Stability of Periodically Driven Topological Phases against Disorder

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In recent experiments, time-dependent periodic fields are used to create exotic topological phases of matter with potential applications ranging from quantum transport to quantum computing. These nonequilibrium states, at high driving frequencies, exhibit the quintessential robustness against local disorder similar to equilibrium topological phases. However, proving the existence of such topological phases in a general setting is an open problem. We propose a universal effective theory that leverages on modern free probability theory and ideas in random matrices to analytically predict the existence of the topological phase for finite driving frequencies and across a range of disorder. We find that, depending on the strength of disorder, such systems may be topological or trivial and that there is a transition between the two. In particular, the theory predicts the critical point for the transition between the two phases and provides the critical exponents. We corroborate our results by comparing them to exact diagonalizations for driven-disordered 1D Kitaev chain and 2D Bernevig-Hughes-Zhang models and find excellent agreement. This Letter may guide the experimental efforts for exploring topological phases.

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The dynamics of nonequilibrium quantum systems has been a subject of active and recent study with experiments involving several dozens of qubits [1,2]. A promising technique for creating nonconventional states of matter is by the application of a time-periodic field (e.g., to interacting cold atoms). These nonequilibrium states of matter are frequently referred to as *Floquet* phases [3,4]. Their propositions and realizations include Floquet topological insulators [5–10], anomalous Floquet-Anderson insulators [11–13], discrete time crystals [14,15], etc. Remarkably, the controlled periodic driving helps create Majorana modes with non-Abelian braiding statistics potentially useful in topological quantum computation [16–18].

Local disorder is inevitable in realizing such nonequilibrium phases. Yet engineered systems can utilize artificial disorder as a tool for control [12,15]. For example, disorder leads to many-body localization [19,20], preventing uncontrolled heating [14,21] and stabilizing topological phases of matter [22–25]. Disorder is also responsible for phase transitions [26–30]. Even though topological phases in equilibrium are universally robust against disorder, their Floquet counterparts may not be. In low-dimensional systems, the stability is typically granted by the Anderson localization preserving the bulk mobility gap, even if the disorder closes the bulk spectral gap [31,32]. The same mechanism protects Floquet topological phases at high frequencies [33]. However, if the driving frequency is finite, Anderson localization may break down depending on the driving amplitude and disorder strength [34–37]. In this regime, nothing can preserve the topological phase if the bulk spectral gap is closed by disorder.

Despite the numerical frontiers [26,27,33], it is very difficult to quantify disordered Floquet systems in general. Even though in the limits of high driving frequency and weak disorder one can use techniques such as perturbation theory, many current realizations operate outside these limits [14,16,17]. This raises the following questions: Are Floquet topological phases preserved under finite frequency and strong disorder? And if there is a disorder-induced transition into a trivial phase, can one quantify the critical point in the thermodynamic limit?

In this Letter, we leverage on modern free probability theory and ideas inspired by random matrices to answer

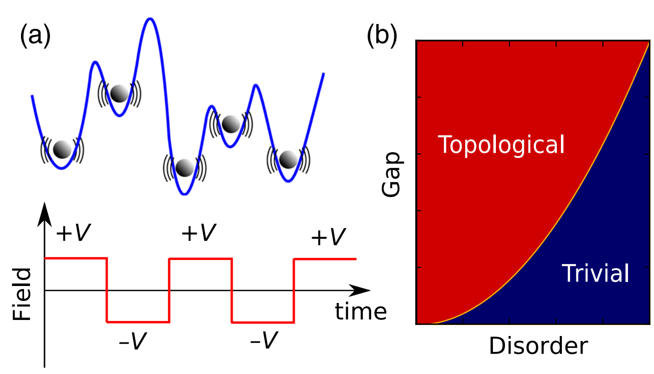

FIG. 1. Schematics. (a) An isolated disordered quantum system represented by trapped cold atoms. The time-periodic field $V(t)$ induces a transition from a trivial to a topological phase. (b) The phase diagram for the system in the presence of local disorder. An increase of the disorder strength W induces a phase transition at $W_c \sim \Delta_0^{1/2}$, where Δ_0 is the gap of the clean system.

