlation, however, exists between the orbital period and the mass of a helium WD which is produced in a LMXB system [44,45]. This correlation has been confirmed both observationally [46,47] and using the latest detailed binary stellar models including diffusion processes and rotational mixing [48]. Since only post-LMXB NS + WD binaries with orbital periods less than about 9 h are able to coalesce within a Hubble time (and thereby becoming visible LISA sources), the masses of all these WDs turn out to be the same within a narrow range ($M_{\mathrm{WD}} = 0.162 \pm 0.005 M_\odot$; see Supplemental Material [12]). This fortunate circumstance enables an accurate determination of NS masses within ~4% (Fig. 4), provided precise measurements of chirp masses in pre-UCXB systems. For the best cases, it is found that the uncertainty of the measured $\mathcal{M}$ will be 0.5%–1% (see Supplemental Material [12]). The resulting precise NS mass determinations may then yield a new upper mass limit of a NS accretor [49] which helps to constrain the long-sought-after equation of state of NS matter [50]. A similar approach can be applied to infer the mass of the first-formed WD in a double WD system, which originates from stable RLO in a CV system.

A caveat is that LISA will only be able to measure $\dot{f}_{\mathrm{GW}}$ for nearby GW binaries with a very large SNR and which are close to their minimum orbital period where the rate of change in frequency is largest (see Fig. S1 in the Supplemental Material [12]). However, this is where multi-messenger astronomy [52] combining GWs and electromagnetic radiation is beneficial (Fig. 1), including distance measurements of nearby sources using GAIA which can be combined with GW strain amplitude measurements to constrain $\dot{f}_{\mathrm{GW}}$ [53]. Optical observations of the WD [54] and searches for radio pulses from the NS can identify the nature of the LISA sources and help infer the chirp mass by measuring $\dot{f}$. Similar targeted searches for radio pulsations from NSs in Fermi detected γ-ray sources have proven quite successful [55]. Combined radio and optical observations of binary pulsars and WDs in close-orbit LISA (progenitor) sources will enable further tests of, e.g., WD formation, tidal effects, and general relativity [49].

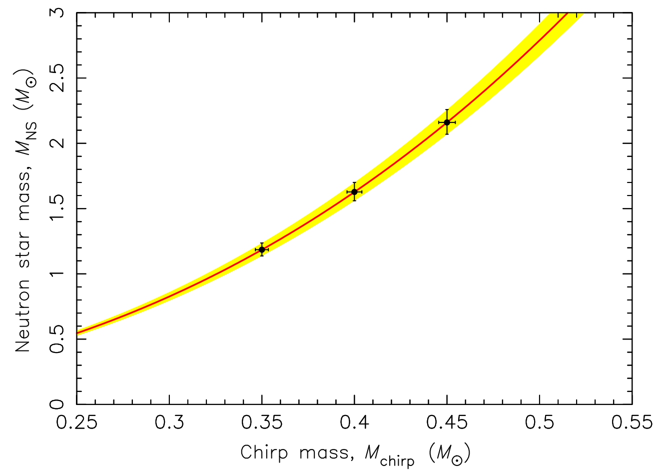

FIG. 4. NS mass vs measured chirp mass. For NS + WD binaries, there is a unique correlation between the orbital period and helium WD mass [44,45] after the LMXB stage, and all such LISA (progenitor) systems have a well-defined WD mass of $0.162 \pm 0.005 M_\odot$ before the two compact objects coalesce and initiate mass transfer. Therefore, given a precisely measured chirp mass in such a binary (three examples indicated with an uncertainty of 1%; see Supplemental Material [12]), it is possible to derive the NS mass to an accuracy of about 4%. The plotted relation is also valid for double WD systems (pre-AM CVn systems), which evolved from a stable CV system [51].

It is anticipated that measuring $\dot{f}_{\mathrm{GW}}$ is possible in about 25% of the thousands of resolved LISA sources expected to be found [56]. The precision of the measured values of f_{GW} and $\dot{f}_{\mathrm{GW}}$ increases over time, and a few sources may even have high enough SNR that allow for a measurement of $\ddot{f}_{\mathrm{GW}}$ [56]. Although $\dot{f}_{\mathrm{GW}} \simeq 0$ for sources *very* near to their minimum orbital period (which prevents a measurement of $\mathcal{M}_{\mathrm{dyn}}$), this epoch is short lasting [see Fig. 2(b) and Fig. S1 in the Supplemental Material [12]] and will thus only affect a few systems. For UCXBs and AM CVns, secular effects from tidal and mass-transfer interactions may introduce short-term variations in the measured values, but these effects will most likely not prevent detection of $\dot{f}_{\mathrm{GW}}$ [8,57] and thereby $\mathcal{M}_{\mathrm{dyn}}$.