these questions. The local disorder in the Hamiltonian introduces a correction to the Floquet Hamiltonian [Eq. (2)]. At finite driving frequencies, this correction is the sum of (potentially infinitely) many noncommuting terms in the Magnus expansion. Due to its nonlocality and randomness, we find that this correction has level statistics very similar to the Gaussian orthogonal (GOE) or unitary (GUE) ensembles depending on the problem [Figs. 2(a) and 2(b)]. We propose an effective model for the disordered Floquet Hamiltonian, in which the correction is replaced by a single generic random matrix proportional to the strength of disorder [Eq. (4)]. We use free probability theory to analytically demonstrate that the effective Floquet Hamiltonian does indeed exhibit a topological phase at a finite strength of disorder and finite driving frequency. We also find a critical strength of disorder beyond which the spectral gap closes. Consequently, a transition is induced from a topological into a trivial metallic phase. The resulting phase diagram is shown in Fig. 1(b). We compare the universal analytical results against exact diagonalization for the disordered Kitaev chain and the 2D Bernevig-Hughes-Zhang (BHZ) model (see Fig. 3).

Consider the problem of noninteracting particles on a lattice. It is useful to divide the Hamiltonian into three parts: the translationally invariant static part H_0, the static local disorder δV, and the applied external time-periodic driving field $V(t)$ [Fig. 1(a)]. Therefore,

$$H(t) = H_0 + \delta V + V(t), \qquad V(t) = V(t+\tau), \tag{1}$$

where τ is the driving period. Since topological phases in the integer quantum Hall universality class are often understood in terms of free particles, we leave the effects of many-body interactions for future work.

Let us first focus on the clean system. By the Floquet-Bloch theorem, the total time evolution at discrete times $t = n\tau$ is given by the unitary operator $U_n = (U_F)^n$, where $U_F \equiv \exp(-i\tau H_F) = \mathcal{T}\exp(-i\int_0^\tau dt' H(t'))$, $\mathcal{T}$ denotes chronological time ordering, and H_F is the Floquet Hamiltonian. The Hamiltonian H_F defines a new quantum (Floquet) phase [22]. Depending on the field $V(t)$, this phase can be equivalent to the initial phase of H_0, or be different. Here we focus on the latter case where the field $V(t)$ is designed to convert a trivial into a topological phase [3,38].

Next, we look at the role of disorder, δV, which may be represented by a diagonal random matrix. Periodic driving $V(t)$ and δV dress the bare Floquet Hamiltonian into a disordered Floquet Hamiltonian H'_F defined by

$$H'_F = H_F + \delta V_F, \tag{2}$$

where $\delta V_F \equiv \sum_{\ell \geq 1} \delta V_\ell \tau^{\ell-1}$ with the coefficients

$$\delta V_1 = \delta V, \qquad \delta V_\ell = \frac{1}{\tau^\ell}[K_\ell\{H(t) + \delta V\} - K_\ell\{H(t)\}], \tag{3}$$

denoting by $\{.\}$ a functional. K_ℓ is the ℓth term in the Magnus expansion (Ref. [3] and the Supplemental Material [39]). In contrast to the random on-site potential δV, in principle, each δV_ℓ contributes nonzero off-diagonal entries to the matrix δV_F, making the effective disorder nonlocal.

If the high driving frequency limit $\Omega = 2\pi/\tau \to \infty$, the higher-order corrections can be neglected. As a result, δV_F acts similarly to the local disorder δV, leading to the localization of eigenstates. In this situation, $H_F + \delta V_F$ always describes the topological phase as soon as a mobility gap is present in the system.