A remaining issue for refining the solutions presented here is calculating the exact evolution of $\dot{f}_{\rm GW}$ related to the torque balance arising from angular momentum advected from the donor to the accretion disk along with the transferred matter and the return of angular momentum from the disk to the orbit by means of a tidal torque between the outer disk and the donor [5,58,59]. Whereas detailed modeling of the accretion disk is left for future studies, I performed trial computations with MESA including tidal effects, diffusion, and rotational mixing of the WD, following Istrate *et al.* [48]. The resulting change in entropy (inflated WD envelope) is found to be very limited (at the level of a few percent). Furthermore, the cooling properties of the WD also change when including diffusion and rotational mixing [48], and at the onset of the UCXB stage, $f_{\rm GW}$ is slightly *larger* (2.06 vs 1.63 mHz).

Number of Galactic NS + WD *LISA sources.*—The number of UCXBs (and detached NS + WD systems prior to the UCXB phase) that LISA will detect is expected to be significantly smaller than the number of AM CVn and detached double WD systems [3]. Simple estimates based on known numbers of binary radio MSPs (see Supplemental Material [12]) nevertheless reveal an expected LISA population of at least a hundred sources with NSs in the Milky Way.

Dual-line gravitational wave system.—With capabilities to calculate through two phases of mass transfer, the LMXB and the UCXB phases, it is possible to develop better models to follow the evolution of the accreting NSs and make improved theoretical predictions for their distribution of spin rates—with applications to potential LIGO and Virgo detections of continuous high-frequency GWs from rapidly spinning NSs [60,61]. With a bit of luck, a Galactic dual-line GW frequency system can be detected from a combination of NS spin and orbital motion. The LIGO and Virgo detectors may detect a high-frequency GW signal from a rapidly spinning NS (note, recycled MSPs reside in these binaries) with some ellipticity ε and a resulting strain amplitude [60,61],

$$h_{\rm spin} = \frac{4\pi^2 G I_{zz} f_{\rm GW}^2 \varepsilon}{c^4 d_L}, \tag{5}$$

where I_{zz} is the principal moment of inertia, and d_L is the luminosity distance. LISA may then measure the low-frequency GW signal arising from the orbital motion $h_{\rm orb}$ with a strain amplitude given by [62]

$$h_{\rm orb} = \left(\frac{32}{5}\right)^{1/2} \frac{\pi^{2/3} G^{5/3} f_{\rm GW}^{2/3} \mathcal{M}^{5/3}}{c^4 d_L} \tag{6}$$

generated by a binary for an average orbital orientation and polarization. Combining these two expressions yields

$$I_{zz}\varepsilon = \left(\frac{32}{80}\right)^{1/2} \pi^{-4/3} G^{2/3} f_{\rm GW}^{-4/3} \mathcal{M}^{5/3} \left(\frac{h_{\rm spin}}{h_{\rm orb}}\right). \tag{7}$$

Once the right-hand side of this equation is determined observationally, and assuming that the NS mass $M_{\rm NS}$ can be determined from the chirp mass $\mathcal{M}$ (under the assumption of $M_{\rm WD} = 0.162 \pm 0.005 M_\odot$), constraints can be made on the NS moment of inertia and, thus, the NS radius [63] (although only in combination with the ellipticity ε), and thereby help pin down the long-sought-after equation of state of NS matter.

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*tauris@phys.au.dk

[1] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari *et al.*, Phys. Rev. Lett. **116**, 061102 (2016).
[2] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya *et al.*, Phys. Rev. Lett. **119**, 161101 (2017).
[3] G. Nelemans, L. R. Yungelson, and S. F. P. Zwart, Astron. Astrophys. **375**, 890 (2001).
[4] P. Amaro-Seoane, H. Audley, S. Babak, J. Baker, E. Barausse *et al.*, arXiv:1702.00786.
[5] L. M. van Haaften, G. Nelemans, R. Voss, M. A. Wood, and J. Kuijpers, Astron. Astrophys. **537**, A104 (2012).
[6] D. Hils and P. L. Bender, Astrophys. J. **537**, 334 (2000).
[7] D. L. Kaplan, L. Bildsten, and J. D. R. Steinfadt, Astrophys. J. **758**, 64 (2012).
[8] K. Kremer, K. Breivik, S. L. Larson, and V. Kalogera, Astrophys. J. **846**, 95 (2017).
[9] R. Sengar, T. M. Tauris, N. Langer, and A. G. Istrate, Mon. Not. R. Astron. Soc. **470**, L6 (2017).
[10] A. G. Istrate, T. M. Tauris, N. Langer, and J. Antoniadis, Astron. Astrophys. **571**, L3 (2014).
[11] B. Paxton, P. Marchant, J. Schwab, E. B. Bauer, L. Bildsten, M. Cantiello, L. Dessart, R. Farmer, H. Hu, N. Langer, R. H. D. Townsend, D. M. Townsley, and F. X. Timmes, Astrophys. J. Suppl. Ser. **220**, 15 (2015).
[12] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.121.131105 for further calculation details, comments, and Figs. S1–S5, which includes Refs. [13–34].
[13] A. G. Istrate, T. M. Tauris, and N. Langer, Astron. Astrophys. **571**, A45 (2014).
[14] T. M. Tauris and E. P. J. van den Heuvel, Formation and evolution of compact stellar x-ray sources," in *Compact Stellar X-Ray Sources* (Cambridge University Press, Cambridge, England, 2006), Chap. 16, pp. 623–665.