On the other hand, if Ω is finite, the higher-order terms in Eq. (3) cannot be ignored, as τ may exceed the radius of convergence of Eq. (3). Consequently, the off-diagonal entries in δV_F are not negligible. Physically, this corresponds to emergence of driving-induced Landau-Zener transitions between localized states responsible for the breakdown of Anderson localization. In this regime, if the spectral gap closes, the Floquet topological phase is breaking with following disorder-induced transition to a trivial phase.

In general, obtaining the exact spectral properties of Eq. (2) analytically is formidably difficult, mainly limited by the noncommutativity. Further numerical simulations are limited for large system sizes. However, there are many nondiagonal corrections appearing in Eq. (3); the disorder δV added to $H(t)$ smears all over the effective Floquet Hamiltonian [i.e., δV_F in Eq. (2)]. It then seems plausible to assume that the resulting δV_F should mimic a generic Hermitian random matrix. Indeed, in Figs. 2(a) and 2(b), we show the accuracy by which the level statistics of δV_F are represented by the standard Gaussian ensembles. We will return to this below.

Therefore, the *effective Hamiltonian* we propose is

$$H_F^{\text{eff}} = H_F + \lambda M, \tag{4}$$

where the matrix M is chosen from the Gaussian ensemble with eigenvalues in $[-2, 2]$, which in the limit of infinite size would follow the semicircle law [40], and $\lambda = \sqrt{\varphi(\delta V_F^2)}$, with $\varphi(A) = \mathbb{E}\mathrm{Tr}(A)/\dim(A)$ denoting the empirical mean of the matrix A. Physically, H_F^{eff} describes a competition between the topological phase ($\lambda < \lambda_c$) and a featureless chaotic phase ($\lambda > \lambda_c$), where λ_c is a critical point. This model describes the transitions in a finite range of disorder strength and may not retain the precision in the limit of high disorder $W \gg \Omega$ in which the target Floquet Hamiltonian is expected to exhibit Poissonian quasienergy level statistics.

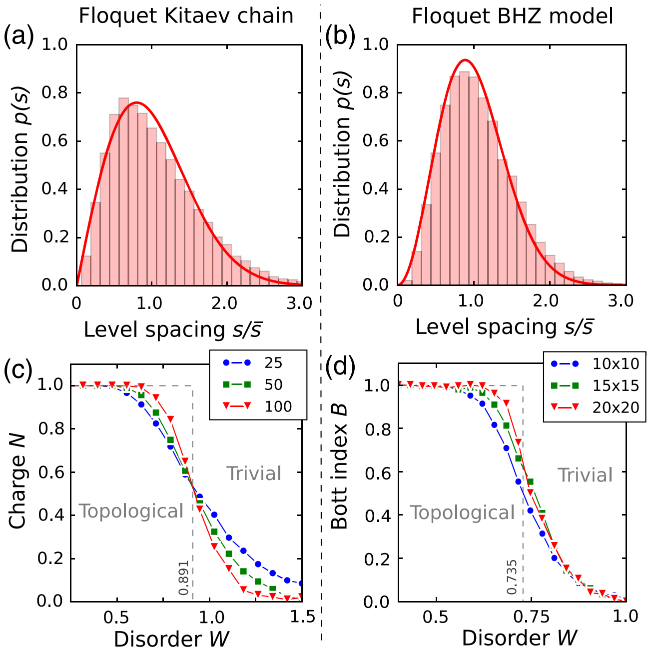

FIG. 2. Effect of disorder. (a),(b) Level spacing distribution for the middle of the spectrum of δV_F for the Floquet Kitaev chain and Floquet BHZ models, respectively, for $W = 0.5$. Red curves are the level spacing distribution for GOE and GUE, respectively. (c),(d) The topological charge and Bott index as a function of the disorder for the Kitaev chain and BHZ models, respectively. The dashed step function is the expected behavior in the thermodynamic limit. The parameters of the models are as in Fig. 3.

The value of the parameter λ depends on both disorder strength W and driving period τ (see the Supplemental Material for details [39]). In the weak disorder limit, $\lambda \approx \alpha(\tau)W/\sqrt{3}$, where $\alpha(\tau)$ is a dimensionless parameter. At high driving frequencies, $\alpha(\tau) = 1 + O(\tau^2)$. This approximation is valid if the period of driving is less than the radius of convergence of Eq. (3) [41]. At low frequencies, $\lim_{\tau\to\infty}\alpha(\tau) = \alpha_0$, where α_0 is a constant that depends on the model. In the strong disorder limit, assuming that the eigenvalues of $\delta V_F \tau$ are evenly distributed in the interval $[-\pi, \pi]$, we get $\lambda\tau \approx \pi/\sqrt{3}$. The value of λ plays the role of a phenomenological parameter in the model.

To this end, and before presenting the analytical machinery, we demonstrate our results in the context of two widely studied models, the Kitaev chain and the 2D Bernevig-Hughes-Zhang (BHZ) model. For numerical simulations, both models can be represented as particular cases of a fermion hopping on a lattice:

$$H_0 = \sum_{\mathbf{r},\mathbf{a}\in A} (\Gamma_{\mathbf{a}}|\mathbf{r}\rangle\langle\mathbf{r}+\mathbf{a}| + \mathrm{H.c.}) + \mu\Gamma_0, \qquad (5)$$

where $A = \{\mathbf{a}\}$ is the set of primitive vectors on the lattice, $\Gamma_{\mathbf{a}}$ and Γ_0 are Hermitian matrices, and μ is the chemical potential. We choose the driving field and disorder to be

$$V(t) = F\theta(t), \qquad \delta V = \Gamma_0 \sum_{\mathbf{r}} h_{\mathbf{r}}|\mathbf{r}\rangle\langle\mathbf{r}|, \qquad (6)$$

where $\theta(t) = \mathrm{sgn}(\sin\Omega t)$, and $h_{\mathbf{r}}$ is uniformly random in $[-W, W]$.

As the first example, we take the Bogoliubov–de Gennes Hamiltonian for the Kitaev chain [43], which has the form of Eq. (5) with $\Gamma_x = i\Delta\sigma_y + J\sigma_z$, $\Gamma_0 = \sigma_z$, and $F = f\sigma_z$. The σ_i's are the Pauli matrices, Δ is the superconducting pairing, $J > 0$ is the hopping constant, and f is the amplitude of the external driving. In the absence of driving, the clean system is an archetypal example of a 1D topological superconductor, exhibiting a topological phase transition at $|\mu| = 2J$. When $|\mu| < 2J$, the system is in the topological phase hosting two Majorana zero-energy modes at each end of the chain, and is in the trivial phase otherwise. However, recent proposals [16,17] suggest that when $|\mu| > 2J$, the Kitaev chain may also exhibit topological states if a local periodic field is applied ($f \neq 0$). In this case, Majorana modes can exist for quasienergies $\varepsilon = 0$ and $\varepsilon = \Omega/2$. We focus on the stability of the $\varepsilon = 0$ Majorana mode against disorder present in the system, and we assume similar behavior for the $\varepsilon = \Omega/2$ mode.

Numerically, we find that strong disorder destroys the induced topological Floquet phase by closing the spectral gap. Figure 2(c) demonstrates this transition for the average topological charge $N = -\langle \mathrm{sgn}[\mathrm{Pf}(i\tilde{H}_F)]\rangle_{\mathrm{dis}}$, where $\tilde{H}_F$ is the Floquet Hamiltonian in the Majorana representation (it is purely imaginary), and Pf is the Pfaffian. If $N = 1$, the system is in a topological phase, while in the disordered trivial phase $N = 0$. Vanishing of the gap corresponds to the transition from $N = 1$ to $N = 0$. Figure 3(a) shows the closing of the gap at the critical disorder strength. The analytical predictions of the effective theory [Eq. (4)] with $\lambda = W$ are also presented in Fig. 3(a). The analytical calculation of the gap (white solid curve) and zero-energy mode (white dashed line) show good agreement with exact diagonalization.