[15] O. G. Benvenuto, M. A. De Vito, and J. E. Horvath, Astrophys. J. **753**, L33 (2012).
[16] J. Antoniadis, M. H. van Kerkwijk, D. Koester, P. C. C. Freire, N. Wex, T. M. Tauris, M. Kramer, and C. G. Bassa, Mon. Not. R. Astron. Soc. **423**, 3316 (2012).
[17] G. Nelemans, S. F. P. Zwart, F. Verbunt, and L. R. Yungelson, Astron. Astrophys. **368**, 939 (2001).
[18] C. J. Deloye, L. Bildsten, and G. Nelemans, Astrophys. J. **624**, 934 (2005).
[19] C. J. Deloye, R. E. Taam, C. Winisdoerffer, and G. Chabrier, Mon. Not. R. Astron. Soc. **381**, 525 (2007).
[20] L. Bildsten, D. M. Townsley, C. J. Deloye, and G. Nelemans, Astrophys. J. **640**, 466 (2006).
[21] M. A. Ruderman and J. Shaham, Astrophys. J. **289**, 244 (1985).
[22] R. G. Martin, M. Livio, and D. Palaniswamy, Astrophys. J. **832**, 122 (2016).
[23] M. Sun and P. Arras, Astrophys. J. **858**, 14 (2018).
[24] J. J. Hermes, M. H. Montgomery, A. Gianninas, D. E. Winget, W. R. Brown, S. T. Harrold, K. J. Bell, S. J. Kenyon, M. Kilic, and B. G. Castanheira, Mon. Not. R. Astron. Soc. **436**, 3573 (2013).
[25] D. L. Kaplan, T. R. Marsh, A. N. Walker, L. Bildsten, M. C. P. Bours, E. Breedt, C. M. Copperwheat, V. S. Dhillon, S. B. Howell, S. P. Littlefair, A. Shporer, and J. D. R. Steinfadt, Astrophys. J. **780**, 167 (2014).
[26] G. Nelemans, L. R. Yungelson, S. F. P. Zwart, and F. Verbunt, Astron. Astrophys. **365**, 491 (2001).
[27] S. Toonen, J. S. W. Claeys, N. Mennekens, and A. J. Ruiter, Astron. Astrophys. **562**, A14 (2014).
[28] R. Takahashi and N. Seto, Astrophys. J. **575**, 1030 (2002).
[29] S. Shah and G. Nelemans, Astrophys. J. **791**, 76 (2014).
[30] L. S. Finn and K. S. Thorne, Phys. Rev. D **62**, 124021 (2000).
[31] P. Ajith, S. Babak, Y. Chen, M. Hewitson, B. Krishnan, J. T. Whelan, B. Brügmann, P. Diener, J. Gonzalez, M. Hannam, S. Husa, M. Koppitz, D. Pollney, L. Rezzolla, L. Santamaría, A. M. Sintes, U. Sperhake, and J. Thornburg, Classical Quantum Gravity **24**, S689 (2007).
[32] R. N. Manchester, G. B. Hobbs, A. Teoh, and M. Hobbs, Astron. J. **129**, 1993 (2005).
[33] D. R. Lorimer and M. Kramer, *Handbook of Pulsar Astronomy*, by D. R. Lorimer and M. Kramer, (Cambridge University Press, Cambridge, UK 2012).
[34] K. Kremer, S. Chatterjee, K. Breivik, C. L. Rodriguez, S. L. Larson, and F. A. Rasio, Phys. Rev. Lett. **120**, 191103 (2018).
[35] Q. Z. Liu, J. van Paradijs, and E. P. J. van den Heuvel, Astron. Astrophys. **469**, 807 (2007).
[36] C. O. Heinke, N. Ivanova, M. C. Engel, K. Pavlovskii, G. R. Sivakoff, T. F. Cartwright, and J. C. Gladstone, Astrophys. J. **768**, 184 (2013).
[37] D. Bhattacharya and E. P. J. van den Heuvel, Phys. Rep. **203**, 1 (1991).
[38] M. Bailes, S. D. Bates, V. Bhalerao, N. D. R. Bhat, M. Burgay, S. Burke-Spolaor, N. D'Amico, S. Johnston, M. J. Keith, M. Kramer, S. R. Kulkarni, L. Levin, A. G. Lyne, S. Milia, A. Possenti, L. Spitler, B. Stappers, and W. van Straten, Science **333**, 1717 (2011).
[39] M. Maggiore, *Gravitational Waves: Volume 1: Theory and Experiments* (Oxford University Press, New York, 2008).
[40] P. C. Peters, Phys. Rev. **136**, B1224 (1964).
[41] N. Cornish and T. Robson, arXiv:1803.01944.
[42] T. Kupfer, V. Korol, S. Shah, G. Nelemans, T. R. Marsh, G. Ramsay, P. J. Groot, D. T. H. Steeghs, and E. M. Rossi, Mon. Not. R. Astron. Soc. **480**, 302 (2018).
[43] S. A. Hughes, Annu. Rev. Astron. Astrophys. **47**, 107 (2009).
[44] G. J. Savonije, Nature (London) **325**, 416 (1987).
[45] T. M. Tauris and G. J. Savonije, Astron. Astrophys. **350**, 928 (1999).
[46] M. H. van Kerkwijk, C. G. Bassa, B. A. Jacoby, and P. G. Jonker, in *Binary Radio Pulsars*, Astronomical Society of the Pacific Conference Series edited by F. A. Rasio and I. H. Stairs (Astronomical Society of the Pacific, San Francisco, 2005), Vol. 328, p. 357.
[47] T. M. Tauris and E. P. J. van den Heuvel, Astrophys. J. **781**, L13 (2014).
[48] A. G. Istrate, P. Marchant, T. M. Tauris, N. Langer, R. J. Stancliffe, and L. Grassitelli, Astron. Astrophys. **595**, A35 (2016).
[49] J. Antoniadis, P. C. C. Freire, N. Wex, T. M. Tauris, R. S. Lynch, M. H. van Kerkwijk, M. Kramer *et al.*, Science **340**, 1233232 (2013).
[50] F. Özel and P. Freire, Annu. Rev. Astron. Astrophys. **54**, 401 (2016).
[51] P. C. Joss, S. Rappaport, and W. Lewis, Astrophys. J. **319**, 180 (1987).
[52] T. B. Littenberg, S. L. Larson, G. Nelemans, and N. J. Cornish, Mon. Not. R. Astron. Soc. **429**, 2361 (2013).
[53] K. Breivik, K. Kremer, M. Bueno, S. L. Larson, S. Coughlin, and V. Kalogera, Astrophys. J. **854**, L1 (2018).
[54] J. J. Hermes, M. Kilic, W. R. Brown, D. E. Winget, C. A. Prieto, A. Gianninas, A. S. Mukadam, A. Cabrera-Lavers, and S. J. Kenyon, Astrophys. J. **757**, L21 (2012).
[55] A. A. Abdo, M. Ackermann, M. Ajello, B. Anderson, W. B. Atwood *et al.*, Science **325**, 840 (2009).
[56] P. Amaro-Seoane *et al.*, Classical Quantum Gravity **29**, 124016 (2012).
[57] A. Stroeer and G. Nelemans, Mon. Not. R. Astron. Soc. **400**, L24 (2009).
[58] D. N. C. Lin and J. Papaloizou, Mon. Not. R. Astron. Soc. **186**, 799 (1979).
[59] T. R. Marsh, G. Nelemans, and D. Steeghs, Mon. Not. R. Astron. Soc. **350**, 113 (2004).
[60] B. Haskell, M. Priymak, A. Patruno, M. Oppenoorth, A. Melatos, and P. D. Lasky, Mon. Not. R. Astron. Soc. **450**, 2393 (2015).
[61] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya *et al.* (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. D **97**, 102003 (2018).
[62] C. R. Evans, I. Iben, Jr., and L. Smarr, Astrophys. J. **323**, 129 (1987).
[63] D. G. Ravenhall and C. J. Pethick, Astrophys. J. **424**, 846 (1994).