Similar results can be obtained in 2D by choosing a square lattice with $\Gamma_x = -i(A/2)\sigma_x + B\sigma_z$, $\Gamma_y = -i(A/2)\sigma_y + B\sigma_z$, and $M_{\mathbf{r}} = h_{\mathbf{r}} + [\Delta - 4B + f\mathrm{sgn}(\sin\Omega t)]\sigma_z$. Here A is a velocity parameter, B defines the inverse kinetic mass, Δ is the bulk band gap, and $h_{\mathbf{r}}$ is the static disorder. The long-wavelength limit of this model coincides with the seminal BHZ theory [44] $H_0 = \sum_{i=1}^{3} d_i(\mathbf{k})\sigma_i$, where $d(\mathbf{k}) = [Ak_x, Ak_y, \Delta - B(k_x^2 + k_y^2)]$. For $\Delta/B > 0$, the disordered system is characterized by the Bott index [45,46] $C = 1$ and hosts topologically protected states at the edge. Similar to the Kitaev chain, the trivial phase $\Delta/B < 0$ can be converted into a topological phase by applying a periodic driving field $f \neq 0$. The effect of disorder is shown in Figs. 2(d) and 3(c). In Fig. 3(c), the white solid curve and dashed white curves are the gap and edge states, respectively; both are analytically computed. The discreteness of edge states is due to the finite size.

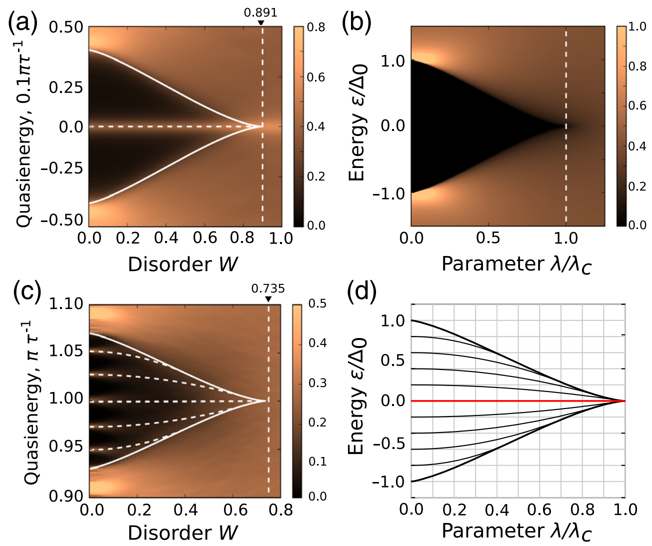

FIG. 3. Density of states (DOS). (a) DOS of a Floquet 1D Kitaev chain of size $L = 10^2$ for $\Delta = 1, J = 1$ $\mu = 4.5$, $f = 1.5$, and $\tau = 2\pi/\Omega = 1.1$ (level width applied $\gamma = 10^{-2}\tau^{-1}$). The white solid curve and the dashed white line are the analytical gap prediction [Eq. (11)] with $\lambda = W$ and the Majorana state, respectively. (b) DOS of a Floquet BHZ model on a square lattice 20×20 with mixed periodic (x-direction) and open (y-direction) boundary conditions near quasienergy $\varepsilon = \Omega/2$. (Level widening applied is $\gamma = 0.2 \times 10^{-2}\tau^{-1}$.) The white curve is the analytical gap prediction given by Eq. (11) with $\lambda = 0.9W$, and the dashed curves are the analytical prediction for behavior of the midgap states given by Eq. (12). (c) Analytic calculations of bulk DOS for the model Eq. (4), as described below Eq. (9). (d) Analytic result for the midgap states given by Eq. (12).

The efficacy of $H_F^{\rm eff}$ [Eq. (4)] in capturing the exact H_F' [Eq. (2)] is easily demonstrated in the models we studied by examining the matrix $\delta V_F = H_F - H_F|_{W=0}$. Remarkably, δV_F turns out to be a nonlocal matrix with level statistics close to the Wigner-Dyson law [Figs. 2(a) and 2(b)]. Interestingly, the renormalized disorder δV_F approximately follows the GOE and GUE statistics for the 1D Kitaev chain and 2D BHZ Hamiltonian, respectively. Whether GOE or GUE level statistics is dictated by the dimension of the lattice is a question we leave for future work.

We proceed with our main goal of analytically solving spectral properties of $H_F^{\rm eff}$ [Eq. (4)]. The main tool enabling this is free probability theory [47–50], which we now introduce (see the Supplemental Material [39] and Ref. [50] for an applied overview). Free probability theory (FPT) extends the conventional probability theory to the non-commuting random variables setting. Recall the φ notation $\varphi(A) = (\mathbb{E}{\rm Tr}A)/\dim(A)$ and $\overline{A^k} = A^k - \varphi(A^k)$. Two random matrices A and B are freely independent (or free) if all expectation values of cross-term correlators vanish in the infinite size limit. That is, $\varphi(\overline{A^{k_1}}\,\overline{B^{l_1}}\ldots\overline{A^{k_n}}\,\overline{B^{l_n}}) = 0$ for any integers k_i, $l_i \neq 0$ (see Refs. [50,51] for a comprehensive definition). The free independence is immediate if either A or B is chosen independently from the Gaussian ensemble. Therefore, in Eq. (4), H_F and λM are free.

The input to the theory is the Cauchy transform of the DOS of the summands $G_A(z) = \varphi[(z - A)^{-1}]$ and $G_B(z) = \varphi[(z - B)^{-1}]$. The integral representation of DOS, $\rho_A(\varepsilon)$, of matrix A (and similarly for B) is

$$G_A(z) = \frac{1}{2\pi i}\int_{\mathbb{R}} d\varepsilon \frac{\rho_A(\varepsilon)}{z - \varepsilon}. \tag{7}$$

The R transform is defined by $R_A(w) = G_A^{-1}(w) - w^{-1}$, where G_A^{-1} is the functional inverse (similarly for B). Recall that in standard probability theory, the additive quantity for sums of scalar random variables is the log characteristics. In FPT, the analogous additive quantity is the R transform, which in turn defines the Cauchy transform of the sum $G_{A+B}(\varepsilon)$. One then obtains the DOS from $G_{A+B}(\varepsilon)$, with the caveat that the technical challenge often is the inversion of $G_{A+B}(\varepsilon)$ to obtain the density. Below, H_F and λM replace A and B, respectively (see the Supplemental Material for technical details of what follows [39]).

The R transform of $H_F^{\rm eff}$ in Eq. (4) is $R_{H_F^{\rm eff}}(w) \equiv R_{H_F}(w) + R_{\lambda M}(w)$. This is equivalent to (see the Supplemental Material [39])

$$G^{-1}_{H_{\rm eff}}(w) = G^{-1}_{H_F}(w) + G^{-1}_{\lambda M}(w) - \frac{1}{w}. \tag{8}$$

At energies not much larger than the Floquet band gap Δ_0, the bulk DOS of the topological Hamiltonian H_F is approximated by $\rho_{H_F}(\varepsilon) \approx \rho_0 \varepsilon/\sqrt{\varepsilon^2 - \Delta_0^2}$, where ρ_0 is the DOS in the vicinity of the gap. The DOS of λM is the well-known semicircle law $\rho_{\lambda M}(\varepsilon) = (2\pi\lambda^2)^{-1}\sqrt{4\lambda^2 - \varepsilon^2}$. The Cauchy transform $w \equiv G_{H_{\rm eff}}(\varepsilon)$ can be derived from the condition Eq. (8), which is equivalent to

$$\varepsilon = \lambda^2 w + \frac{w^2\Delta_0^2}{\sqrt{\pi^2\rho_0^2 - w^2}}. \tag{9}$$

The DOS is then obtained from the imaginary part of the Cauchy transform, $\rho(\varepsilon) = \pi^{-1}{\rm Im}w$. Energies ε for which w is real in Eq. (9) correspond to zero density of states—i.e., the band gap. The real solutions of w occur for $\lambda < \lambda_c$, with

$$\lambda_c = \sqrt{\Delta_0/\pi\rho_0}. \tag{10}$$

λ_c defines the critical point for the phase transition, where two bands merge and the gap vanishes [Fig. 3(b)]. Let $\Delta(\lambda)$ be the band gap as a function of the effective disorder strength λ. For $|\varepsilon| < \Delta$, one has

$$\Delta(\lambda) = \Delta_0[1 - (\pi\rho_0\lambda^2/\Delta_0)^{2/3}]^{3/2}, \qquad \lambda < \lambda_c, \tag{11}$$

and $\Delta(\lambda) = 0$ for $\lambda \geq \lambda_c$.

We turn our attention to the behavior of the surface states energies ε_μ situated in the bulk band gap, where μ can be either a discrete or a continuous quantum number. To evaluate $\varepsilon_\mu(\lambda)$, one can use the fact that the number of surface states is small compared to bulk ones. This allows us to derive the spectrum, considering them as small corrections to the Cauchy transform [Eq. (7)].

In the Supplemental Material [39], we show that the resulting spectrum of midgap states satisfies $G_{H_F}(\varepsilon_\mu) = G_{H_{\text{eff}}}[\varepsilon_\mu(\lambda)]$, which leads to

$$\varepsilon_\mu(\lambda) = \varepsilon_\mu\left(1 - \frac{\pi\rho_0\lambda^2}{\sqrt{\Delta_0^2 - \varepsilon_\mu^2}}\right), \qquad \lambda < \lambda_c^\mu, \tag{12}$$

where λ_c^μ denotes smallest positive of $\varepsilon_\mu(\lambda_c^\mu) = \Delta(\lambda_c^\mu)$. The plots for $\varepsilon_\mu(\lambda)$ for different initial values ε_μ are shown in Fig. 3(d). As seen there, the continuous spectrum of surface states never opens up a spectral gap.

Remark.—The theory is universal, in that the details of the underlying model, such as the dimension of the lattice, the period τ, or the DOS of the clean system, only enter through ρ_0 and Δ_0.

To summarize, we demonstrated that the disorder effects on finite-frequency Floquet phases can be well approximated by generic random matrices [Eq. (4)]. Using this and free probability theory, we analytically show that the topological phases in this regime are generically stable against disorder for a range of strength. The breakdown into the trivial phase typically happens at a critical disorder strength that is potentially many times larger than the spectral gap. The proposed theory allows us to compute the critical gap behavior and the corresponding critical exponents. The analytical prediction of the critical point can serve as a guide in experiments to search for topological phases in the presence of disorder more systematically and irrespective of the underlying model.

The utility of free probability theory for approximating spectral properties of physical systems extends beyond this Letter. On the one hand, it works in the more general settings in which perturbative analysis fails (e.g., in the current study, the regime of strong disorder and/or moderate frequency of driving). On the other hand, free convolution is an entirely new technique that can be added to the arsenal of the existing tools. We emphasize that the success of free probability theory does not rely on the disorder being generic (cf. Refs. [48,49]).

Future research may include applying our techniques to time crystals [52] and other disordered systems, especially with many-body interactions—for example, the treatment of the self-energy in self-consistent Born approximations [53]. We anticipate these methods to provide a new angle of attack on problems of disordered superconductivity and many-body localization.

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